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A

PRACTICAL ARITHMETIC.

BY

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PREFACE.

THE object of a text-book on Arithmetic should be to teach the pupil to cipher, — to learn by doing. The shortest and surest road to a knowledge of Arithmetic is by solving problems, not by memorizing rules or by demonstrating propositions. The pupil should be trained to obtain results rapidly and correctly. He should be taught, in questions involving decimal fractions, to limit the answers to the number of decimals required by the nature of the examples, and to avoid all superfluous work. He should not be expected to discover the reason of a process until he fully understands the process; then he should be allowed to state the reason in his own language.

This Arithmetic is not intended for beginners; but it is presumed that pupils will have a thorough knowledge of our "Lessons in Number," and be at least twelve years of age, before entering upon the study of this book.

Decimal fractions are introduced at the beginning of the book. Experience proves that when thus taught they present no difficulty. The difficulty of decimal fractions arises solely from comparing them with common fractions, and is avoided by teaching decimals first. The pupil learns the notation on both sides of the decimal point as easily as on one side; provided the notation on both sides is presented at the same time. Much time is saved by strict adherence to the motto, "Decimal fractions as soon as possible, thoroughly mastered; common fractions postponed as long as possible."

The Metric System in a few years will be in common use, and will supersede other systems, as dollars and cents have superseded pounds, shillings, and pence. Taught immediately after decimal fractions, the system is easily learned. A great number of examples is given to show the simplicity of the system in its application to questions of common occurrence, and to furnish additional practice in operations with decimal fractions. The abbreviations used are such as have been adopted throughout Germany.

Many of the problems are original, but some have been obtained from French, English, and German sources. Though the problems are very numerous, it has been found, by actual trial, that a class of pupils fourteen to fifteen years old can accomplish the whole work of this Arithmetic, with one recitation a day, in a school year. The examples are intended to convey, incidentally, a great deal of accurate and valuable information; so that, by means of the index, the book becomes a book of reference for many physical and mathematical constants.

The introduction of logarithms will be welcomed by all who know the ease of learning the practical use of a four-place table, and the increased power given by it over mathematical questions. Teachers who have never taught or learned logarithms are assured that they will find no difficulty in the subject as here presented.

The method of "Supposition," called in old Arithmetics "Position," has been restored to its rightful place, and is fully explained in the chapter on Approximations. This method is applicable to a large variety of problems, and is made very simple by logarithms.

We gladly acknowledge our obligations to many friends who have improved this work by their advice; and we also give assurance that any suggestions for its further improvement will be thankfully received.

THOMAS HILL.
G. A. WENTWORTH.

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VOCABULARY.

Abstract number. This phrase is employed to designate numbers used without reference to any particular unit, as 8, 10, 21. But *all numbers are in themselves abstract whether the kind of thing numbered is or is not mentioned.*

Addition. The process of combining two or more numbers so as to form a single number.

Aliquot part. A number which is contained an integral number of times in a given number. Thus, 5, $6\frac{1}{4}$, $12\frac{1}{2}$, $16\frac{2}{3}$, are aliquot parts of 100.

Amount. The sum of two or more numbers. In Interest, the sum of principal and interest.

Analysis. The separation of a question into parts, to be examined each by itself.

Annuity. A sum of money that is payable yearly or in parts at fixed periods in the year.

Antecedent. The first of the two terms named in a ratio.

Area of a surface. The ratio of the surface to another surface assumed as the unit of measure; usually the square of the linear unit.

Arithmetic. The science that treats of numbers and the methods of using them.

Assets. All the property belonging to an estate, individual, or corporation.

Average. The mean of several unequal numbers, so that, if substituted for each, the aggregate would be the same.

Bank. An establishment for the custody, loaning, and exchange of money; and often for the issue of money.

Bank discount. An allowance received by a bank for the loan of money, paid at the time of lending as interest on the sum lent.

Bonds. Written contracts under seal to pay specified sums of money at specified times, issued by national governments, states, cities, and other corporations.

Cancellation. The striking out of a common factor from the dividend and divisor.

Combination. An arrangement of different things without reference to their order of sequence.

Commission. Compensation for the transaction of business, generally reckoned at some per cent of the money employed in the transaction.

Common denominator. A denominator common to two or more fractions.

Common factor. A factor common to two or more numbers.

Common multiple. A multiple common to two or more numbers.

Complex fraction. A fraction that has a fraction in one or both of its terms.

Composite number. The product of two or more integral factors, each factor being greater than unity.

Compound denominations. Several denominations used to express parts of one quantity.

Compound interest. When the interest due is left unpaid, and considered as an increase made to the principal, the whole interest, accruing in any time, is called compound interest.

Compound fraction. A fraction of another fraction.

Concrete number. A phrase without meaning. Things numbered are concrete, but the number is abstract.

Consequent. The second of the two terms named in a ratio.

Consignee. The person or firm to whom goods are sent.

Consignor. The person or firm who sends goods to another.

Corporation. An association of individuals authorized by law to transact business as a single person.

Couplet. The two terms of a ratio taken together.

Coupon. A certificate of interest attached to a bond, to be cut off when due and presented for payment.

Creditor. A person or firm to whom money is due.

Cube root. One of the three equal factors of a number.

Customs. Duties or taxes imposed by law on merchandise imported, and sometimes on merchandise exported.

Debtor. A person who owes money to another.

Decimal. A tenth. In the ordinary notation, a figure in combination with others has only the tenth part of the value it would have if removed one place towards the left. Thus, in 476, the 4 means 4 times 100; the 7, 7 times 10; the 6, simply 6.

Decimal fractions. Fractions of which only the numerators are written, and the denominators are ten or some power of ten.

Decimal point. A dot placed after the *units'* figure to mark its place.

Decimal system. The common system of numbers founded on their relations to *ten*, *ten tens*, etc.

Denominator. The number which shows into how many equal parts a unit is divided.

Difference. The number which, added to a given number, makes a sum equal to another given number.

Discount. Allowance made for the payment of money before it becomes due. Also, the amount which the market value is *below* the face or nominal value.

Dividend. In division, the given number which is equal to the product of a given factor (called divisor) and required factor (called quotient). In business, the share of profits which belongs to each owner of stock, according to his proportion of the whole capital.

Division. The operation by which, when a product and one of its factors are given, the other factor is found.

Divisor. The number by which a given dividend is to be divided.

Draft. A written order directing one person to pay a specified sum of money to another.

Drawee of a draft. The person to whose order the sum of money named in a draft is to be paid.

Drawer of a draft. The person who signs the draft.

Duty. A sum of money required by government to be paid on the importation, exportation, or consumption of goods.

Equation. A statement that two expressions of numbers are equal.

Equation of payments. The finding of an average time at which several payments may be justly made.

Exchange. A system of paying debts, due to persons living at a distance, by transmitting drafts instead of money.

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and last terms of a proportion or of a series.

Evolution. The process of finding the root of a number.

Factors. The factors of a number are a set of numbers whose product is the given number; they are assumed to be integral except in the extraction of roots. In commerce, agents employed by merchants to transact business.

Figures. Symbols used to represent numbers in the common system of notation. Also diagrams used to represent geometrical forms.

Firm. The name or title under which a company transact business.

Fractions. One or more of the equal parts into which the unit is divided.

Geometrical series. A series in which each term is obtained from the one preceding it by multiplying the preceding term by a constant factor.

Grace. An allowance of three days, after the date a note becomes due, within which to pay the note.

Gram. The unit of weight in the metric system, equal to 15.43235 troy grains.

Greatest common measure. The greatest number which is a common factor of two or more given numbers.

Improper fraction. A fraction whose numerator equals or exceeds the denominator.

Index. A figure written at the left and above the radical sign to show what root of the number under the radical sign is required. A fraction written at the right of a number, of which the numerator shows the required power of that number and the denominator the required root of that power.

Instalment. A payment in part.

Insurance. A guarantee of a specified sum of money in the event of loss of property by fire, storm at sea, or other disaster; or of loss of life.

Integral number. A number which denotes whole things.

Interest. The sum paid for the use of money.

Involution. The process of finding a power of a number.

Latitude of a point. The angle made by the vertical line at that point with the plane of the equator.

Least common multiple. The least number which is a common multiple of several given numbers.

Liability. A debt, or obligation to pay.

Line. Length without breadth or thickness. The path of a moving point.

Liter. The unit of capacity in the metric system equal in volume to a cube each edge of which is one-tenth of a meter; it is equivalent to 1.05671 liquid quarts.

Logarithm of a number. The exponent of the power to which 10 must be raised in order to obtain the number.

Long division. The method of dividing in which the processes are written in full.

Longitude of a point. The angle between two planes which are supposed to pass through the centre of the earth and contain, the one the meridian of that point, and the other the standard meridian.

Loss. In business, the excess of the cost price above the selling price or net proceeds of sale.

Maturity of a note. The date at which a note legally becomes due.

Mean proportional. A number which is both the second and third terms of a proportion. The square of a mean proportional is equal to the product of the extremes.

Means. The terms of a proportion or of a series intervening between the extremes.

Meter. The unit of length in the metric system, equal to 39.37043 inches.

Minuend. The given number in subtraction which is equal to the sum of another given number called the subtrahend, and a required number called the difference or remainder.

Mixed number. A number that expresses both entire things and parts of things taken together.

Multiple of a number. The product obtained by taking the given number an integral number of times.

Multiplicand. The number to be multiplied by another.

Multiplication. The operation of finding a number bearing the same ratio to the multiplicand which the multiplier bears to unity.

Multiplier. The number by which the multiplicand is multiplied.

Net proceeds. The amount that remains of the money received for property after paying all expenses incurred in disposing of it.

Notation. A system of expressing numbers by symbols.

Note. A written agreement to pay a specified sum of money at a specified time.

Number. The answer to the question, How many?

Numeration. A system of naming numbers.

Obligation. A debt, or liability to pay.

Order of number. A name used to designate the number of things in a group, as *tens*, *hundreds*, *thousands*, etc.

Oncometrics. The measurement of volumes.

Partial payment. Part payment on a note.

Partnership. An association of two or more persons to carry on business.

Par value. Face or nominal value.

Pendulum. A body suspended by a straight line from a fixed point, and moving freely about that point as a centre.

Percentage. A part of any given number reckoned at some rate per cent.

Period. A group of three figures.

Permutation. The changing of the order of sequence in a given number of things.

Planet. A celestial body revolving about the sun.

Policy. The written contract of insurance.

Poll tax. A tax levied by the head or poll.

Power. The product of two or more equal factors.

Premium. The sum paid for insurance computed at some rate per cent of the amount insured. Also the excess of market value above par value.

Present worth. The present value of a debt due at some future day.

Prime number. A number which has no integral factors except itself and one.

Principal. The sum of money drawing interest.

Problem. A question to be solved.

Product. The result obtained by multiplying the multiplicand by the multiplier.

Profit. The excess of selling price or net proceeds above cost.

Progression. A series.

Proof. The evidence by which the accuracy of any result is established.

Proper fraction. A fraction the numerator of which is less than the denominator.

Proportion. A statement that two ratios are equal.

Quantity. The answer to the question, How much?

Quotient. The number sought in division.

Rate per cent. Rate by the hundred.

Ratio. The *relative magnitude* of two numbers or of two quantities compared.

Reciprocal of a number. *One* divided by that number.

Reduction. The process of changing the *unit* in which a quantity is expressed without changing the *value* of the quantity.

Remainder. The number which, added to the subtrahend, gives a sum equal to the minuend.

Roman notation. The system of expressing numbers adopted by the Romans, which employs seven letters of the alphabet.

Root of a number. One of the equal factors of the number.

Rule. The statement of a prescribed method.

Security. Property used to guarantee the payment of any obligation.

Series. A set of terms any one of which can be derived from one or more of the terms that precede, according to some law.

Share. One of a certain number of equal parts into which the capital of a company is divided.

Short division. The method of dividing in which the operations of multiplying and subtracting are performed mentally.

Solid. A magnitude which has length, breadth, and thickness.

Solution. The process by which the answer to a question is obtained.

Specific gravity of a substance. The ratio of the weight of a given volume of it to that of an equal volume of water.

Square root. One of two equal factors.

Stock. Capital invested in business.

Subtraction. The process of finding a number which added to one of two given numbers will produce the other.

Sum. The number which results from combining two or more numbers together.

Surd. An indicated root the value of which cannot be exactly expressed in figures.

Surface. That which has only length and breadth.

Thermometer. An instrument for measuring heat.

Unit. A single thing. Also, an arbitrary length, adopted as a standard of measure, in terms of which all measurements are expressed.

Verify. To establish, by experiment, the truth of any statement.

Volume of a solid. The ratio of the solid to an assumed unit of measure; usually a cube of the linear unit.

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ARITHMETIC.

CHAPTER I.

NUMBERS.

1. A FUNDAMENTAL IDEA, like that of number, cannot be defined. A number is the simple, direct answer to the question, How many?

2. A system of naming numbers is called **numeration,** and a system of representing numbers by symbols is called **notation.**

The first twelve numbers, formed by adding one thing at a time, are named:

One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve.

These numbers are represented by Arabic figures, or digits, as follows:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

3. The digit 0, called naught, zero, or cipher, is used as a symbol of the words "none" or "nothing." A digit written in the **second** place to the left of another digit signifies **tens.** Thus, 10 is one ten and nothing; 11 is one ten and one; 12 is one ten and two; 23 is two tens and three; etc., etc.

4. The next seven numbers, 13, 14, 15, 16, 17, 18, 19, are the teens: thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

5. Two tens are called **twenty**; **three tens**, **thirty**; **four**, **forty**; **five**, **fifty**; **six**, **sixty**; **seven**, **seventy**; **eight**, **eighty**; **nine**, **ninety**.

Thus, 57 is fifty-seven; 63, sixty-three; 89, eighty-nine; etc.

6. Read the numbers:

14, 13, 11, 12, 19, 18, 17, 16, 15, 10, 20, 22, 21, 27, 26, 25, 28, 23, 24;

30, 32, 31, 36, 38, 34, 35, 37, 33, 29, 44, 55, 66, 77, 43, 52, 61, 42, 41, 45, 49, 57, 47, 60, 56;

67, 76, 53, 39, 58, 75, 77, 83, 87, 92, 91, 96, 87, 99, 98, 85, 74, 84, 48, 46, 44, 64, 65, 63.

7. Ten tens are one **hundred**; and hundreds are written in the **third** place toward the left; thus:

100, one hundred; 700, seven hundred; 823, eight hundred twenty-three; 941, nine hundred forty-one; 607, six hundred seven.

8. Read the numbers:

121, 222, 78, 780, 778, 708, 827, 741, 531, 712, 231, 365, 474, 439, 143;

534, 587, 932, 239, 392, 617, 761, 176, 873, 246, 789, 879, 987, 978, 934, 543, 897;

707, 680, 704, 518, 405, 903, 319, 309, 104, 870, 801, 807, 708, 610, 909, 120, 201.

9. Write in figures:

Seventeen, seventy, nineteen, ninety-three, sixteen, sixty-seven, fourteen, forty-five, fifty-nine;

One hundred three, seven hundred nineteen, three hundred three, six hundred eight, six hundred eighteen, six hundred eighty;

Nine hundred ten, five hundred six, eight hundred eighty-eight, four hundred twenty-seven, seven hundred sixty-five, two hundred eleven.

10. Ten hundred make one **thousand**, and thousands are written in the **fourth** place to the left.

Tens of thousands are written in the **fifth** place, **hundreds of thousands** in the **sixth** place.

A comma is usually placed among the figures at the point where, in reading, the word thousand comes.

Thus, 621,453 is 621 thousand, 453; and 700,006 is 7 hundred thousand, six.

11. Read the numbers:

1000; 7000; 7842; 1067; 5043; 8375; 18,757; 75,764; 234,567; 34,561; 123,456;

654,089; 600,897; 607,804; 2753; 704,608; 900,008; 806,042; 709,053; 350,709; 240,682; 118; 537; 3094;

5000; 50,000; 500,005; 682,000; 246,001; 753,110; 703,101; 930; 2020; 200,200; 708,056; 70,856; 870,890.

12. Write in figures:

Eight hundred thousand eight hundred eight; six hundred thousand six; seven hundred thirteen thousand three hundred nine.

Five hundred seventeen thousand six hundred thirteen; seven thousand eight hundred fifty-four; seven hundred eighteen thousand eight hundred eighty-nine.

13. A **thousand thousands** make a **million**; and millions up to 999 millions **are written in** the **seventh, eighth,** and **ninth** places.

Thus, 728,651,423 is 728 million, 651 thousand, 423.

14. Read the numbers:

21,978,564; 17,756,423; 300,200,100; 707,303,202; 312,406,780; 908,765,487; 2,632,561.

15. Digits are thus divided by commas into **periods** of three places each. Each period contains its own units' place, tens' place, and hundreds' place; but the **unit** in the **first** period, on the right, is **one**; in the **second** period, a **thousand**; in the **third** period, a **million**; and so on.

The **first** period is called the **units'** period; the **second** is the **thousands'** period; the **third** is the **millions'** period. A **fourth** period is the **billions'** period; a billion being a thousand millions; and the **fifth** period the **trillions'** period. The next periods, taken in order, are quadrillions, quintillions, sextillions, septillions, octillions, and so on.

16. We may read the number 1736 either as 1 thousand 736, or as 17 hundred 36. In like manner, we may read a larger number in a still greater variety of ways; thus, 2,736,000,000, may be read, 2 billions, 736 millions; or, 2 thousand 736 millions; or, 27 hundred 36 millions. To put this into the form of a rule:

When numbers are expressed in words, the name of a smaller number following a larger signifies that the two

numbers are to be added; but the name of a smaller number coming before a larger, or before an equal number, signifies that the second is to be multiplied by the first.

17. Read the numbers:

764,123,897; 40080; 795,013; 103,547,020; 71,003,054; 3,125,476,890; 79,501,346,081; 3,001,574.

18. Write in order the numbers from 1 to 25; from 95 to 115; from 195 to 215; from 985 to 1011; from 9995 to 11111; from 99999 to 100,011.

19. Write in figures, then read from the figures:

One hundred ten; one hundred one; two hundred seven; three hundred seventy; six hundred forty-one; seven hundred thirty-two; eight hundred eight; eight hundred eighty; nine hundred sixty-four; nine hundred ninety-three; one hundred seven; one thousand seven; one thousand ten.

Six million one thousand one; three million five; five million six; four million three hundred thousand three hundred three; seven million six hundred thousand eight hundred twenty-nine; eighty-one thousand ninety-five; seventeen hundred thousand millions; twenty-one thousand thousands; eighty-three million millions.

One hundred ten million two hundred seventy-nine; nineteen trillion four million three hundred three; one quadrillion one hundred twelve trillion three hundred thirty-four million two hundred eleven; ten quintillion two trillion three hundred billion five thousand seven.

CHAPTER II.

DECIMAL FRACTIONS.

20. Those things of which we do not naturally ask, How many? but, How much? we endeavor to measure; and we answer the question, How much? by answering, How many measures?

21. The measure of any kind of quantity is readily conceived as divided into ten smaller measures. A dollar, for example, as a measure of value, is divided into ten dimes; each dime into ten cents; each cent into ten mills.

22. Dollars are signified by the mark $ written at the left of the figures.

When part of a dollar is to be written, a full point is written after the dollars, the dimes are written to the right, then the cents and mills.

For example: $17 is seventeen dollars; $18.20 is eighteen dollars, two dimes, or eighteen dollars, twenty cents; $35.875 is 35 dollars, 87 cents, 5 mills; $0.08 is 8 cents.

23. Read as dollars, cents, and mills: $76.375; $163.58; $241.185; $357.34; $12.50; $0.875; $0.125; $1.01; $10.10.

24. A dime is the tenth of a dollar, a cent the hundredth of a dollar, a mill the thousandth of a dollar.

Parts of other measures than those of value may be written in the same way; with tenths, hundredths, etc., to the right of a point. Thus, if we omit the mark $ from $ 5.375, it may stand for 5 quarts, yards, bushels, or any other full measures, and 375 thousandths of another measure.

25. Parts thus written are called *Decimal Fractions.* We write and number to the right of the units' place, precisely as we do to the left, first carefully marking the units' place with a decimal point to its right. Thus, in the figures

9,876,543,210.123,456,789

the full point after 0 shows that 0 stands in the units' place. The 1 to the left is 1 **ten,** the one to the right is 1 **tenth;** the 2 to the left is 2 **hundreds,** the 2 to the right is 2 **hundredths;** the 3 to the left is 3 **thousands,** the 3 to the right is 3 **thousandths;** the 4 to the left is 4 **ten-thousands,** the 4 to the right is 4 **ten-thousandths;** the 5 to the left is 5 **hundred-thousands,** the 5 to the right is 5 **hundred-thousandths;** the 6 to the left is 6 **millions,** the 6 to the right is 6 **millionths;** and so on.

In like manner, the 210 is the units' period; the 543 is the **thousands',** the 123 the **thousandths'** period, etc.; so that the number may be read 9 billions, 876 millions, 543 thousands, 210, and 123 thousandths, 456 millionths, 789 billionths.

26. In reading decimal fractions in this manner, we are obliged, if the right-hand period contains less than three places, to fill the missing places mentally with naughts. For example, .0004 would be read 400 millionths. This is sometimes objectionable (for reasons which will hereafter be plain to the student), and **therefore the more usual way of reading is:**

Read the decimal precisely as if a whole number, and add the fractional name of the lowest place.

For example, 5.17 is read 5 and 17 hundredths; 5.0017, five and 17 ten-thousandths; 6.0203107, six and 203 thousand 107 ten-millionths.

In either of these ways of reading decimals, the word "and" is distinctly pronounced at the decimal point and carefully omitted in all other places. Thus, one hundred forty-seven means 147; but one hundred *and* forty-seven thousandths means 100.047; and .147 must be read one hundred forty-seven thousandths.

Another ambiguity in this way of reading can be avoided only by a pause; thus, .300 is three hundred . . . thousandths, while .00003 is three . . . hundred-thousandths.

27. To avoid these ambiguities, practical computers introduce the word "decimal" at the place of the point, and then pronounce the digits in succession to the right. Thus, 203.07051 is read two hundred three, decimal, naught, seven, naught, five, one.

28. Read the mixed numbers:

17.23; 18.41; 27.49; 341.07; 1.52; 0.52; .52; .1357; 201.106; 11.111; 13.013; 17,000.017; 6132.0173; .0609; .00613; 26.7; 2.67; .267; .00267; 195.123; 0.83; 0.0087; .00091; 3.1416; 3.14159; 3.14159265.

29. The necessity for putting the decimal point in its right place may be seen on comparing $312.50, $31.25, $3.125, and $0.3125. *To move the point one place is to multiply or divide by ten.*

30. Write in figures, then read from the figures:

75 hundredths; 8 thousandths; 7 tenths; sixty hundredths; 77 thousandths; 83 ten-thousandths; eight and three ten-

thousandths; six, decimal, naught, naught, one, naught, three; five, decimal, six, naught, seven, three, one;

Nine and 43 millionths; 143 millionths; one hundred and 43 millionths; one hundred forty and three millionths; nine hundred forty-three thousand and nine hundred forty-three thousandths; 722 ten millionths; thirteen, decimal, naught, one, four, six, eight.

31. Point 6753241 in eight different ways, and read each. Point 8957 in eight different ways, prefixing or annexing naughts, and read each.

32. Write in figures:

Three hundred seven; three hundred and seven thousandths; three hundred seven thousandths; three and one hundred seven thousandths; three hundred seven and fourteen thousandths.

33. Read as dollars, cents, and mills:

$24.073; $16.187; $35.625.

Read the same figures as whole numbers and decimal fractions without naming any unit.

34. Write in figures:

81 thousand and 345 thousandths; 37 hundred 41 and 675 thousandths; 4 hundred 13 and 8 hundredths; 96 and 96 thousandths.

Read them as dollars, cents, and mills.

35. Write as money, then read as whole numbers and fractions without naming any unit:

Five and a half dollars; thirteen dollars and twenty-five cents; 83 dollars and 14 cents; 60 dollars and 12 and a half cents; 6 dollars, 9 cents, and 9 tenths of a mill.

REVIEW I.

Numeration is a system of *naming* numbers.

The names of numbers from one to twelve are all different. The names of numbers between twelve and twenty are formed by adding ten to names of numbers less than ten; as, thirteen, fourteen, etc.

Tens are counted like simple units. Thus, two tens, three tens, called twenty, thirty, etc.

Ten tens are called a *hundred*, just as ten units are called a ten. That is, we regard a ten as a *single group*, and call it a *unit of the second order;* a hundred as a *group of ten tens*, and call it a *unit of the third order.*

The names of numbers between any two tens, as from twenty to thirty, are formed by adding to the name of the smaller number of tens the names of numbers less than ten; as, twenty-one, twenty-two, etc.

Hundreds are counted like tens; as, one hundred, two hundred, etc., up to ten hundreds, which is called a *thousand.*

The names of numbers between any two hundreds are formed by adding to the name of the smaller number of hundreds the names of the numbers less than a hundred; as, one hundred one, one hundred two, etc.

A thousand is regarded as a single group of ten hundreds, and is called a *unit of the fourth order.*

Thousands are counted precisely like hundreds, tens, and simple units, up to ten thousands.

The names of numbers between any two thousands are formed by adding to the name of the smaller number of thousands the names of numbers less than a thousand; as, one thousand one, one thousand two, etc.

Ten thousand is regarded as a *unit of the fifth order;* a hundred thousand as a *unit of the sixth order.*

Ten hundred thousand is called a *million*, and is regarded as a *unit of the seventh order;* and so on, always forming from *ten units of the same order one unit of the next higher order.*

It is necessary to observe that we say a *unit of thousands*, a *ten of thousands*, a *hundred of thousands*, just as we say a simple unit, a ten of units, a hundred of units. That is, we regard the thousands as a *second class* of units composed of *three orders*, just as the *first class* of units is composed of *three orders.*

Likewise we regard the million as a *third class* of units, containing the units of millions, the tens of millions, the hundreds of millions.

The following table presents the names and the succession of the units of the different orders and different classes:

FOURTH CLASS. BILLIONS.			THIRD CLASS. MILLIONS.			SECOND CLASS. THOUSANDS.			FIRST CLASS. SIMPLE UNITS.		
Twelfth Order. Hundreds of Billions.	Eleventh Order. Tens of Billions.	Tenth Order. Units of Billions.	Ninth Order. Hundreds of Millions.	Eighth Order. Tens of Millions.	Seventh Order. Units of Millions.	Sixth Order. Hundreds of Thousands.	Fifth Order. Tens of Thousands.	Fourth Order. Units of Thousands.	Third Order. Hundreds.	Second Order. Tens.	First Order. Simple Units.

That is, **the common system of numeration** is founded upon the following principle: *Ten units of any order are equal in value to one unit of the next higher order;* and *one unit of any order is equal in value to ten units of the next lower order.*

Decimal fractions are formed by dividing the unit into **ten equal parts,** called *tenths;* the tenth into ten equal

parts, called *hundredths;* **the hundredth into** ten equal parts, called *thousandths;* the thousandth into ten equal parts, called *ten-thousandths*, etc.

By comparing these different decimal fractions, we see that the formation of decimal fractions depends, like that of whole numbers, upon the following principle: *Ten units of any order are equal in value to one unit of the next higher order;* and *one unit of any order is equal in value to ten units of the next lower order.*

Notation is a system of *writing* numbers.

The common system of notation employs ten figures, or digits: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The first nine figures represent the first nine numbers; the last, which is called zero, or naught, is used to denote the *absence of units* of the order in which it stands. These ten figures express all numbers by the artifice of making the value of each figure increase *tenfold for every place that it is moved to the left.*

To write a number in figures, we write successively the number of units of each *order* from left to right, beginning at the highest order, taking care, if the number contain a decimal fraction, to put a full point at the right of the units' figure, and to supply by zero the units of any order that may be lacking. If the number contain no whole number, we put a *zero in the units' place*, and the decimal point to the right of the zero.

To read a number written in figures, we divide the number into periods of three figures each, from right to left. This done, we begin to read at the last period on the left, and read *as if the figures of that period stood alone*, adding the *name* of the period. Then the next period to the right is read, with the *name* of that period, and so on.

If the number contain a decimal fraction, we first read the whole number; then the decimal as a whole number, taking care to add the *fractional name of the lowest place.*

CHAPTER III.

ADDITION.

36. Addition means putting together. The Saint George's cross $+$ is read **plus**, and means that the numbers between which it stands are to be added together.

The result obtained by adding together two or more numbers is called their **sum**.

The sign of equality $=$ stands for the words "equals," or "equal."

Thus, $8 + 1 = 9$ is read, eight plus one equals nine.

37. Add 2 to each number from 0 to 9; add 3 to each number from 3 to 9.

Add 4 to each number from 4 to 9; add 5 to each number from 5 to 9.

Add 6 to each number from 6 to 9; 7 to 7, to 8, and to 9; 8 to 8, and to 9; 9 to 9.

Repeat these additions until thoroughly familiar with them.

38. Name, as fast as you can talk, the even numbers, 2, 4, 6, etc., up to 102.

Name the odd numbers, 1, 3, 5, etc., up to 101.

39. Name every third number, 0, 3, 6, etc., up to 102.

Name every third number, 1, 4, 7, etc., up to 103.

Name every third number, 2, 5, 8, etc., up to 101.

40. Name every alternate even number, 0, 4, 8, 12, etc., up to 100.

Name every alternate even number, 2, 6, 10, etc., up to 102.
Name every alternate odd number, 1, 5, 9, etc., up to 101.
Do the same, beginning 3, 7, 11, etc., and go to 103.

41. Name every fifth number under 100, beginning 5, 10, 15, etc.; beginning 1, 6, 11; beginning 2, 7, 12; beginning 3, 8, 13; and beginning 4, 9, 14.

42. In like manner, add by sixes up to a number exceeding a hundred, beginning 0, 6, 12; beginning 1, 7, 13; beginning 2, 8, 14; and so on.

43. Add by sevens up to a number exceeding 100, beginning 0, 7, 14; 1, 8, 15; and so on; by eights, beginning 0, 8, 16; 1, 9, 17; and so on; by nines, beginning 0, 9, 18; 1, 10, 19; and so on.*

44. For the addition of numbers in general, the following mode has been found most convenient.

Write the numbers in columns, units under units, tens under tens, tenths under tenths, etc.

Add the digits in the right-hand place; set the units of the sum in that place, but carry the tens mentally to the next place to the left, to be added to the digits there, and so proceed.

The study of three or four examples will make the process understood.

---

* The teacher may take ten small cards. On each side of the first write 0; of the second, 1; etc. Shuffle, and dictate the numbers on all but one for the class to add. Subtract the reserved number from 45; the remainder is the sum. With two sets of cards at once, subtract the reserved number from 90. For advanced classes, use cards with larger numbers; and complemental cards, which may be obtained of the publishers, furnishing unlimited examples, with the answers to all.

4128 3789 2667 5821 ——— 16405	The following wording, *and no more*, is to be used: 1, 8, 17, **25** (emphasize 5, and write it down while pronouncing it), carry 2; 4, 10, 18, **20**, carry 2; 10, 16, 23, **24**, carry 2; 7, 9, 12, **16**.

271.188	2.7113	314.
51.27	.51278	.0561
1003.684	10.03684	27.53
1326.142	13.26092	341.5861

45. Find the following sums:

231 + 764; 341 + 57.8; 430.31 + 58.61; 512.87 + 36.84 + 12.78 + 711.56 + 415.86.

46. Add 1543.1 to 164.7; to 1728; to 402.56; to 1897.3; to 475.34; to 6897.65.

47. Add 1897.3 to 475.34; to 6897.65; to 1728; to 402.56; to 164.7; to .5236; to 2.71828.

48. Find the following sums:

.7854 + 3.1416 + 2.71828; .7854 + 3.1416 + 30,103;
2.71828 + 402.56 + 1897.3; 2.7113 + 27.53 + 341.586.

49. Add 737.87 to each of the following numbers:

111; 1011; 2304; 222; 263; 373; 262.13; 561.2; 32.35; 604.3.

50. Find the five sums:

230.8 + 223 + 2.63 + 373.8 + 56.123; 32.358 + 821.9 + 23.04 + 73.7; 202.3031 + 71.575 + 65.813 + .0078 + 7.377; 653.03 + 65.303 + 6.5033; 939.303 + 65.746 + 8.2794 + 681.28.

51. In this article are given 24 decimal fractions, arbitrarily numbered, for convenience in referring to them.

No.	Decimal.	No.	Decimal.
1.	0.4771213	13.	1.6093295
2.	2.7182818	14.	0.6213768
3.	3.1415927	15.	3.785
4.	0.7853982	16.	0.264
5.	0.5235988	17.	0.8450980
6.	0.4342945	18.	15.4323487
7.	0.2908882	19.	1.4142136
8.	4.8104774	20.	1.7320508
9.	2.5399772	21.	2.2360680
10.	0.3937043	22.	0.3819660
11.	0.3047973	23.	0.6180340
12.	3.2808693	24.	0.3010300

52. Add together numbers 2, 3, 4, from the last article; numbers 6, 8, 9; numbers 12, 9, 8; numbers 13, 18, 15; numbers 11, 24, and 14; numbers 10, 22, and 6.

53. Add the numbers marked 24, 23, 22; 23, 21, 20; 22, 20, 19; 21, 19, 18; 18, 17, 16; 15, 14, 13; 14, 12, 11; 13, 12, 10; 11, 10, 9; 10, 12, 8; 11, 6, 5; 2, 3, 5.

54. Add the numbers marked 2, 3, 4, 5, 6; 3, 5, 8, 9, 10; 4, 6, 8, 10, 11; 10, 12, 13, 6, 8; 13, 11, 10, 5, 6; 13, 14, 15, 16, 18; 14, 16, 18, 10, 12; 15, 16, 18, 19, 20; 18, 19, 21, 22, 23.

55. Add the numbers marked 1, 7, 13, 17, 22, 23, 24; 2, 3, 4, 6, 8, 9, 10; 24, 23, 22, 21, 20, 19, 18; 13, 14, 15, 16, 18, 19, 20; 10, 11, 12, 13, 14, 15, 16; 15, 18, 14, 19, 12, 11, 8.

56. A simple test of the correctness of an addition is to add a second time, beginning at the top instead of the bottom of the columns, or to add two columns at once.

It is of great advantage to educate the eye to take in at a glance digits enough to make 10 or more, and then these *sums* can be added instead of the separate digits.

To illustrate, take the example in the margin. Adding from the bottom, the computer says to himself (seeing $8+2=10$, and $9+3=12$), 10, 12, **22**; **8**; 8, 12, **20**; 11, 4, **15**; 11, 10, **21.**

$$\begin{array}{r} 61803 \\ 43429 \\ 47712 \\ 62138 \\ \hline 215082 \end{array}$$

In the two-column mode he says, 50, 32, **82**; and writes down the 82. Then he continues, 98, 52, **150**, and writes the 50 at the left of 82; then he proceeds, 11, 10, **21**, and writes the 21.

Many bookkeepers and merchants strongly recommend the addition of two columns at once, as the most expeditious and the least liable to error.

Many computers begin at the bottom of the right-hand column in adding, and write on a piece of waste-paper the full sum of each column or double column; then they begin at the top of the left-hand column, and add each column or double column, also writing the full sum; finally, they add the sums obtained in the first addition, and the sums obtained in the second addition, and compare the results. Thus, in the example above,

By Single Columns:

$$\begin{array}{r|l} 22 & 20 \\ 6 & \;13 \\ 20\; & \;\;20 \\ 13\;\; & \;\;\;\;6 \\ 20\;\;\; & \;\;\;\;22 \\ \hline 215082 & 215082 \end{array}$$

By Double Columns:

$$\begin{array}{r|r} 82 & 213\;\;\; \\ 150\;\; & 206\; \\ 20\;\;\;\;\; & 22 \\ \hline 215082 & 215082 \end{array}$$

57. Add the following by double columns, and test by adding with single columns:

45.68	154.31	73.86
73.91	296.85	453.71
78.54	736.48	137.64
534.69	345.19	98.87
134.70	782.34	643.48
581.43	78.43	462.71

498.50	65.42	621.65
17.37	638.34	167.32
684.29	763.43	856.96
231.56	809.31	718.83
210.10	798.83	501.49
671.54	835.78	315.72
643.53	356.47	768.44

791.52	32.54	763.89
504.83	254.63	78.23
879.26	63.27	345.61
243.97	131.56	26.73
732.86	506.72	489.56
47.95	283.54	812.35
856.43	845.83	607.28
497.65	643.46	219.07
541.26	708.91	68.72
616.72	463.73	216.78
857.94	67.74	436.74

CHAPTER IV.

SUBTRACTION.

58. SUBTRACTION means taking away. The sign $-$ is read **minus**, and means that the number before which it is placed is to be subtracted.

For example, $8-5=3$ is read, 8 minus 5 equals 3; $17-8=9$.

59. The number to be subtracted is called the **subtrahend**; the number from which it is taken, the **minuend**; the resulting number is called the **difference** or **remainder.**

60. Count 50 backward. Name the even numbers from 50 down to 0. Name the odd numbers from 51 down to 1.

61. Subtract by threes: beginning 60, 57; beginning 61, 58; beginning 59, 56.

62. Subtract 4 from every number between 8 and 14; between 38 and 44. Subtract 5 from every number between 5 and 15; between 85 and 95.

63. Subtract 6 from each number between 6 and 16; between 46 and 56. Subtract 7 from each number between 17 and 27.

64. Subtract 8 from each number between 18 and 28; 9 from each number between 59 and 69.

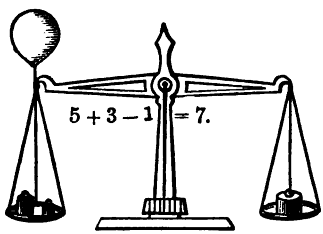
65. A number with the sign minus before it is called a **minus** number.

When the subtrahend is larger than the minuend, the remainder obtained by subtracting the minuend from the subtrahend is a minus number. Such results are common in Algebra, but are avoided in Arithmetic.

The meaning of a minus number is generally manifest; as in the example, The thermometer was at 17° and fell 25°, how high was it then? This would be written, $17° - 25° = -8°$, and shows that the mercury fell to 8° *below* zero.

66. An expression containing the sign of equality (=) is called an **equation.** It is like a balanced scale-beam: the plus numbers may be represented by weights, the minus numbers by balloons lifting up the beam of the scale.

The one rule in working with equations is, *Keep the balance true.* In other words, *Do to one side whatever you do to the other.*



67. When numbers are connected by the signs + or −, part of them are also sometimes joined by parentheses, brackets, or bars; as,

$$7 + (5 - 8) = 4; \quad 7 - (5 - 8) = 10; \quad 7 - (8 - 5) = 4.$$

$$7 + \overline{5 - 8} = 4; \quad 7 - \overline{5 - 8} = 10; \quad 7 - \overline{8 - 5} = 4.$$

The operations on the joined numbers must be performed first, and the result treated as a single number, plus or minus.

The sign outside the parenthesis must also be prefixed; and thus arise four cases of a double sign: $++$, $+-$, $-+$, and $--$.

The $++$ is equivalent to a single $+$; the $+-$ and the $-+$ are each equivalent to a single $-$; while the $--$ means the *taking away of a minus;* it is equivalent to the *removal of a balloon*, and this is the same as *adding the weight* which the balloon was lifting; that is, $--$ is equivalent to $+$.

68. The parenthesis, therefore, may be, according to the signs, useless or needful; it may or may not affect the result, and must be handled carefully. To illustrate:

$$8-3+5=(8-3)+5=10; \text{ but } 8-(3+5)=0.$$
$$12-(6-3)=12-3=9.$$
$$12-(3-6)=12-(-3)=15.$$

69. Write the second members to the following equations:

$8-3-2=$	$(8-3)-2=$	$8-(3-2)=$
$8-2-3=$	$(8-2)-3=$	$8-(2-3)=$
$18-(7-3)=$	$18-(3-7)=$	$18-3+7=$

70. The following questions will illustrate the meaning of minus numbers:

Starting 90 miles south of Chicago, I go 50 miles due north; and the next day, 80 miles still north. How far from Chicago am I now?

With only 67 dollars I undertake to pay three bills, of \$47, of \$13, and of \$11. Can I pay the bills? How much shall I lack?

71. *If we add the same number to a minuend and to a subtrahend, we do not alter the remainder.* For example, $27-15=12$; and if we add any number whatever, both

to 27 and to 15, the difference of the sums will be twelve; thus, adding 7 to 27 and 15, we have $34-22=12$; adding 2 to 27 and 15, we have $29-17=12$, etc.

To add 10 to any digit in a number is the same as adding 1 to the next left-hand digit. This will explain the last clause in the following rule for subtraction:

Write the subtrahend under the minuend, point under point, and each order under its corresponding order. If the number of places in the two be unequal, mentally consider the missing places filled with naughts.

Begin at the right, and subtract each figure of the subtrahend from the figure over it, setting the remainder below. But,

If the minuend figure be the smaller, add ten to it mentally; and, after subtracting, add 1 mentally to the next left-hand place in the subtrahend.

For example, in the margin we have $7-0=7$; $11-8=3$; $2-2=0$; $6-5=1$. The addition of 10 to the 1 hundredth in the minuend is exactly balanced by the subsequent addition of 1 to the 1 tenth in the subtrahend. In precisely the same manner,

$$\begin{array}{r} 6.217 \\ 5.18 \\ \hline 1.037 \end{array}$$

the following subtraction is performed, by saying, mentally, 1 from 10, 3 from 8, 7 from 11, 5 from 10, 2 from 10, 9 from 11, 1 from 7; each time setting down the remainder.*

$$\begin{array}{r} 71.0018 \\ 8.14721 \\ \hline 62.85459 \end{array}$$

72. It is plain, that *the remainder added to the subtrahend equals the minuend; and that the remainder subtracted from the minuend leaves the subtrahend.*

These are two tests of accuracy, or checks on subtraction.

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* This addition of ten to both minuend and subtrahend is, for practical purposes, far superior to the method of transposing values in the minuend, and is as readily explained.

73. Subtract 123 from each of the numbers:
234, 343, 424, 555, 676, 725, 839, 999, 1000, 10101, 5120.

74. Subtract 456 from each of the numbers:
789, 879, 978, 6378, 6855, 6853, 7797, 7006, 3542, 4334, 9790, 3455.

75. What is the difference between 779 and 974? 368 and 249? 479 and 2301? 2731 and 929? 708 and 394? 1123 and 1072? 891 and 773? 5621 and 8103? 19,001 and 3456? 792 and 2180?

76. Subtract:
$76.47 from $183.45; $628.74 from $716.43; $549.64 from $647.51; $128.31 from $270.04; $101.50 from $125; $129.47 from $247.93; $333.95 from $641.87; $29.89 from $56.27.

77. Subtract from 7854 each of the numbers:
788, 879, 567, 5006, 6107, 578, 867, 894, 463, 4603.

78. Subtract:
2.7182818 from 3.1415927; 0.5235988 from 0.7853982; 0.4342945 from 4.8104774; 0.3937043 from 2.5399772; 0.3047973 from 0.3937043; 0.3047973 from 3.2808693; 1.6093295 from 3.2808693; 0.6213768 from 3.785; 0.264 from 15.4323487; 1.4142136 from 1.7320508; 0.3819660 from 2.2360680; 0.3010300 from 0.6180340; 1.7320508 from 2.2360680; 0.6180340 from 2.2360680; 0.3010300 from 0.3819660.

79. Subtract 0.7853982 from 3.1415927; from 2.3561945; from 1.5707963.

Subtract 0.5235988 from 3.1415927; from 2.6179939; from 2.0943951; from 1.5707963; from 1.0471975.

80. Subtract 0.3819660 from 1; 0.6180340 from 1.4142136. Add 0.3819660 to 0.6180340.

81. In a school of 83 pupils, 37 are girls; the rest, boys. How many boys are there?

82. Take 1787 from 21,205, and what is the remainder?

83. Into a bowl containing 338 fine shot I poured a handful more, and the bowl then contained 720. How many did I pour in?

84. From a box containing 209 oranges I took a basketful, and left 163 oranges. How many did I take in the basket?

85. The minuend being 1718.754, and the subtrahend 1389.328, what is the remainder?

86. If the minuend was 6532.18, and the remainder 1916.47, what was the subtrahend?

87. How many must be taken from 729,434 in order to leave 613,488?

88. How many must be taken from 1,000,000 to leave 817,259?

89. Subtract 4187.94 from 8010.101.

90. Find the difference between 8,765,420 and 9,873,210.

REVIEW II.

Addition is the process of combining two or more numbers so as to form a single number. The result obtained by adding two or more numbers is called their **sum.**

For convenience, numbers to be added are written so that figures expressing the same order shall fall in the same column.

In adding numbers, first add all the units of the lowest order; then all the units of the next higher order, and so on, reserving from the sum of the numbers of any order the *tens* of that order, to be added as *ones* to the next higher order.

Only **units of the same kind** can be added; so many things of one kind added to so many things of the *same* kind.

To test the accuracy of the work, add in the inverse order, or by double columns.

The sign + is read "plus," and is used to indicate addition. The sign = is read "is equal to," and is placed between expressions which are equal.

Subtraction is the process of finding the difference between two numbers, or of finding the number that must be added to one of the two numbers in order that their sum shall be equal to the other.

The **subtrahend** is the number to be subtracted; the **minuend** is the number from which the subtrahend is to be taken; the **difference** is the number sought.

The addition of the same number to both the subtrahend and minuend does not affect the difference. Hence, we may add ten to any order of the minuend if we add one to the next higher order of the subtrahend.

For convenience, the figures of the subtrahend are written under those of the minuend so that those of the same order shall fall in the same column. If a decimal extend to the right more places in one than in the other, the missing places are supplied mentally with zeros.

First subtract the number of the lowest order in the subtrahend from that of the same order in the minuend. If the number in any order in the subtrahend be greater than the number in the corresponding order in the minuend, add *ten* to that number in the minuend before subtracting, and then add *one* to the number of the next higher order in the subtrahend.

The sign — is read "minus," and is used to indicate subtraction.

Only **units of the same kind** can be subtracted; so many units of one kind subtracted from so many units of the *same* kind.

To test the accuracy of the work, add the subtrahend and difference together; and if the work be correct, their sum will be equal to the minuend.

Whenever a question arises in which the minuend is smaller than the subtrahend, we may subtract the minuend from the subtrahend, and prefix the minus sign to the result. Such a result is called a minus number.

CHAPTER V.

APPLICATIONS OF ADDITION AND SUBTRACTION.

91. In a till are \$391 in bills, \$67.50 in gold, \$39.75 in silver, and \$2.77 in copper and nickel. How much money is in the till?

92. Starting out with \$315.75 in one wallet and \$54.37 in another, I pay the grocer \$127.38; the butcher, \$64.17; the shoemaker, \$21.40; the landlord, \$50; the tailor, \$35. What ought I to have left?

93. On a bill of \$753.43, I pay \$517.87. How much do I still owe? If I owe \$817.87, and have but \$637.50, how much do I lack of being able to pay?

94. If a man was born January 1, 1812, how old was he January 1, 1878? How old December 31, 1857?

95. America was discovered in 1492. How many years after its discovery was each of the following events?

Settlement of Florida, 1565; of Virginia, 1607; of Massachusetts, 1620; of Quebec, 1608; French and Indian War, 1756; Declaration of Independence, 1776; inauguration of Washington, 1789; war with England, 1812; Mexican War, 1846; Civil War, 1861.

96. How many days in common years, and in leap-years, between January 1 and March 1? January 4 and April 4?

February 5 and May 5? February 7 and October 7? January 4 and July 4? March 4 and July 4?

97. The sum of two numbers is 3; their difference, 1. What are the numbers? The sum of two numbers is 5; their difference, 1. Required the numbers. What two numbers added together make 8, if the difference of the numbers is 2? If the difference is 0? if 4? if 6?

98. If the minuend is 9874, and remainder 3185, what is the subtrahend? The subtrahend being 7659, and remainder 675.68, what is the minuend?

99. The smaller of two numbers is 7.95764328; their difference is .00087692. What is the larger number?

100. The larger of two numbers is 7.95764328, and their difference is 7.153485. What is the smaller number?

101. A hired man pumps out of my cistern in one hour 243.75 gallons; in the next hour, 227.5 gallons; in 45 minutes more, an additional 137.75 gallons; and the cistern is empty. How much was in it?

102. From what number must I subtract 5 to leave 7? 8 to leave 9? From what number must I subtract 5.1736 to leave 8.1964? 6.231 to leave 9.6648? 74.213 to leave 25.787?

103. What must be subtracted from 1 to leave .5? to leave .53? to leave .532? to leave .5236? to leave .5235988?

104. I start on a journey of 3433 miles. The first day I make 428 miles; the second day, 511 miles; the third,

497 miles; the fourth, 513. How many miles of my journey remained for me at the close of each day? How many miles had I gone at the close of each day?

105. Subtract 76,343 from the sum of 61,932, 51,387, 5193, 4674, and 8199; then subtract 23,657 from the remainder.

106. J. bought a farm and stock for $7633.90; sold off the stock for $305.75; then sold the farm for $7325. What did he lose?

107. If I gave $4375 for my land, and paid for house, barn, sheds, and fences, $2789.50; also $973.75 for horses, cattle, tools, etc.; what did my farm and stock cost?

If I sold part of the land for $675, and some cattle, etc., for $217.50, what may I estimate as the cost of what I have left?

108. Alfred the Great died at the age of 52, A.D. 901. In what year was he born? William the Conqueror began to reign A.D. 1066, and reigned 21 years. In what year did he die? Socrates was born B.C. 469, and died at the age of 70. In what year did he die? Plato was born B.C. 429, and died at the age of 82. In what year did he die? Demosthenes died at the age of 60, B.C. 322. In what year was he born? The battle of Marathon was fought B.C. 490; 560 years later Jerusalem was destroyed by Titus. In what year was Jerusalem destroyed?

109. John has 158 cents, James has 271 cents; James gives John 56 cents. Which has more than the other, and how many more?

CHAPTER VI.

MULTIPLICATION.

110. A ST. ANDREW'S cross $\times$ between two numbers means that one of the numbers is to be repeated as many times as is indicated by the other number. The number to be repeated is called the **multiplicand**; the number which shows how many times the multiplicand is to be repeated is called the **multiplier**; and the result is called the **product.**

The sign $\times$ is read **times,** or **multiplied by,** according as the multiplier precedes or follows the multiplicand. Thus, 5×4 cents $= 20$ cents is read, five times four cents equals twenty cents; but, 4 cents $\times\, 5 = 20$ cents is read: four cents multiplied by five equals twenty cents.

111. The multiplier and multiplicand are often called **factors** of the product. The product of two or more factors is the same in whatever order they are taken. Thus, $3 \times 4 = 4 \times 3$. The dots in the margin, read horizontally, make 3 fours; read vertically, make 4 threes.

● ● ● ●
● ● ● ●
● ● ● ●

112. The sign $\times$ cannot extend its power, forward or backward, beyond a $+$ or $-$, without the aid of a parenthesis. To illustrate:

$$2+3\times 4-1=13; \qquad 2+3\times(4-1)=11;$$
$$(2+3)\times 4-1=19; \qquad (2+3)\times(4-1)=15.$$

113. The products, in all cases in which neither factor exceeds ten, should be thoroughly committed to memory. They will be found in the following table:

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108

114. In the above table, take the multiplier in the upper line, the multiplicand in the left-hand column; the products will be found directly under the multiplier, and opposite the multiplicand; as, 12×7 is 84.*

115. To multiply any multiplicand by a multiplier less than 13, the work may be written as in the margin. Beginning at the right, 4×8 is **32**; the 2 is written, and the 3 carried mentally and added to 4×6, making **27**; and the process is thus continued to the left.

$$\begin{array}{r} 2.236068 \\ 4 \\ \hline 8.944272 \end{array}$$

$12 \times 8 = \mathbf{96}$; $12 \times 6 = 72$, and 9 makes **81**; $12 \times 6 = \mathbf{72}$; $12 \times 3 = 36$, and 7 makes **43**; $12 \times 2 = 24$, and 4 makes **28**; $12 \times 2 = 24$, and 2 makes **26**.

$$\begin{array}{r} 2.236068 \\ 12 \\ \hline 26.832816 \end{array}$$

* The table should be learned, not by lines but by squares; that is, first learn 2×2, 2×3 and 3×2, 3×3; next learn 2×4, 3×4, 4×2, 4×3, 4×4; thirdly, 2×5, 3×5, 4×5, 5×2, 5×3, 5×4, 5×5; fourthly, all products under 36, etc.

The cards referred to in the footnote to § 43 may be advantageously used for practice in multiplying two digits. Shuffle them, and pass them in couples from one hand to the other, naming the two factors, while the pupil names the products.

116. Multiply 111 by 5; 123 by 3; 231 by 2; 114 by 3; 421 by 4; 512 by 5; 4328 by 4; 1187 by 6; 1782 by 8; 8.287 by 7; 9.6198 by 3; 62.818 by 7; 9.2758 by 8; 52.134 by 9.

117. Multiply .5235988 by 6; .7853982 by 4; 3.14159265 by 5, and the product by 5.

118. Multiply 3.1416 by 11; by 12; by 10 and by 3, and add the two results; by 10 and by 4, and add the results; by 9 and by 6, and add the results. Multiply 2.236068 by 11; by 6 and by 7, and add the results; by 8 and by 9, and add the results; by 10 and by 7, and add the results (compare the sum of these two products with the sum of the last two products); by 10 and by 8, and add the results; by 12 and by 7, and add the results.

119. To multiply by 10, 100, 1000, etc., it is enough to move the decimal point of the multiplicand* as many places to the *right* as there are *naughts* after the one; annexing naughts to the multiplicand, if necessary.

Thus, $10 \times 15.4323 = 154.323$; $10{,}000 \times 15.43 = 154{,}300$.

120. How much is 10 times 3.14159265? 100 times? a million times? What will 10 barrels of apples cost, at \$3.75 a barrel? at \$2.17? at \$5.875? How much will 100 barrels cost at each of these prices, and at \$3.375? at \$5.125?

121. It is plain, that to multiply by .1, .01, .001, etc., it is necessary only to move the point of the multiplicand as many places to the *left* as there are *decimal places* in the

* When the multiplicand is a whole number, the decimal point is not written, but understood, after the units' figure.

multiplier, prefixing naughts to the multiplicand, if necessary.

Thus, $.1 \times 15.43 = 1.543$; $.001 \times 15.43 = .01543$.

122. What is a tenth of 2.36? a hundredth of 2.36? a thousandth of .63? Write the second members of the following equations, and then read them:

$.01 \times 7.8 =$ $\quad .001 \times 4.31 =$ $\quad .0001 \times 23.31 =$

$.1 \times .065 =$ $\quad .01 \times .012 =$

123. It is evident that 30 times is 3 times as much as 10 times, or 10 times as much as 3 times. Hence (§§ 115, 119),

Find the cost of 30 barrels of flour, at $3.27 a barrel; of 70 barrels, at $4.58; of 90 barrels, at $6.76; of 100 barrels, at $7.84; of 120 barrels, at $8.57.

124. It is evident that $.03 = 3 \times .01$. Hence (§ 121),

Find the cost of .03 of a barrel of oil, at $27.875 a barrel; of .7; of .009; of .17*; of .019; of .13; of .8; of .83; of .014 of a barrel.

It is plain from the above examples that,

The decimal places in the product are as many as the sum of the decimal places in the multiplicand and the multiplier.

125. What is the numerical value of the expressions:

30×8.75? $\quad 700 \times 7.81$? $\quad 300 \times 7.85$?

$.07 \times 6.975$? $\quad 8000 \times 65.432$? $\quad .0009 \times 10356.78$?

126. Ex. 1. Multiply 6957 by 463. $463 = 400 + 60 + 3$.

If we wished to find the result by addition, it would be necessary to write 463 numbers, each equal to 6957, under each other, and find the sum of all these numbers. (The

* Multiply first by .1, then by .07, and add the results.

necessity of abridging such an operation has given rise to multiplication.)

Or, we might make three additions, the first of 3 numbers, each equal to 6957; the second of 60 numbers, each equal to 6957; the third of 400 numbers, each equal to 6957; and then add together the three sums. In other words, we might multiply 6957, first by 3, then by 60, and then by 400, and find the sum of the three products obtained. Thus,

$$\begin{array}{rr}
\text{Multiplicand,} & 6957 \\
\text{Multiplier,} & \underline{463} \\
\end{array}$$

$$\left.\begin{array}{rr}
\text{3 times the multiplicand} = & 20871 \\
\text{60 times the multiplicand} = & 417420 \\
\text{400 times the multiplicand} = & \underline{2782800} \\
\end{array}\right\} \text{Partial Products.}$$

$$\begin{array}{rr}
\text{463 times the multiplicand} = & 3221091
\end{array}$$

It is evident that the zeros at the right of the second and third partial products do not affect the result of the addition; we may, then, omit them, if we observe *to what place in respect to the next preceding product* the right-hand figure of each succeeding product belongs.

Therefore, *multiply by each digit of the multiplier, place the right-hand figure of each product under the multiplier used, and add the partial products.*

127. Ex. 2. Multiply 1231 by 2007.

Solution of Ex. 2.

$$\begin{array}{r}
1231 \\
\underline{2007} \\
8617 \\
\underline{2462} \\
2470617
\end{array}$$

Ex. 3. Multiply 964.73 by 123.8.

Solution of Ex. 3.

$$\begin{array}{r}
964.73 \\
\underline{1238} \\
771\ 784 \\
2894\ 19 \\
19294\ 6 \\
\underline{96473} \\
119433.574
\end{array}$$

In multiplication, write the units' digit of the multiplier under the right-hand digit of the multiplicand. Put the right-hand digit of each partial product under the multiplying digit used; and put the decimal point in the product directly under the decimal point in the multiplicand.

The importance of putting each partial product in its proper place is seen from Ex. 2, and the method of dealing with the decimal point from Ex. 3.

128. A test of the accuracy of the pointing is furnished, by observing that (by § 124),

The fractional places in the product are as many as the sum of the fractional places in the factors.

Care must be taken lest the omission of superfluous naughts confuse us. Thus, $6000 \times .5236 = 3141.6$, which appears like a violation of the rule until we notice that three 0's are omitted in the product.

Another test is furnished by making a rough estimate of what the product should be. Thus, in the last example, we have 6000 times a number a little greater than .5; the product, therefore, should be something over 3000.

129. Multiply .785398 by each of the following numbers: 2; 20; 3; 300; 5; .5; .005; 737; 7.37; 856; 85.6; .0856; 10; 1001; 1.001; 954; .00954.

130. Multiply 2150.42 by .1; by .001; by .75; by .075; by .083.

131. Multiply 1.4142136 by .7; by .707; by .7071; by .707107. Multiply 1.41421 by 1.4; by 1.4142; by 1.41422. Multiply 1.732 by 1.732; 2.23607 by 2.236; .618 by 618; .618034 by .618035. Subtract this last product from 1.

132. When the multiplier is the product of factors, it is occasionally a saving of labor to multiply by the factors successively; as, for example, to multiply by 289, we may multiply by 17 and then multiply the product by 17; or, to multiply by 8.4, we may multiply by .7 and by 12.

133. Find the value of the expressions:

88×718.54; 96×6.8193; 6.3×71.569; 1.32×234.769.

134. Multiply 291.47 by 16, and the product by 625. In like manner, find the continued products:

$8 \times 125 \times 278.56$; $8 \times 3.75 \times 3.33333$; $8 \times 625 \times 1.5708$.

135. One mile measures 5280 feet. How many feet in 3 tenths of a mile? in .7? in .17? in .573? in .846 of a mile?

136. When a number is subtracted from 1 standing in the *place next to the left of the highest figure in the number*, the remainder may be called a **complement** of the number; but if the 1 is put in the unit's place, the remainder is called **the arithmetical complement.**

For example, *the arithmetical* complement of .097 is .903; but .003 may also be called *a* complement of .097.

137. When a complement of a multiplier is a simpler factor than the multiplier, we may multiply by that complement, and subtract the product from the product obtained by multiplying by 1 in the proper *order* of units.

For example, .097 of a multiplicand is found by subtracting .003 of the multiplicand from .1 of the multiplicand; and to multiply by 9.98, we need but to subtract .02 of the multiplicand from 10 times the multiplicand.

138. Multiply (using complements) .7854 by 9.9; by .99; by .099. Multiply .5236 by 99.7; by 9.989; by 9.87. Multiply 8537 by .0097; by .9995.

139. Multiply .61803 by 147; by 373; by 7.56; by 8.93; by 9.93. Multiply .5236 by 5.99; by 7.99; by 8.997; by 699.98.

140. Multiply .7854 by .618; by .382; by .7854; by .302. Multiply 2.718 by .618; by .382; by .7854; by .607.

141. Find the continued products:
$.477 \times 101 \times .708$; $15.43 \times .4343 \times 3$; $4 \times .175 \times 3.28$; $.615 \times .771 \times 10$; $3.2809 \times 5 \times .71$; $.785 \times .7 \times .202$; $.471 \times .807 \times 22$; $3.28 \times 25 \times .909$.

CONTRACTED MULTIPLICATION OF DECIMALS.*

142. In ordinary calculations we seldom use a fraction smaller than .00001 of the unit.

Ex. 1.

	.123456789
	1.23456789
	1111111101
	987654312
	864197523
	740740734
6	27283945
49	3827156
370	370367
2469	13578
12345	6789
.15241	675750190521

The 6 at the left of the vertical line is obtained by multiplying the 1 in the multiplicand by 5 in the multiplier, and carrying 1 from 5×2. The 9 below the 6 is obtained by multiplying 2 by 4, and carrying 1 from 4×3. The 0 below the 9 is obtained by multiplying the 3 by 3, and carrying 1 from 3×4. The 9 below the 0 is obtained by multiplying 4 by 2, and carrying 1 from 2×5. The 5 below the 9 is obtained by multiplying 5 by 1.

* Contracted Multiplication and Division of Decimals may be taken or omitted, at the option of the teacher.

It is evident that, if five decimal places only are required, all the work to the right of the vertical line is wasted.

To shorten the work, write the multiplier under the multiplicand, so that each digit of the multiplier shall fall directly under the digit of the multiplicand into which it is multiplied to produce the first figure to the left of the vertical line in each partial product. Thus:

```
 0.123456789
 987654321
 ----------
     12346
      2469
       370
        49
         6
         1
    ------
    .15241
```

Notice that the figures of the multiplier are reversed; that the units' figure of the multiplier falls under the last decimal of the multiplicand which is to be retained, and that the decimals in the result are correct. In order, however, to have the required number of decimal places correct, it will generally be necessary to take one more than the required number of decimal places.

143. Hence, in contracted multiplication of decimals:

Reverse the multiplier, and put the *units' figure* under the last place of decimals to be retained.

Multiply each figure of the multiplier into the figure next to the right above it. Do not write *this* result, but carry the *nearest ten* to the next result, multiplying as usual.

Write the first figures of the partial products in a vertical column.

Add the products, and point off from the sum as many decimal places as were taken in the multiplicand.

If the multiplier has no units' figure, supply its place with zero. To insure accuracy, take one decimal place more than the required number. To detect errors that may arise from displacement of the decimal point, or from an erroneous arrangement of the factors, test the result by a rough estimate of what the product should be.

Ex. 2. Multiply 4.65440758 by 1609.3295 correct to 3 places.

Take 4 places.

```
 4.65440758
 59239061
 ----------
 46544076
 27926445
   418896
    13963
      931
      419
       23
 ----------
 7490.4753 Ans.
```

7490.475$\cancel{3}$ *Ans.*

Ex. 3. Multiply 1.7320508 by 0.0618034 to 5 places.

Take 6 places.

```
 1.7320508
43081600
---------
103923
  1732
  1386
     5
---------
.107046 Ans.
```

.10704$\cancel{6}$ *Ans.*

144. Find the product, to the fifth fractional place, of 3.14159265 by 2.236. Find 1414.2136 × 14142.136, to the second place; .618034 by .618034, to the sixth place; 2.236068 by 2236.068, to the third place; 1.73205 by 1732.0508, to the second place.

145. Whenever absolute accuracy of a long decimal is not required, in curtailing the decimal 1 must be added to the figure in the lowest place retained, if the figure in the next place is 5, or greater.

Thus, 2.2360679775 must be represented by 2.236067978, 2.23606798, 2.2360680, 2.236068, 2.23607, 2.2361, 2.236, 2.24, or 2.2, according to the degree of accuracy required.

Give the nearest values to 1.732050808 for each place of decimals from the first to the eighth.

146. When the multiplier contains many more places of significant* digits than the multiplicand, we may reverse

* The digits 1, 2, 3, 4, 5, 6, 7, 8, 9, are called significant digits.

the factors and use the multiplicand as multiplier, provided we carefully notice what we are doing, and interpret the result correctly.

1578 sheep, at \$3 a head, are worth \$4734; and, although in numbers we may obtain the result more easily by saying $3 \times 1578 = 4734$, yet we must remember that the \$4734 is really $1578 \times \$3$.

147. What will a man earn in a year if he has \$2 a day, omitting Sundays? Suppose that the year begins on Sunday? Suppose the year to be leap-year, and not begin on Sunday? Suppose it leap-year, and to begin on Saturday?

2. If a field of corn averages 2 ears to a stalk, how many ears on 673 stalks?

3. At 27 bushels an acre, how much wheat to the square mile of 640 acres, deducting 47 acres for roads and waste land?

4. How much money would be required to give \$7000 to each of 7568 men?

5. In a certain book of 378 pages, the words average 7 letters to a word. and 10 words to a line. There are, on an average, 29 lines to a page. How many letters in the book?

6. How many bushels of wheat in a township of 37 square miles, if we deduct 47 acres to the square mile for roads and waste, and suppose that half the remainder is in wheat averaging 23 bushels to an acre?

7. If 5700 persons, each paying 1 cent toll, and 324 carriages, each paying 5 cents toll, pass over a bridge in a day, how much money will be received?

8. A merchant bought 960 pounds of cheese at 7 cents a pound, and 147 pounds of butter at 20 cents. He gave in payment 12.5 yards of cloth at 1 dollar a yard, 2 barrels of sugar, each weighing 226 pounds, at 9 cents a pound,

and the remainder in cash. How much money had he to pay?

In each question of this article, distinguish between the real multiplicand and number used as multiplicand.

148. If a product consist of **equal** factors, it is called a **power** of that factor, and one of the equal factors is called a **root** of the product.

The power is named according to the *number* of equal factors taken. Thus,

$5 \times 5 = 25$; 25 is the *second power*, or **square**, of 5.
$5 \times 5 \times 5 = 125$; 125 is the *third power*, or **cube**, of 5.
$5 \times 5 \times 5 \times 5 = 625$; 625 is the *fourth power* of 5.

Likewise,

5 is called the *second root*, or **square root**, of 25.
5 is called the *third root*, or **cube root**, of 125.
5 is called the *fourth root* of 625.

To avoid the necessity of writing long rows of equal factors, a small figure, called the **exponent**, is written at the right of a number to show how many times the number is taken as a factor. Thus,

5^5 means the same as $5 \times 5 \times 5 \times 5 \times 5$, and is read the fifth power of 5.

3^3 times 3^4 means $3 \times 3 \times 3$ times $3 \times 3 \times 3 \times 3 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$.

From this last example, it is evident that *the product of two or more powers of the same number is expressed by writing the number with an exponent equal to the sum of the exponents of the given powers.*

Express the product of,

1. $7^5 \times 7^3$; $8^2 \times 8$; $2^8 \times 2$; $5^4 \times 5^2$.
2. $3.01^2 \times 3.01$; $0.67^2 \times 0.67^3$; $.208 \times .208^3$.
3. $2.003^2 \times 2.003^4$; $20.03^3 \times 20.03$; 20.03×20.03^2.

REVIEW III.

Multiplication, strictly speaking, is the operation of finding the sum of two or more **equal** numbers.

With reference to this operation, this sum is called the **product**; one of the equal numbers is called the **multiplicand**; and the number which shows *how many times* the multiplicand is taken is called the **multiplier.**

In multiplication *we must always have* **two different kinds of units**: *So many* **things** *taken or repeated so many* **times.**

It is sometimes required to take or repeat the **whole** of the multiplicand, sometimes a **certain part** of it, sometimes the **whole and a certain part** of it; and the meaning of multiplication is *extended* to cover all of these cases.

To multiply by a **whole number** is to take the multiplicand as many times as is indicated by the whole number.

To multiply by a **fraction** is to take **such a part of** the multiplicand as is indicated by the fraction.

To multiply by a **whole number and a fraction** is to take the multiplicand as many times as is indicated by the whole number, and such a part of the multiplicand as is indicated by the fraction.

The product, therefore, will be equal to, greater than, or less than the multiplicand, according as the multiplier is equal to, greater than, or less than one.

The multiplicand and the multiplier are often called **factors** of the product.

When a product consists of more than two factors, it is called the **continued product** of the factors.

When a product consists of two or more **equal** factors, it

is called a **power** of that factor; and one of the equal factors is called a **root** of the product.

The **index** or **exponent** of a power is a small figure placed at the right of a number to show how many times the number is taken as a factor, and is read the *first* power, *second* power, *third* power, etc., of the number.

The second power of a number is generally called the **square** of the number, and the third power is called the **cube** of the number.

The product of two or more powers of the same number may be expressed by writing the number with an exponent equal to the **sum of the exponents** of the given powers.

The numerical result of multiplying one number by another is the same whichever is taken as the multiplicand, the other being taken as the multiplier; *but the product will always denote the same kind of units as the true multiplicand.*

A St. Andrew's cross × placed between two numbers means that one of the numbers is to be multiplied by the other, and it is read "times," or "multiplied by."

To multiply by 10, 100, 1000, etc., it is necessary only to move the decimal point in the multiplicand as many places to the *right*, annexing ciphers, if necessary, as there are *ciphers* in the multiplier.

To multiply by .1, .01, .001, it is necessary only to move the decimal point in the multiplicand as many places to the *left*, prefixing ciphers, if necessary, as there are *places* in the multiplier.

The decimal places of a product are equal in number to the decimal places in the multiplicand and multiplier counted together.

To multiply by the product of two or more factors gives the same result as to multiply by one of the given factors, this product by another of the factors, and so on.

If we have to multiply one number by another, **we may separate the multiplicand into parts,** multiply each part by the multiplier, and add the results to form the complete product. Or, **we may separate the multiplier into parts,** find the product of the multiplicand by each part, and add these partial products to form the complete product.

When the **multiplier is a single digit,** the product is obtained by multiplying each *order* of the multiplicand, beginning with the lowest order, care being taken to write down the ones of each partial product, and to reserve the *tens* to be added as *ones* to the product of the next higher order.

When the **multiplier consists of two or more digits,** multiply by each digit separately, arrange the partial products so that figures of the *same order* shall fall in the *same column*, and add.

To test the accuracy of the work, interchange multiplicand for multiplier. Or, separate the multiplier into parts, multiply the multiplicand by each part separately, and add the results. If the work be correct, their sum will be equal to the original product.

In contracted multiplication of decimals:

Reverse the multiplier, and place the units' figure under the last place of decimals in the multiplicand to be retained.

Multiply by each figure of the multiplier the figure next to the right above it. Do not write the result, but carry the *nearest ten* to the next product and write this result, multiplying as usual.

Write the first figures of the products in a vertical column.

Add the several products, and point off from the sum as many places for decimals as were retained in the multiplicand.

CHAPTER VII.

DIVISION.

149. Division is the operation by which, when a product and one of its factors are given, the other factor is found.

With reference to this operation, the product is called the **dividend**; the given factor is called the **divisor**; and the required factor is called the **quotient.**

150. When the given factor is the multiplicand, the factor sought is the multiplier.

In this case, the question is: *What must we* **multiply the divisor by** *to get the dividend?*

To answer this question, it is necessary to find *how many times* the divisor is contained in the dividend, and the answer will be **so many times.**

Thus, in the question, How *many times* are 6 cents contained in 30 cents? the factor sought is the multiplier, 5; 5 times 6 cents are 30 cents.

When the given factor is the multiplier, the factor sought is the multiplicand.

In this case, the question is: *What must we* **multiply by the divisor** *to get the dividend?*

To answer this question, it is necessary to divide the dividend into as many **equal parts** as is indicated by the number in the divisor, and the answer will be **so much to each part.**

Thus, in the question, How *much* will each boy receive if 30 cents be divided among 6 boys? the factor sought is the multiplicand, 5 *cents;* 6 times 5 cents are 30 cents.

The *arithmetical process* and the *numerical result* are the same in both cases; but the *name* to be attached to the result depends upon the nature of the question.

151. Division is indicated by the sign $\div$, by the colon, :, or by writing the dividend over the divisor, and drawing a line between them; thus, each of the expressions, $15 \div 3$, $15 : 3$, $\frac{15}{3}$, means and is read, "Fifteen **divided by** three."

SHORT DIVISION.

152. When the divisor does not exceed 12, the work may be written in the following manner: the divisor is placed to the left of the dividend, the quotient under the dividend, and each remainder is added as so many *tens* to the next figure of the dividend not divided.

(1) Divide 2736 by 4.

$$\begin{array}{r} 4\underline{)2736} \\ 684 \end{array}$$

Wording. 4 in 27, **6**, carry 3; in 33, **8**, carry 1; in 16, **4**. *Answer*, 684.

(2) Divide 2736 by 9.

$$\begin{array}{r} 9\underline{)2736} \\ 304 \end{array}$$

Wording. 9 in 27, **3**; in 3, **0**, carry 3; in 36, **4**. *Answer*, 304.

(3) Divide 3696 by 12.

$$\begin{array}{r} 12\underline{)3696} \\ 308 \end{array}$$

Wording. 12 in 36, **3**; in 9, **0**, carry 9; in 96, **8**. *Answer*, 308.

The pupil will observe that, the divisor being a whole number, each quotient figure is of the same **order of units** *as the right-hand figure of the partial dividend used in obtaining it.*

153. Divide 963 by 3; 846 by 2; 846 by 3; 846 by 6; 848 by 4; 52.05 by 5; 84.028 by 7; 13.31 by 11; 1.728 by 12.

When the divisor is a whole number, the pupil will be careful to put the decimal point in the quotient as soon as the decimal point in the dividend is reached.

154. If the divisor is not contained in the dividend without a remainder, ciphers may be mentally annexed to the dividend, and the division continued until the required number of decimal places is obtained. *If the remainder, when the last quotient figure is obtained, is greater than half the divisor, increase the quotient figure by* 1.

Divide 3.1 by 4, by 5, and by 7, carrying the division to the third place of decimals.

4)3.1	5)3.1	7)3.1
.775	.620	.443

155. If the dividend and divisor are both multiplied, or both divided, by the same number, the quotient is not changed.

Thus, $12 \div 4 = 3$, and (when both dividend and divisor are multiplied by 2) $24 \div 8 = 3$. Again (when both dividend and divisor are divided by 2) $6 \div 2 = 3$.

Hence, if we have to divide 2.24 by 35, we may first divide by 7, and then by 5, as follows:

7)2.24	Again,	5).32	
.32		.064	*Answer*, .064.

Dividing both dividend and divisor by the same number is called **cancelling** equal factors in dividend and divisor.

Find the quotients, to five decimal places, of: $3 \div 7$ $4 \div 11$; $3.17 \div 14$; $7.85 \div 21$.

156. Since the quotient is not altered if the dividend and divisor are both multiplied, or both divided, by the same number, it follows that,

If the divisor contains decimal places, we may remove the decimal point from the divisor, provided we carry the decimal point in the dividend as many places to the right as there are decimal places in the divisor.

If the divisor is a whole number and ends in zeros, the zeros may be cut off, provided the decimal point in the dividend is carried to the left as many places as there are zeros in the divisor.

Divide 78.52 by .008, by .8, and by 8000, first making the required changes of the decimal point.

8)78520	8)785.2	8).07852
9815	98.15	.009815

157. From the last three examples it is seen that,

If the first figure of the quotient is placed under the right-hand figure of the first partial dividend, the decimal point in the quotient will be written directly under the decimal point in the dividend.

158. Divide,*

1. .003 by .07;	.003 by 110;	110 by .003.
2. .07 by .003;	110 by .07;	1.3 by .07.
3. 1.7 by .07;	.07 by 110;	1.3 by 110.
4. 1.7 by 110;	.07 by 1.2;	.003 by 1.2.
5. 110 by 1.2;	1.7 by 1.2;	17 by 1.2.
6. 136 by .06;	136 by .12;	136 by 1100.
7. 256 by .8;	2.56 by .08;	.0256 by .008.
8. 256 by 8000;	1.06 by .9;	1.06 by 9000.

* If the divisor is not contained in the dividend without a remainder, carry the quotient to the fifth decimal place.

LONG DIVISION.

159. The process of Long Division is precisely like that of Short Division, except that the work is written in full, and the quotient is written *over* the dividend.

Divide 3.1415927 by 1.73.

Suppress the decimal point in the divisor, and move the point two places to the right in the dividend.

```
      1.81595
173)314.15927
    173
    1411
    1384
      275
      173
      1029
       865
       1642
       1557
         857
         692
         165
```

173 is contained in 314 once. The one is written over the 4, and, by § 156, the point is placed over the decimal point in the dividend. The process, which has been gone through with, mentally, in short division, is written out below the dividend. 173 is subtracted from 314; to the remainder, the 1 in the dividend is annexed; then 8×173 is subtracted; then to the remainder, 5 in the dividend is annexed, and so on. The fifth place in the quotient is 4, but written 5, because the remainder, 165, is more than half 173.

Divide 23.13685 by 7.843.

Suppress the decimal point in the divisor, and move the point three places to the right in the dividend.

```
          2.95
7843)23136.85
     15686
      74508
      70587
       39215
       39215
```

Determine each figure of the quotient carefully before writing it. The first figure of the quotient is 2; for, though 7 is contained 3 times in 23, 78 is contained only 2 times in 231. The second figure is obviously 9; the third figure is 5, for 7 is contained 5 times in 39.

NOTE. As a test of the accuracy of each quotient figure, observe that if the product of the divisor by the quotient figure is greater than the partial dividend, the quotient figure is too large; and if the difference between the partial dividend and the product of the divisor by the quotient figure is greater than the divisor, the quotient figure is too small.

From these examples the following rule is deduced:

Change the decimal point so as to make the divisor a whole number, and find, by trial, the first quotient figure.

Subtract from the partial dividend the product of the divisor by this quotient figure.

Consider the remainder with the next figure of the dividend annexed as a **new partial dividend,** *and proceed as before.*

160. Divide,

1. 1.6093295 by .479; by .917; by .017; by .0087.
2. 3. by 1.7; by 1.73; by 1.732; by 1.7321.
3. 1.6093295 by 5280, and the quotient by 12.
4. 2 by 1.4142; 5 by 2.236.

161. Two numbers are **reciprocals** of each other when their product is equal to 1. Thus, .5 is the reciprocal of 2; 4 is the reciprocal of .25; 3 is the reciprocal of .33333; .75 is the reciprocal of 1.33333; .8 is the reciprocal of 1.25, etc.

162. When the dividend is 1, the quotient is the reciprocal of the divisor. And if the dividend is any other number, as 5, 7, etc., the quotient will be 5, 7, etc., times the reciprocal of the divisor; or, in other words, *the reciprocal times* 5, 7, etc.; that is,

To divide a number by a divisor gives the same numerical result as to multiply it by the reciprocal of the divisor.

163. It is equally plain, that *to multiply by a factor gives the same result as to divide by the reciprocal of the factor.*

164. The processes of multiplication and division are frequently made much simpler by using reciprocals of the multiplier or of the divisor.

It is easier to use 8, .8, .08, etc., than to use .125, 1.25, 12.5, etc. If required to divide 2.71828 by 37.5, we may (if we notice that 37.5 is 3×12.5) divide 2.71828 by 3, and multiply the quotient by .08.

165. Perform the work in the following questions by the use of reciprocals:

1.	$8 \times .25 =$	**10.**	$1764 \times .025 =$
2.	$171 \div .25 =$	**11.**	$5381 \div .025 =$
3.	$876 \times 1.25 =$	**12.**	$7452 \div .875 =$
4.	$132 \times 2.5 =$	**13.**	$651 \times .33333 =$
5.	$591 \div 2.5 =$	**14.**	$456 \times 6.66667 =$
6.	$756 \div .125 =$	**15.**	$1554 \times .16667 =$
7.	$268 \times 25 =$	**16.**	$432 \div 1.33333 =$
8.	$753 \div 25 =$	**17.**	$375 \div 16.667 =$
9.	$567 \div 625 =$	**18.**	$225 \div 6.6667 =$

166. If 8375 mills be divided by 10 (the number of mills in a cent), the quotient, 837.5, is the number of cents in 8375 mills. Divide the same number by 1000, and the quotient, 8.375, is the number of dollars.

In like manner, if we divide 173 inches by 12 (there being 12 inches in a foot), the quotient, 14.417, is the number of feet in 173 inches.

And in general, *to express a given number in terms of a second number as* **unity**, *we have only to divide by the second number.*

167. Taking 7 as unity, what would be the value of 14? of 28? of 35? of 3.5? of 2.8105? of 6.31415?

2. If the side of a square is 10 inches, and its diagonal 14.14214, express the side in terms of the diagonal as unity.

3. If the diagonal of a square is one foot, what decimal of a foot must its side be?

4. If the diameter of a circle is 11.3 inches, and its circumference 35.5 inches, what is the circumference in terms of the diameter? What is the diameter in terms of the circumference?

5. What decimal fraction of 87 is 47? 53? 43.5? 29?

6. How many times 393 is 587? 7857? 131? 196.5?

7. How many 684's are there in 1368? in 1760? in 342? in 77? in 6.84? in .0785?

All the examples in § 167 are illustrations of § 166.

CONTRACTED DIVISION OF DECIMALS.

168. Bringing down a digit to the right of a partial *dividend* multiplies the partial dividend by 10 and adds to it the number expressed by the digit.

Cutting off a digit from the right of the *divisor* subtracts from the divisor the number expressed by the digit and divides the divisor by 10. The quotient, therefore, will be the same whichever we do; and this principle can be applied to shorten the labor of division, as may be illustrated in finding the quotient of

$$15.4323487 \div 1.4142136, \text{ to four decimal places.}$$

It is necessary to determine, first, the number of significant figures required in the quotient.

Since 1.4 is contained in 15 ten times, it is evident that the quotient will have two integral places. The two integral figures and the four decimal figures make six, which will be the number of figures required in the quotient.

In general, to determine the number of significant figures required in the quotient, move the decimal point of the divisor to the right of the first significant figure, and move the decimal point of the dividend as many places, and in the same direction, as the decimal point of the divisor has been moved. The number of integral places in the quotient can then easily be determined.

It is advisible not to cut off figures from the right of the divisor until the number of figures still required in the quotient is two less than the number of digits in the divisor. In multiplying the divisor by each quotient figure, multiply the figure of the divisor cut off, and carry the nearest ten.

The work may be arranged as follows:

$$\begin{array}{r} 10.9123 \\ 14\cancel{1}\cancel{4}\cancel{2}\cancel{1}\cancel{3}\cancel{6})\overline{154323487.} \\ \underline{1414214} \\ 129021 \\ \underline{127279} \\ 1742 \\ \underline{1414} \\ 328 \\ \underline{283} \\ 45 \\ \underline{42} \end{array}$$

We first cut off the 6. The first product is increased by 1 for the 1×6 omitted. The first remainder is increased by 1 for the 8 in the dividend. Cut off the 3. As the divisor is not contained in the partial dividend, we also cut off the 1. The product by 9 is increased by 1 for the 9×1 omitted. Cut off the 2. As 1×2 is less than 5, the product by 1 is not increased. Cut off the 4. The product by 2 is increased by 1 for 2×4 omitted. Cut off the 1. The product by 3 is not increased, for 3×1 is less than 5.

In like manner divide,

1. 11.4285285 by 3.1415927 to six decimal places.
2. .004239239 by 3.2783278 to five decimal places.
3. 437 by 215.253 to three decimal places.
4. .0053 by 72.654 to eight decimal places.
5. 6 by .1573 to three decimal places.
6. .11 by 1937.43 to eight decimal places.
7. 46 by .00751515151 to three decimal places.

DIVISION OF POWERS.

169. Since $5^5 = 5 \times 5 \times 5 \times 5 \times 5,$

and $5^3 = 5 \times 5 \times 5,$

therefore, $\dfrac{5^5}{5^3} = \dfrac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5 \times 5 = 5^2.$

That is,

(1) *The quotient of two powers of the same number is expressed by writing the number with an exponent equal to the exponent of the dividend minus that of the divisor.*

Also, $\dfrac{5^3}{5^5} = \dfrac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = \dfrac{1}{5 \times 5} = \dfrac{1}{5^2}.$

But by (1), $\dfrac{5^3}{5^5} = 5^{-2};$

therefore, $5^{-2} = \dfrac{1}{5^2}.$ That is,

(2) *Any number with a* **minus** *exponent is the* **reciprocal** *of the number with an equal* **plus** *exponent.*

Again, $\dfrac{5^3}{5^3} = 1.$

But by (1), $\dfrac{5^3}{5^3} = 5^0;$

therefore, $5^0 = 1.$ That is,

(3) *Any number with* **zero for an exponent** *is equal to* 1.

Find the value of

1. 10^0; 10^1; 10^2; 10^3; 10^4; 10^5; 10^6.
2. 10^0; 10^{-1}; 10^{-2}; 10^{-3}; 10^{-4}; 10^{-5}; 10^{-6}.
3. $10^0 \times 10^0$; $10^1 \times 10^{-1}$; $10^2 \times 10^{-2}$; $10^3 \times 10^{-3}$; $10^4 \times 10^{-5}$.
4. $10^3 \div 10^{-1}$; $10^{-2} \div 10^2$; $10^{-1} \div 10^{-3}$; $10^{-2} \div 10^{-4}$.
5. $10^{-3} \times 10^2$; $10^3 \div 10^2$; $10^{-3} \div 10^2$; $10^{-2} \div 10^{-3}$.
6. $10^2 \div 10$; $10^2 \div 10^3$; $10^0 \div 10^{-1}$; $10^{-1} \div 10^{-1}$.
7. $1.01^2 \div 1.01^{-1}$; $1.01^2 \times 1.01^{-1}$; $1.01^{-2} \div 1.01^{-1}$.

REVIEW IV.

Division is the operation by which, when a product and one of its factors are given, the other factor is found.

With reference to this operation, the product is called the **dividend**; the *given* factor, the **divisor**; and the *required* factor, the **quotient.**

When the given factor is the multiplicand, the factor sought is the multiplier.

In this case, it is required to find *what we must* **multiply the divisor by** *to get the dividend.*

It is, therefore, necessary to find *how many times* the divisor is contained in the dividend; and the answer will be, **so many times.**

When the given factor is the multiplier, the factor sought is the multiplicand.

In this case, it is required to find *what we must* **multiply by the divisor** *to get the dividend.*

It is, therefore, necessary to divide the dividend into as many *equal parts* as is indicated by the number in the divisor; and the answer will be, **so much to each part.**

The **arithmetical process** and the **numerical result** are the same in both cases, *but the name to be attached to the quotient depends upon the nature of the question.*

Division is indicated by the sign $\div$, the colon, :, or by writing the dividend over the divisor and drawing a line between them. Whatever symbol of division is employed, it is read, "divided by."

The dividend may be separated into parts, and each part divided by the given divisor; the sum of the results will then be the entire quotient.

Hence, in dividing,

Find by trial the first figure of the quotient.

Subtract from the partial dividend the product of the divisor by this quotient figure.

Consider the remainder with the next figure of the dividend annexed **as a new partial dividend,** and proceed as before.

When the divisor is so small that the necessary multiplications and subtractions can be carried on mentally, the operation is called **Short Division.**

When the work is written in full, the operation is called **Long Division.**

The decimal point may be suppressed in any divisor, if the point in the dividend be moved as many places to the right as there are decimal places in the divisor. And if the first quotient figure be written over the right-hand figure of the first partial dividend, the position of the decimal point in the quotient will be directly over that in the dividend.

To divide by 10, 100, 1000, etc., it is necessary only to move the decimal point in the dividend as many places to the *left* as there are *ciphers* in the divisor.

To divide by .1, .01, .001, etc., it is necessary only to move the decimal point in the dividend as many places to the *right* as there are *decimal places* in the divisor.

To divide by the product of two or more divisors gives the same result as to divide first by one divisor, the quotient by another divisor, and so on. That is,

Equal factors of the dividend and divisor *may be suppressed without altering the value of the quotient.*

Suppressing equal factors in dividend and divisor is called **cancelling.**

When the dividend is 1, the divisor and quotient are **reciprocals** of each other. Hence,

Multiplying by the reciprocal of a number is equivalent to dividing by the number.

Dividing by the reciprocal of a number is equivalent to multiplying by the number.

The quotient of a power by another power of the same number may be expressed by writing the number with an exponent equal to the exponent of the dividend diminished by the exponent of the divisor.

Any number with a **minus** exponent is the **reciprocal** of the number with an equal **plus** exponent.

The quotient of two equal powers of the same number may be expressed by unity, or by the number with zero for an exponent. That is,

The **zeroth** power of any number is equal to 1.

To test the accuracy of the work in Division, multiply the divisor by the quotient. The product should be equal to the dividend.

When a parenthesis includes two or more numbers, the included numbers must first be reduced to a single number, and the result put in place of the parenthesis.

In contracted division of decimals:

First, determine the number of significant figures required in the quotient.

Secondly, begin to cut off figures from the right of the divisor, when the number of figures still required in the quotient is two less than the number of digits in the divisor.

Thirdly, in multiplying the divisor by each quotient figure, multiply the figure of the divisor cut off, and carry the nearest ten.

CHAPTER VIII.

MISCELLANEOUS EXERCISES.

BEFORE giving any more examples, we shall give a method of testing the accuracy of results in the four operations by what is called

CASTING OUT NINES.

170. A product of two integral factors is called a multiple of either of its factors.

Every power of 10 is one more than some multiple of 9. Thus, $10 = 9 + 1$; $10^2 = 11 \times 9 + 1$; $10^3 = 111 \times 9 + 1$, etc.

Every multiple of a power of 10 by a single digit is, therefore, some multiple of 9, plus that digit. For example, $500 = 5 \times 11 \times 9 + 5$; $7000 = 777 \times 9 + 7$, etc.

But as every number consists of the sum of such multiples of powers of 10, every number is a multiple of 9, plus the sum of its own digits.

Thus, 24,573 is a multiple of 9 plus $2 + 4 + 5 + 7 + 3$. If a number, therefore, be divided by 9, the remainder will be the same as that arising from dividing the sum of its digits by 9.

In finding the remainder from dividing the sum of the digits by 9, we may, of course, omit the nines, or any two or three digits which we see at a glance will make 9. Thus, to find the remainder on dividing 1,926,754 by 9, we see at once that 1, 2, 6, and 5, 4, make nines, and the single 7 will be the remainder. So in 254,786, we reject 5, 4, and 2, 7, and only add $8 + 6$, from the sum of which reject 9, and there is left 5.

171. This truth may be applied to test the accuracy of results obtained in addition, subtraction, multiplication, and division.

ADDITION.

$$\begin{array}{llllll} & (1)\ 81{,}364 = & \text{a number of nines} & +4 & \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{Add.} & \\ & 27{,}632 = & \text{"} \quad \text{"} & +2 & & \\ & 38{,}507 = & \text{"} \quad \text{"} & +5 & & \\ & \underline{67{,}549} = & \text{"} \quad \text{"} & \underline{+4} & & \\ 6 \ldots\ldots & 215{,}052 & & 15 \ldots\ldots 6 & & \end{array}$$

From the several numbers to be added, the nines are cast out; then, from the sum of the several remainders, a final remainder of 6 is obtained, which corresponds with remainder obtained by casting out the nines from the sum. Therefore, the work may be presumed to be correct.

SUBTRACTION.

$$\begin{array}{lllll} & (2)\ 176{,}543 = & \text{a number of nines} & +8 & \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{Subtract.} \\ & \underline{85{,}674} = & \text{"} \quad \text{"} & \underline{+3} & \\ 5 \ldots\ldots & 90{,}869 & & 5 & \end{array}$$

The nines are cast out from the minuend and subtrahend, and the remainder of the subtrahend is subtracted from that of the minuend, giving a final remainder, 5, which corresponds with the remainder obtained by casting out the nines from the difference.

It is obvious, that a number of nines + a remainder, subtracted from a number of nines + a remainder, will give an exact number of nines + the difference of the remainders.

$$\begin{array}{lllll} & (3)\ 51{,}786{,}531 = & \text{a number of nines} & +0 & \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{Subtract.} \\ & \underline{23{,}456{,}780} = & \text{"} \quad \text{"} & \underline{+8} & \\ 1 \ldots\ldots & 28{,}329{,}751 & & 1 & \end{array}$$

Since the remainder from casting out nines from the minuend is less than that of the subtrahend, it is necessary to add a nine to the remainder of the minuend.

Multiplication.

172. (4) Multiply 47 by 61.

$$47 = 45 + 2.$$
$$61 = 54 + 7.$$

Therefore, $47 \times 61 = (45 + 2) \times (54 + 7)$
$= 45 \times 54 + 2 \times 54 + 45 \times 7 + 2 \times 7.$

45×54, 2×54, and 45×7 are a number of nines. That is, the entire product is a number of nines $+ 2 \times 7$; or, since $2 \times 7 = 9 + 5$, the entire product is a number of nines $+ 5$. And the product of 47×61, namely, 2867, is a number of nines $+ 5$; therefore, the work is presumed to be correct.

The work may be arranged as follows:

$$\begin{array}{rl}
47 \;\ldots\; 2 & \multirow{2}{*}{\} Multiply.} \\
\underline{61} \;\ldots\; \underline{7} & \\
47 \qquad 14 & \ldots\; 5 \\
\underline{282} & \\
2867 \;\ldots\ldots\ldots & \ldots\; 5
\end{array}$$

$$\begin{array}{rl}
(5)\quad 4567 \;\ldots\; 4 & \multirow{2}{*}{\} Multiply.} \\
\underline{293} \;\ldots\; \underline{5} & \\
13701 \qquad 20 & \ldots\; 2 \\
41103 & \\
\underline{9134} & \\
1338131 \;\ldots\ldots\ldots & \ldots\; 2
\end{array}$$

It is important to observe, that if there be an error which this test does not indicate, the error must be a multiple of nine.

DIVISION.

173.

$$\begin{array}{r}
2708 \\
498\overline{)1348708} \\
\underline{996} \\
3527 \\
\underline{3486} \\
4108 \\
\underline{3984} \\
124
\end{array}$$

That is, $1348708 = 498 \times 2708 + 124.$

But $1348708 = \text{a number of nines} + 4,$

by (5) $498 \times 2708 = \quad \text{“} \quad \text{“} \quad + 6,$

and $124 = \quad \text{“} \quad \text{“} \quad + 7.$

Therefore,

$$\begin{aligned} 498 \times 2708 + 124 &= \text{number of nines} + 6 + 7. \\ &= \quad \text{“} \quad \text{“} \quad + 4. \end{aligned}$$

Therefore, the two members of the equation $1348708 = 498 \times 2708 + 124$, when divided by 9, give the same remainder, namely, 4.

The work may be arranged as follows:

$$\begin{array}{rlrrl}
 & & 8 \;\ldots\ldots\ldots & 2708 & \\
 & & \underline{3} \;\ldots\ldots & 498\overline{)1348708} & \ldots\ldots\;\mathbf{4} \\
 & \left\{\begin{array}{l} 6 \;\ldots\ldots\; 24 \\ \\ \\ \text{} \\ \\ \\ \underline{7} \;\ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right. & & \begin{array}{r} \underline{996} \\ 3527 \\ \underline{3486} \\ 4108 \\ \underline{3984} \\ 124 \end{array} & \\
\text{Add} & & & & \\
\mathbf{4} \;\ldots\ldots & 13 & & &
\end{array}$$

CASTING OUT ELEVENS.

174. Even powers of 10 are multiples of 11, plus 1; odd powers of 10 are multiples of 11, minus 1. For example,

$100 = 9 \times 11 + 1$; $10{,}000 = 909 \times 11 + 1$, etc.
$10 = 11 - 1$; $1000 = 91 \times 11 - 1$; $10^5 = 9091 \times 11 - 1$, etc.

Therefore, if a number expressed by a digit in an odd (1st, 3d, etc.) place be divided by 11, the remainder will be equal to that digit. And a number expressed by a digit in an even (2d, 4th, etc.) place will lack that digit of being a multiple of 11. Hence, if a number expressed by two digits be divided by 11, the remainder equals digit in the odd place minus digit in the even place. (The digit in the odd place, when less than that in the even place, must be increased by 11).

Therefore, if *any* number be divided by 11, the remainder will be the same as if the sum (increased by a multiple of 11 if necessary) of the digits in the odd places minus the sum of the digits in the even places were divided by 11.

The proof by casting out elevens is similar to that by casting out nines; and if a process stands both tests, the only error possible would be a multiple both of 9 and of 11.

Multiply 67,853 × 2976, and test by casting out the elevens.

$$\begin{array}{r l}
67853 \;.\;.\;.\;.\; 5 & \multirow{2}{*}{$\left.\right\}$ Multiply.} \\
\underline{2976} \;.\;.\;.\;.\; \underline{6} & \\
407118 \qquad 30 \;.\;.\;.\;.\; 8 & \\
474971 & \\
610677 & \\
\underline{135706} & \\
201930528 \;.\;.\;.\;.\;.\;.\;.\;.\;.\; 8 &
\end{array}$$

In applying either test to decimals, disregard the decimal point. In questions involving large numbers, always apply one or both tests to the work.

Exercise I.

Express in words:

1. 327.244.
2. 80.9056.
3. .390012.
4. 20000.002.
5. .0000008.
6. 41.27105.

Write in figures:

7. Two hundred thirty-five and eight hundred thirty-five thousandths.
8. Seventy-four and two hundred three thousand six millionths.
9. Twelve hundred and eight thousand three ten-millionths.
10. Five thousand sixty-four millionths.
11. One million and four tenths.
12. Six hundred-millionths.

Multiply and divide:

13. 789.365 by 10; by 100; by 100,000.
14. .004 by 100; by 10,000; by 1000.
15. 436. by 1,000,000; by 1000; by 10.
16. .1 by ten; by ten millions.

Find the value of:

17. $21.3706 + 15.243 + 1.8954 + .026891 + 5.328 + 29.74$.
18. $57. + .0057 + 6.8 + 1200 + .847 + 159.2 + 3$.
19. $.0012 + 10 + 5.8281 + 5 + 39.43 + .6827 + 1$.
20. $23.9875 - 12.4764$; $35.14732 - 27.62815$.
21. $102.1274 - 83.072$; $39.801 - 17.9645$.
22. $30 - 5.2817$; $1.7 - .8469$.
23. $1 - .54237$; $100 - .00176$.
24. $24.271 - 3.6485 + 15.271 - 13.256 - 14.125$.
25. $52 + .52 - 17.8946 - 30.254 - .5 + 21.12$.

26. $41.289 \times .5$; $.268 \times .9$; $.112 \times .2$.
27. 2.435×4.23; 71.651×3.37; $.251 \times .04$.
28. $.0012 \times .005$; 2.26823×200; $5.6125 \times .0768$.
29. $.7 \times 7 \times .07$; $.15625 \times 23.7 \times .00192 \times 5$.
30. $(2.465 + 1.21) \times (3.2 - 2.89)$.
31. $(3.01)^2$; $(.045)^2$; $(.0081)^2$; $(5.1004)^3$; $(.76)^3$.
32. $(.125)^2 \times (.32)^3$.

Divide:

33. 291.84 by 6; .12936 by 12; 7.92801 by .9.
34. 58.383 by .39; .28744 by .08; 491.205 by .065.
35. 68.325 by 6.25; .732 by 1.6; 1208.88 by .438.
36. 498 by .0125; 7 by .007; 1000 by .0001.
37. .235 by 10.24; 27 by 12; .00507702 by .0283.
38. 89.3 by .00752; 74.1 by .0256; 1 by .128.
39. .39842 by 3.7164; 281.5 by 13.789; .0005 by .0028.
40. 63.04128 by 912.85; 287.209 by .00493; 2000 by .0059.

Exercise II.

Find the second members of the following equations:

1. $1.4 + 2.08 + 3.895 =$
2. $2.8 + 2.08 + .28 + .028 + .812 =$
3. $1.667 + 0.4 + 0.286 + 6.08 + .636 + 0.931 =$
4. $6.125 - .57 =$
5. $(4.625 + 1.146) - (1.2 + 3.571) =$
6. $6.913 - (2.85 - 0.937) =$
7. $24 - 2.4 + (5 - 3.508) - 3.092 =$
8. $10 - (4.25 - 2.5 + 2 - 0.625 - 0.4 - 2.02) - 0.295 =$
9. $1.5 \times .08 \times .5 =$
10. $.1204 \times .0168 \times 100 =$
11. $.04 \times 3.25 \times .06 =$
12. $36 \times .002 \times 2.05 \times .00765 =$
13. $.139 \times 28 + 42 \times .002 + 6 \times .004 - .05 \times 20 =$

14. $(10-1.25)\times 0.2+0.02\times 2.8+(80.3\times 0.1-5.3)\times 10 - 805.3\times .02=$

15. $28.3696\div 1.49=$

16. $.27\div .00225=$

17. $8.8779\div 175.8=$

18. $.0427\div 92.3=$

19. $.28744\div 800=$

20. $491.205\div 650=$

21. $68.325\div 6250=$

22. $.732\div 16{,}000=$

23. $1208.88\div .438=$

24. $2\div .01-(.2\div .02+.8\div 10)+36.48\div 8-(4\div .05 -2+.6\div 1.25)=$

25. $72.2\div 10-2\div(.5\div 1.60)+2.125\div(1.75-.5)=$

Exercise III.

1. What number subtracted 88 times from 80,005 will leave 13 as a remainder?
2. If 7 men can build a wall in 16 days, how many men will it take to build a wall three times as long in half the time?
3. How many minutes are there between 25 minutes past 8 in the morning and midnight?
4. The velocity of sound being 1090 feet per second, at what distance is a gun fired the report of which I hear 11 seconds after seeing the flash? (5280 feet make a mile.)
5. How long would it take to travel 30.2375 miles at the rate of 8.85 miles per hour?
6. The circumference of a circle being 3.1416 times the diameter, find the circumference of a circle whose diameter is 6.8 feet; also, find the diameter of a circle whose circumference is 20 inches.
7. How much wire will be required to make a hoop 30 inches in diameter, allowing 2 inches for the joining?
8. How many times would such a hoop turn in going half a mile?

9. Cork, whose weight is .24 of that of water, weighs 15 pounds per cubic foot. What is the weight of 6 cubic feet of oak, the weight of oak being .934 of that of water?
10. From what number can 847 be subtracted 307 times, and leave a remainder of 49?
11. What is the 235th part of 141,235?
12. What will 343 barrels of flour cost, at $6.37 a barrel?
13. 12 make a dozen, and 12 dozen make a gross. How many steel pens in 28 gross? What will a gross of eggs cost, at 27 cents a dozen?
14. How much must be added to $4429 in order to make the sum $43 \times \$241$?
15. What number deducted from the 26th part of 2262 will leave the 87th part of the same number?
16. At an ordinary rate, 123 words a minute, how long will it take a man to deliver a speech of 15 pages, each of 28 lines, and each line containing 11 words? How long would it have taken Daniel Webster to deliver the same speech, at the rate of 93 words a minute?
17. How long would it take a railway train to go from New York to San Francisco, 3310 miles, at the rate of 1973 feet a minute?
18. How long will it take to count a million, at the rate of 67 a minute?
19. If you put into a box 17 cents a day, including Sundays, beginning January 1 and ending July 4, how much money will there be in the box?
20. If a man's income is $3000 a year, and his daily expenses average $7.68, what does he save in a year?
21. In a question of division the quotient was 87.83, the divisor, 759. What was the dividend?

22. It is 3.1416 times as far round a wheel as across it. How many times will a wheel 4.5 feet across turn round in going 23 miles of 5280 feet each?

23. How many gallons of 231 cubic inches are contained in a cubic foot (1728 cubic inches)? in a bushel of 2150.42 cubic inches? How many cubic feet in a bushel? How many bushels in 31.5 gallons?

24. Seven children had left to them $7186 apiece; one died, and his share was divided among the surviving six. How much had each then?

25. What is the nearest number to 7196 that will contain 372 without a remainder?

26. How long will it take 2 men to do what 1 man can do in 6 days? what 4 men can do in 3 days? what 3 men can do in 4 days?

27. Divide $1.80 among Thomas, Richard, and Henry in such a way that Henry shall receive 3 cents for every 5 cents that Thomas gets, and Richard shall receive 2 cents for every 3 cents that Henry gets.

28. Divide $87.84 between B and C so that C shall get $19 as often as B gets $17.

29. Three partners received for goods: one, $371.63; the second, $285.40; the third, $411.91. They paid for the goods $879.34, and divided the balance equally among them. How much did each receive?

30. At 12 inches in a foot, how many inches long is a wall 35 feet in length? A brick and its share of mortar being 8.4 inches long, how many bricks in length is the wall?

31. A brick and mortar being 2.4 inches in height, how many bricks are required to build the wall 12 feet high, if the wall be two bricks wide.

32. What is the total weight of the wall, if a brick and its share of the mortar weigh 4.13 pounds? What is

the weight after a long rain, when the weight is increased to 4.27 pounds for each brick?

33. How many pounds does each foot in length of the wall weigh?

34. If 60.98 cubic inches of brick weigh 4 pounds, how many cubic inches of brick weigh 1 pound? How many pounds would a cubic foot (1728 cubic inches) weigh?

35. If a cubic foot of water weigh 62.5 pounds, how many times as heavy as water is brick?

36. Light moves through the air at 186,500 miles in a second. How many times can it go around the earth in a second, the distance round the earth being 24,897.714 miles?

37. Light moves through the air at 300,190 kilometers in a second. How many times can it go around the earth in a second, the distance round the earth being 40,007.5 kilometers?

38. A minute is 60 seconds. How many miles and how many kilometers can light travel through air in a minute?

39. An hour is 60 minutes. How many miles and how many kilometers can light travel in an hour?

40. The distance round the earth, given in Ex. 37, is measured on a north and south line. Around the equator the distance is 40,075.45 kilometers. How many times could light move round the equator in one minute?

41. Find the reciprocal of the difference between 31.24 and 31.23768.

42. The Hanoverian mile is 25,400 Hanoverian feet long, each foot being .9542 of an English foot. Find to four places of decimals the fraction that an English mile of 5280 English feet is of a Hanoverian mile.

43. Express in inches the length of a meter, given that a meter is one ten-millionth of a quarter of the earth's circumference, that the circumference is 3.14159 times the diameter, that the diameter is 7911.7 miles, and that a mile is 5280×12 inches.

44. How must a number be altered to double its reciprocal?

45. What effect is produced on the sum of two numbers, if each number is increased by the same number? What effect on the difference?

46. What effect is produced on the product of two numbers, if both numbers are multiplied by the same number? What effect on the quotient?

47. What effect is produced on the *remainder*, if both divisor and dividend are multiplied by the same number? If both are divided by the same number?

48. In going from one planet to another light probably moves faster than in air. Suppose it moves at 309,800 kilometers a second, how long would it take light to perform each of the following journeys:

Moon to Earth	375,500	kilometers.
Sun to Earth	147,250,000	"
Sun to Mercury	56,900,000	"
Sun to Venus	106,400,000	"
Sun to Mars	224,100,000	"
Sun to the Asteroids	400,000,000	"
Sun to Jupiter	765,400,000	"
Sun to Saturn	1,403,000,000	"
Sun to Uranus	2,817,000,000	"
Sun to Neptune	4,421,000,000	"
Sun to the nearest star	24,000,000,000,000	"

49. A kilometer is about .6214 of a mile. How many miles is each of the planets from the sun?

CHAPTER IX.*

MEASURES.

175. Continuous quantities (such as length, surface, bulk, value, heat) cannot be counted; they are measured.

176. To **measure** a quantity is to find how many times it contains a *known quantity* of the same kind, called the **unit of measure.**

177. The unit of measure for lengths is a **meter**; and from this are derived the units of surface, volume, and weight.

The meter was intended to be one ten-millionth of the distance from the equator to the north pole, but more careful measurements of meridians show that this distance is 10,001,887 meters.

178. The **standard meter**, as defined by law, is the length of a bar of very hard metal, carefully preserved at Paris, accurate copies of which are furnished to the governments of all civilized countries.

179. The principal units of measure are:

The meter (^{m}) for lengths;
The square meter (^{qm}) for surfaces;
The cubic meter (^{cbm}) for large volumes;
The liter (^{l}) (*leé-ter*) for smaller volumes;
The gram (^{g}) for weights.

*** Chapters IX. and XIV. may be taken or omitted, at the option of the teacher.**

180. All these units are divided and multiplied decimally, and the size of the measures thus produced is shown by one of seven prefixes; namely, *deka*, meaning 10; *hekto*, meaning 100; *kilo*, meaning 1000; *myria*, meaning 10,000; and *deci*, meaning 0.1; *centi*, meaning 0.01; *milli*, meaning 0.001.*

But, as in United States money we seldom speak of anything else than dollars and cents, so in other measures it is only those printed in **black letter** in this chapter that are in common use.

Money.

181. The unit of commercial values is the **dollar.** It is compared with the gram of gold by laws fixing the weight of gold which shall constitute a dollar; but these laws are changed from time to time. Coins are also made of silver, nickel, and bronze.

182.	Divisions	A mill	= 0.001	of a dollar.
		A **cent** (ct.)	= 0.01	" "
		A dime	= 0.1	" "
	The unit,	A **dollar** ($).		
	Multiple,	An eagle	= 10 dollars.	

Cents are made of bronze; half-dimes of nickel; dimes, quarter-dollars, half-dollars, and dollars, of silver; quarter-eagles, half-eagles, eagles, and double eagles, of gold. But the eagle is usually called ten dollars, and the dime ten cents. That is to say, $137.875 is read 137 dollars 87 and a half cents, and *not* 13 eagles 7 dollars 8 dimes 7 cents 5 mills.

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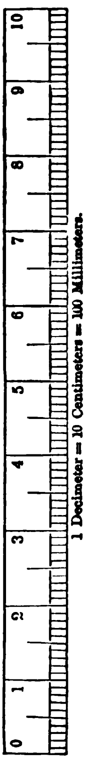
* All the compound names are accented on the first syllable, thus: mil′limeter.

Linear Measures.

183. Divisions	A **millimeter** (^{mm})	= .001	of a meter.
	A **centimeter** (^{cm})	= .01	" "
	A decimeter	= .1	" "
The unit,	A **meter** (^{m}).		
Multiples	A dekameter	= 10	meters.
	A hektometer	= 100	"
	A **kilometer** (^{km})	= 1,000	"
	A myriameter	= 10,000	"



184. A length given in any one of these measures may be expressed in terms of another measure by simply moving the decimal point to the right or left.

Thus, 17,856,342^{mm} may be written as **kilo**-meters by observing that **milli**-meters are changed to meters by moving the point three places to the left; and these meters into **kilo**-meters by carrying it three places further, making, in all, six places. Therefore, 17,856,342^{mm} = 17.856342^{km}.

Again, 4.876326^{km} may be written as centimeters, by observing that **kilo**-meters are changed to meters by moving the point three places to the right, and meters to **centi**-meters by moving it two places further, making, in all, five places. Therefore, 4.876326^{km} = 487,632.6^{cm}.

185. The rule, therefore, for this conversion is: First change the point so as to convert the **given measures** into terms of the **principal unit**; then change the point so as to convert the principal into the **required** units.

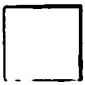
186. Remember that, before adding or subtracting, the quantities must be written in the **same units of measure.**

1. Convert 5427^{m} into kilometers; into millimeters; into centimeters.
2. 6853^{mm} contain how many meters? how many centimeters? what part of a kilometer?
3. Write 49.7^{m} as centimeters; as millimeters; as part of a kilometer.
4. How many centimeters in 12.4^{km}? how many millimeters?
5. Change 1230 meters into kilometers; into centimeters.
6. Write 1230^{cm} as meters; as millimeters.

Find the value of each of the following expressions in meters:

7. $.435^{m} + 852^{cm} + 4263^{mm} + .1595^{km}$.
8. $.927^{km} - 6495^{cm}$; $4.37^{cm} - 42.87^{mm}$.
9. $8 \times .0457^{km}$; 3.04×60.93^{cm}; 5.43×67.2^{mm}.
10. $38{,}019^{mm} \div .097$; $.41^{km} \div 25.625$.
11. At \$1.87 the meter what is the cost of 6.20^{m} of cloth?
12. At \$.75 the meter what is the cost of 60^{m} of cloth?
13. From a piece of cloth containing 47.60^{m} a tailor cuts off three pieces: the first of 3.80^{m}, the second of 1.30^{m}, and the third of 45^{cm}. How much of the cloth is left?
14. What is the value of 60^{cm} of cloth, worth \$5.20 a meter?
15. If \$6.00 are paid for a railroad ticket to travel 440^{km}, what is the fare per kilometer?
16. If a train run 288^{km} in 9 hours, how many meters does it run in a minute?
17. If a man walk at the rate of 6^{km} an hour, what part of an hour will it take to walk 420 meters?
18. A railroad carried 412 passengers 18 kilometers, and received \$88.992; at the same rate, what will it receive for carrying 350 passengers 35 kilometers?

Measures of Surface.

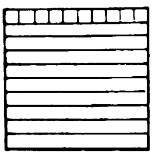


Square Centimeter.

187. The unit of surface is a **square,** each side of which is a linear unit.

The principal unit of surface is, therefore, a **square meter** (^{qm}).

188. But in square measure, the multiplication and division of units is by hundreds and hundredths, instead of by tens and tenths. Suppose the square in the margin to represent a **square meter.** It is divided into ten equal horizontal bands, and each band is one-tenth of the square meter. Each band can be divided, as the upper one is, into ten little squares measuring one-tenth of a meter on a side. Each of these squares will be .1 of the band, or .01 of the whole square. The **square meter,** therefore, contains 10 × 10 or 100 **square decimeters.**



If the square meter were divided into 100 equal horizontal bands, each band would be .01 of the square; and if each of the 100 bands were divided into 100 squares, that is, into 100 square centimeters, the whole square would contain 100 × 100 or 10,000 square centimeters. A **square meter,** therefore, contains 10,000 **square centimeters.**

If the square were divided into 1000 equal bands, each band would be .001 of the square; and if each of the 1000 bands were divided into 1000 squares, that is, into 1000 square millimeters, the whole square would contain 1000 × 1000 or 1,000,000 square millimeters. A **square meter,** therefore, contains 1,000,000 **square millimeters.**

That is,

189.

A square millimeter (^{qmm})	= .000001	of a square meter.		
A **square centimeter** (^{qcm})	= .0001	"	"	"
A square decimeter	= .01	"	"	"
A **square meter** (^{qm})	**Principal Unit.**			
A square dekameter	= 100	square meters.		
A square hektometer	= 10,000	"	"	
A **square kilometer** (^{qkm})	= 1,000,000	"	"	

It will be observed that while centimeters are in the second and millimeters in the third decimal place from meters, **square centimeters** are in the **fourth** and **square millimeters** in the **sixth** decimal place from **square meters.**

190. In the measurement of land, the **square dekameter** is called an **ar** (^{a}), and the **square hektometer** is called a **hektar** (^{ha}).

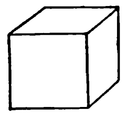
191. Be careful, in converting square measures from one unit to another, to observe that, when the **ar** is the unit, the point is to be moved as in linear measure; but when the **square meter** is the unit, the point is to be moved **twice as far** as in linear measure. Thus, it takes 100 **ars** to make a **hektar**; but 100 × 100, or 10,000 **square centimeters** to make a **square meter**, and 100 × 100, or 10,000 **square meters** to make a square hektometer, or **hektar.**

1. Convert 1,854,276^{qm} into hektars; into square kilometers.
2. How many hektars in 2.7856 square kilometers?
3. Write 1.7431^{qm} as square centimeters; as square millimeters.
4. How many square kilometers in 17,467.5 hektars?
5. How many square meters in 1.3614^{qkm}?
6. How many square meters in 2.25 hektars?

7. How many square centimeters in 0.0137 of a square meter?

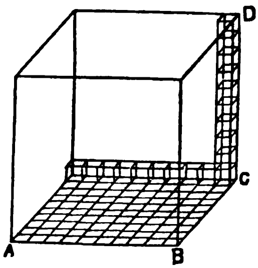
8. Write 3.571^{qcm} as square millimeters.

Measures of Volume.


Cubic Centimeter.

192. The measure of capacity, or volume, is a cube, each face of which is a square unit.

193. It is to be observed that the cubic meter can be divided into 10 layers, each a meter square and a decimeter thick. Each layer will, therefore, be .1 of a cubic meter.



Again, each layer can be divided into 10 equal parts. Each part will, therefore, be .1 of the layer, or .01 of the meter, and will be a decimeter square and a meter long.

Also, each one of these parts can be divided into 10 equal parts, each of which will be a cubic decimeter, and will be .1 of .01, or .001 of the cubic meter.

The **cubic meter,** therefore, contains 1000 **cubic decimeters.**

In like manner, each cubic decimeter can be divided into 1000 cubic centimeters, and each cubic centimeter into 1000 cubic millimeters.

That is,

194.

A cubic millimeter (^{cmm})	= 0.000000001	of a cubic meter.		
A cubic centimeter (^{ccm})	= 0.000001	"	"	"
A cubic decimeter	= 0.001	"	"	"
A cubic meter (^{cbm}).	**Principal unit.**			

It will be seen that in cubic measure, the decimal point is to be moved **three times as far,** when changing the unit, as in linear measure.

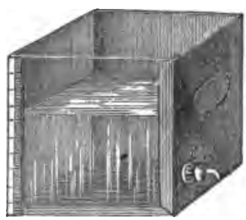
1. How many cubic centimeters in 2.25^{cbm}?
2. How many cubic meters in $2{,}162{,}875^{ccm}$?

Ster of Wood.

195. In measuring wood the cubic meter is called a **ster** (st).

196. In measuring liquids, grain, etc., the **cubic decimeter** is always called a **liter.**

If the water in the liter represented in the margin stands 6^{cm} high, how many *cubic centimeters* of water are there in the measure? How many will be required to fill it? If the faucet be turned and the water allowed to run out until the measure is only half full, how many cubic centimeters will run out? How many will still remain?



Liter = Cubic Decimeter.

When the liter is the unit, the numeral prefixes have the same value as in linear measure. Thus,

197.

Divisions	A milliliter	= .001	of a liter.
	A centiliter	= .01	" "
	A deciliter	= .1	" "
The unit,	A **liter** (^{l}).		
Multiples	A dekaliter	= 10	liters.
	A **hektoliter** (^{hl})	= 100	"
	A kiloliter	= 1000	"

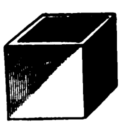
1. How many liters in 1.7^{cbm}? in $157,854^{ccm}$?

Liter (common form).

2. How many cubic centimeters in 9.5^{l}? in $.015^{l}$?
3. Change 1.25^{hl} to cubic centimeters; to the fraction of a cubic meter.
4. Convert 431.88^{l} into hektoliters; into the fraction of a cubic meter.
5. Write $.375^{cbm}$ as liters; as cubic centimeters.
6. Write $734,159.651^{ccm}$ as liters; as hektoliters; as cubic meters.
7. How many cubic meters in $8,573,412.867^{ccm}$?
8. Change the expression $.734578912^{cbm}$ into cubic centimeters; into liters.
9. Change 1731.5 liters into cubic meters; into cubic centimeters.

Weights.

198. The units of weight are the weights of units of pure water taken at its greatest density, that is, a little above the freezing point.



Cubic Centimeter.

Gram Weight.

The principal unit is the **gram**, which is the weight of a **cubic centimeter** of water.

The numeral prefixes have the same value as in linear measure. Thus,

199.

Divisions	A **milligram** (^{mg})	= 0.001	of a gram.
	A centigram	= 0.01	" "
	A decigram	= 0.1	" "
The unit,	A **gram** (^{g}).		
Multiples	A dekagram	= 10	grams.
	A hektogram	= 100	"
	A **kilogram** (^{kg})	= 1000	"
	A **metric ton** (^{t})	= 1000	kilograms.

200. A **cubic centimeter** of water weighs a **gram.**
A **liter** of water weighs a **kilogram.**
A **cubic meter** of water weighs a **ton.**

Kilogram Weight.

1. How many kilos* in 1.73^{t}? in 0.341 of a ton?
2. How many kilos will a hektoliter of water weigh?
3. Convert $13{,}756^{mg}$ into grams; into the fraction of a kilo.
4. What is the weight in grams of 346.1^{ccm} of water?
5. Give the weight in kilograms of 0.37615^{cbm} of water.
6. Change $.6778^{kg}$ into milligrams.
7. How many milligrams in the third part of 17.4 grams?

* Kilogram is generally called kilo.

EXERCISE IV.

1. Add 17.3^{m}, 87.41^{m}, 271^{cm}, 380^{mm}, and 1.79^{m}.
2. What is the sum of \$15.87, \$39.46, \$47.52, \$75.38, \$75.89?
3. Add 187^{cm}, 49.3^{m}, 317^{mm}, and 6.138^{m}.
4. The door-sill being 3^{cm} high; the door, 2.34^{m}; the finish over it, 13.7^{cm}; and the distance from finish to ceiling, 93^{cm}; how far from floor to ceiling?
5. The distance to the post-office is 3.31^{km}; thence to the mill, 1.711^{km}; thence to the store, 3.718^{km}; thence home, 2.543^{km}. How long is the circuit?
6. From Portland, Me., to Boston is about 132^{km}; Boston to Albany, 320^{km}; Albany to Buffalo, 480^{km}; Buffalo to Chicago, 800^{km}; Chicago to Omaha, 800^{km}; Omaha to Cheyenne, 780^{km}; how far from Cheyenne to Portland? to Albany? from Boston to Chicago? from Boston to Cheyenne?
7. If I travel 789.7^{km} a day, how far shall I go in 7 days? in 8.5? in 19.6? in 27.8? in 365 days?
8. How much will 3^{m} of cloth cost, at \$1.37 a meter? How much 5.38^{m}, at \$2.63 a meter?
9. How much will 13.4^{kg} of opium be worth, at \$8.48 a kilo? 28.79^{kg}, at \$7.96 a kilo?
10. A man bought 153 barrels of flour, at \$4.875 a bbl. What did the whole cost him?
11. He gave for it 6 shares of stock, at \$113.50 a share, and the rest in cash. How much money did he pay?
12. He paid \$13.75 for storage; also, 75 cents a barrel for freight. How much do these expenses amount to?
13. It was sold, 49 bbls. at \$6.50 a bbl.; the rest at \$6.25. What were the gross receipts?
14. He paid for commissions, etc., \$17.50; and counts his loss of interest at \$29.30. What then is his net profit?

201. *If the diameter of a circle be multiplied by* 3.1416, *the product is the length of the circumference.* (For great accuracy the diameter is multiplied by 3.1415927.)

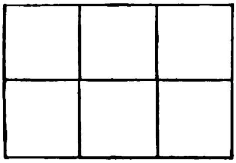
15. Find the circumference of a circle having a diameter of 1^{m}.
16. Find the circumferences of circles of which the diameters are respectively 83^{m}; 3.71^{m}; 32.8^{m}; 10.4^{cm}; 11.8^{cm}; 167.1^{mm}; 39.3^{mm}. Give each to the nearest tenth of a millimeter.
17. What is the length of the earth's orbit, to the nearest meter, if the diameter of the orbit is $294,481,217^{km}$?
18. How far round this world, if its diameter is $12,734^{km}$?
19. If a carriage-wheel is 1.31^{m} in diameter, what is its circumference? How far will it go, if it roll without slipping, in turning once? 17 times?
20. How often must that wheel turn in going 69.429^{m}? 73.513^{m}? 17.27^{km}?
21. Find the reciprocal of 3.14159 to the 5th place.

202. *If, therefore, a circumference be multiplied by* .31831, *the product is the diameter.*

22. What is the diameter of the circle whose circumference is 314.159^{cm}?
23. What is the diameter of the wheel which revolves 19.5 times in going 107.25^{m}?
24. How thick through is a tree which has a girth of 2.97^{m}?
25. What is the diameter of a circular field two kilometers in circumference?
26. What is the diameter of a rope of which the circumference is 20^{cm}?
27. In a park is a fountain whose basin is 75^{m} in circumference. What is the diameter of the basin?

Measures of Surface.

203. When a surface is flat, and has four square corners, it is called a rectangle. Suppose the rectangle in the margin is 3^{cm} long and 2^{cm} wide. If lines be drawn as represented in the figure, the surface will be divided into **square centimeters.** There will be 2 horizontal rows of 3 square centimeters each; that is, in all, 2 times 3 square centimeters. Hence,



Express the length and breadth of a rectangle in the **same linear unit**; *the product of these two numbers will be its area in* **square units** *of the same name as the linear unit of the sides.*

Conversely, the number of square units in a rectangle, divided by the number of linear units in one side, will give the number of linear units in the other side.

Exercise V.

1. Find the area of a rectangle 17^{cm} by 19^{cm}.
2. In a rectangular township 16^{km} by 7^{km}, how many hektars? If there are in it 47.3^{km} of highway, averaging 11.7^{m} wide; how much land is left for other uses?
3. In a rectangular field, 751.3^{m} long and 189.3^{m} wide, is a strawberry bed 31.4^{m} by 17.8^{m}. How many hektars in the field? How many, exclusive of the strawberry bed?
4. If my garden contain 941.65^{qm}, and my neighbor's 748.37^{qm}, what is the area of both in hektars?
5. If a painter can cover 8.786^{qm} in an hour, how much can he cover in 1.78 hours? in 3.86 hours? in 4.57 hours?

6. How many hektars in each of three rectangular fields: one measuring 315.71^m by 78.91^m; a second, 293.6^m by 84.84^m; the third, 346.8^m by 71.82^m. How many in the three?
7. Give the price of each field, and of the whole, at $67.50 a hektar; at $384 a hektar; and at $2.375 a square meter.

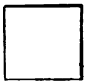
204. If the area of a square be multiplied by .7854 it gives the area of the largest circle which can be drawn in that square. In other words, *multiply the second power of the diameter by* .7854 (that is, by ¼ of 3.1416), *and the product is the area of the circle in* **square units** *of the same name as the* **linear unit** *of the diameter.*

8. What is the area of a circle 27^{cm} in diameter? of one which is 1^m in diameter?
9. What is the area in hektars of a circular field 784^m across?
10. Give the area of a circle 31^{cm} in diameter.
11. Find the length of a rectangle 17^{cm} wide, and containing 306^{qcm}. What length of carpet 75^{cm} wide is required to make 27^{qm}?
12. A room is 16^m long, 8^m wide, and 8^m high; another room is 7^m long, 7^m wide, and 3^m high. How many square meters of painting on the walls of both rooms, if no allowance is made for doors and windows? How many more square meters of painting on the walls of the larger room than on those of the smaller?

205. The surface of a sphere or globe is four times that of a circle of the same diameter. Therefore,

To find the surface of a sphere, multiply the square of the diameter by 3.1416.

Measures of Surface.

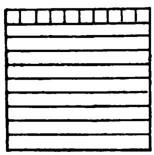


Square Centimeter.

187. The unit of surface is a **square**, each side of which is a linear unit.

The principal unit of surface is, therefore, a **square meter** (^{qm}).

188. But in square measure, the multiplication and division of units is by hundreds and hundredths, instead of by tens and tenths. Suppose the square in the margin to represent a **square meter.** It is divided into ten equal horizontal bands, and each band is one-tenth of the square meter. Each band can be divided, as the upper one is, into ten little squares measuring one-tenth of a meter on a side. Each of these squares will be .1 of the band, or .01 of the whole square. The **square meter,** therefore, contains 10×10 or 100 **square decimeters.**



If the square meter were divided into 100 equal horizontal bands, each band would be .01 of the square; and if each of the 100 bands were divided into 100 squares, that is, into 100 square centimeters, the whole square would contain 100×100 or 10,000 square centimeters. A **square meter,** therefore, contains 10,000 **square centimeters.**

If the square were divided into 1000 equal bands, each band would be .001 of the square; and if each of the 1000 bands were divided into 1000 squares, that is, into 1000 square millimeters, the whole square would contain 1000×1000 or 1,000,000 square millimeters. A **square meter,** therefore, contains 1,000,000 **square millimeters.**

That is,

189.

A square millimeter (^{qmm})	= .000001	of a	square	meter.
A **square centimeter** (^{qcm})	= .0001	"	"	"
A square decimeter	= .01	"	"	"
A **square meter** (^{qm})	**Principal Unit.**			
A square dekameter	= 100	square meters.		
A square hektometer	= 10,000	"	"	
A **square kilometer** (^{qkm})	= 1,000,000	"	"	

It will be observed that while centimeters are in the second and millimeters in the third decimal place from meters, **square centimeters** are in the **fourth** and **square millimeters** in the **sixth** decimal place from **square meters.**

190. In the measurement of land, the **square dekameter** is called an **ar** (^{a}), and the **square hektometer** is called a **hektar** (^{ha}).

191. Be careful, in converting square measures from one unit to another, to observe that, when the **ar** is the unit, the point is to be moved as in linear measure; but when the **square meter** is the unit, the point is to be moved **twice as far** as in linear measure. Thus, it takes 100 **ars** to make a **hektar;** but 100 × 100, or 10,000 **square centimeters** to make a **square meter,** and 100 × 100, or 10,000 **square meters** to make a square hektometer, or **hektar.**

1. Convert 1,854,276^{qm} into hektars; into square kilometers.
2. How many hektars in 2.7856 square kilometers?
3. Write 1.7431^{qm} as square centimeters; as square millimeters.
4. How many square kilometers in 17,467.5 hektars?
5. How many square meters in 1.3614^{qkm}?
6. How many square meters in 2.25 hektars?

Papering and Plastering.

208. The area of the **four walls** of a room is equal to that of a rectangle whose length is the **perimeter** of the room, and whose breadth is the **height** of the room.

Perimeter = twice the length + twice the breadth.
Area = **perimeter × height.**

30. Find the area of the walls of a room whose length is 6.12^{m}, breadth 5.05^{m}, and height 3.5^{m}?

$$\text{Perimeter} = 2\,(6.12^{m} + 5.05^{m}) = 22.34^{m},$$
$$\text{Area} = 22.34^{m} \times 3.5^{m} = 78.19^{qm}.$$

31. How many rolls of paper 45^{cm} wide and 8^{m} long, allowing 11.19^{qm} for doors and windows, will be required to paper this room?
32. Find the cost of papering a room 8^{m} long, 5.5^{m} wide, and 4.5^{m} high, with paper 50^{cm} wide and 7.5^{m} in a roll, at \$1.25 a roll, put on? There is a base-board 25^{cm} wide running round the room, and an allowance of 11^{qm} is made for doors and windows.
33. Find the cost of plastering this room, at \$.50 a square meter.
34. Find the cost of papering a room 5.5^{m} long, 4.8^{m} wide, and 3.2^{m} high, with paper 45^{cm} wide, 7.5^{m} in a roll, at \$.875 a roll, put on, allowing 12^{qm} for base-board, doors, etc.
35. Find the cost of plastering this room, at \$.45 a square meter.
36. Find the cost of papering a room 6^{m} square and 3.5^{m} high, with paper 45^{cm} wide and 7.5^{m} in a roll, at \$.75 a roll, put on; and of putting on a border, at 5 cents per running meter.
37. Find the cost of plastering this room, at \$.36 a square meter.

38. Find the cost of papering a room 13^m long, 12^m wide, and 7^m high, with paper 45^{cm} wide and 7.5^m in a roll, at $1.50 a roll, put on; and of putting on a border, at $.30 a running meter, allowing 115^{qm} for base-boards, doors, etc.
39. Find the cost of plastering this room, at $.60 a square meter.

Board Measure.

209. When boards are 25^{mm} or less in thickness, they are sold by the square meter. Likewise, boards more than 25^{mm} thick, plank and "dimension" lumber are sold by board measure. That is, by calculating the number of square meters of boards 25^{mm} thick to which each piece is equivalent.

Thus, a board 4^m long, 25^{cm} wide, and 25^{mm} or less in thickness, is a meter, board measure; but a plank 4^m long, 25^{cm} wide, and 75^{mm} thick, is three meters, board measure. For, since the plank is three times as thick as an ordinary board, it is equivalent to three boards, each 4^m long and 25^{cm} wide.

If a board tapers regularly, its average width is found by taking one-half the sum of its end widths.

How many meters, board measure:

40. In a board 8^m long and 20^{cm} wide?
41. In a joist 5^m long, 25^{cm} wide, and 75^{mm} thick?
42. In a stick of timber 15^m long and 40^{cm} square?
43. In 2 joists 5^m long, 27.5^{cm} wide, and 50^{mm} thick?
44. In 10 planks, each 4^m long, 45^{cm} wide, and 10^{cm} thick? and what is the value of these planks, at $25 a hundred meters?
45. In 25 box boards, each 4^m long, 42^{cm} wide, and 20^{mm} thick? and what is their value, at $14 a hundred meters?

Find the cost of:

46. Ten joists 4.5^{m} long, 10^{cm} wide, and 7.5^{cm} thick, at $11 a hundred meters.
47. Thirty-six planks, each 4^{m} long, 27.8^{cm} wide, and 75^{mm} thick, at $16 a hundred meters.
48. Three sticks of timber, each 8^{m} long, 22.5^{cm} wide, and 20^{cm} thick, at $17.50 a hundred meters.
49. A board 8.25^{m} long, 28^{cm} wide at one end and 35^{cm} at the other, and 31.25^{mm} thick, at $.30 a meter?
50. A stick of timber 10^{m} long, 25^{cm} thick, 30^{cm} wide at one end and 25^{cm} wide at the other, at $14 a hundred meters.
51. The flooring for a two-story building 16^{m} by 10.5^{m}; the flooring being 32^{mm} thick, and worth $30 a hundred meters.
52. The flooring timbers for this building, the timbers being 25^{cm} by 50^{mm}, and placed on edge 30^{cm} apart, and worth $11.50 a hundred meters.
53. The fencing to enclose a field 150^{m} long and 75^{m} wide; the posts are set 2.5^{m} apart, and cost $.25 apiece; the fence is 5 boards high; the bottom board is 30^{cm}, the top board 25^{cm}, and the other three each 22.5^{cm} wide, and the boards cost $13.25 a hundred meters.

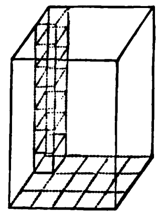
NOTE. Round logs are sold by board measure; .25 is deducted for slabs. A scale is used, the calipers of which are applied to the middle of the log, and the amount of boards for a given length is read off the scale.

Large and heavy timber is sold by the ton; only .20 is allowed for slabs, and the amount is reckoned in cubic measure instead of board measure.

MEASURES OF VOLUME.

To find the volume of a rectangular solid.

210. In the figure represented in the margin, let the length contain 5, the breadth 3, and the height 7 *linear centimeters.*

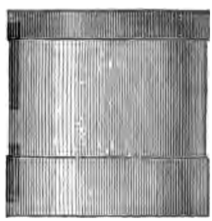


The base may be divided into square centimeters: there will be three rows of 5 square centimeters each; in all, 15 square centimeters. Upon each square centimeter may be placed a pile of 7 cubic centimeters; so that the solid will contain 15×7 cubic centimeters, that is, $3 \times 5 \times 7$ *cubic centimeters.* Hence,

Express the length, breadth, and height of a rectangular solid in the **same linear unit**; *the product of these numbers will be its volume in* **cubic units** *of the same name as the linear unit of the edges.*

EXERCISE VI.

1. How many cubic centimeters in a block 9^{cm} long by 7^{cm} wide, and 6^{cm} deep?
2. If wood is cut into 120^{cm} lengths, and a pile is 43.7^{m} long and 1.4^{m} high, how many sters of wood are there in it?



Hektoliter.

3. In a grain elevator is a bin 11.2^{m} long, 4.34^{m} wide, and 2.83^{m} deep. How many hektoliters of grain will it hold?
4. If a liter of grain weigh .81 of the weight of a liter of water, how much will the grain in that bin weigh?

7. How many kilograms, and how many tons, would 3.6175^{cbm} of brick weigh, at 2 tons to a cubic meter? at 2.34 tons?
8. From a barrel containing 67^{kg} of granulated sugar were taken three parcels of 2.75^{kg} each, and four parcels of 7.50^{kg} each. How much is left in the barrel?
9. Into how many pills of 325^{mg} each can a mass of 7.8^{g} be divided?
10. A mass of 21.8^{g} is divided into 60 pills. What is the weight of each pill?

SPECIFIC GRAVITY.

212. *The* **specific gravity** *of any substance is* **the number of times** *the weight of the substance contains the weight of an equal bulk of water.*

Thus, if a sample of quicksilver has a specific gravity of 13.6, it is 13.6 times as heavy as water; a cubic centimeter of it would weigh 13.6^{g}; a liter of it would weigh 13.6^{kg}; and a cubic meter of it would weigh 13.6^{t}.

Again, if the specific gravity of a certain alcohol is .827, that is, if the alcohol weighs .827 as much as an equal bulk of water, then a cubic centimeter of it would weigh $.827^{g}$; a liter, $.827^{kg}$; and a cubic meter, $.827^{t}$.

213. Therefore, the **specific gravity** of a substance is the **number** that expresses the **weight** of a **cubic centimeter** of it in **grams**; or of a **liter** in **kilograms**; or of a **cubic meter** in **tons**.

214. When a substance heavier than water is under water, the water buoys it up just the amount of the weight of the water displaced by it. I weigh a lump of coal suspended by a thread; it weighs 1017^{g}. I then hang it, still suspended to the balance, in a pail of water, and lower it

until the coal is covered with water; it now weighs 531^{g}, and has lost 486^{g}. The first information I get is, that the lump contains 486^{ccm} of coal, for it displaces 486^{ccm} of water. Then to obtain its specific gravity, that is, its weight compared with that of water as unity (§ 166), we have $1017 \div 486 = 2.092$ as the specific gravity of the piece of coal.

A piece of stone weighing 1.1^{kg} in air and $.6^{kg}$ in water, is bound by a thread to a block of wood; the two together weigh 1.28^{kg} in air and $.54^{kg}$ in water. What is the specific gravity of the wood?

The weight of the wood in air $= 1.28^{kg} - 1.1^{kg} = .18^{kg}$.

The weight of water displaced by stone and wood $= 1.28^{kg} - .54^{kg} = .74^{kg}$.

The weight of water displaced by stone alone $= 1.1^{kg} - .6^{kg} = .5^{kg}$.

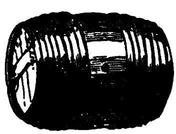
The weight of water displaced by wood, therefore, $= .74^{kg} - .5^{kg} = .24^{kg}$.

Hence, the specific gravity of the wood $= .18^{kg} \div .24^{kg} = .75$.

Ans. 0.75.

Exercise VIII.

1. If a stone weighs 1.3^{kg} in air and $.68^{kg}$ in water, and the the stone and a block of wood together weigh 1.55^{kg} in air and $.63^{kg}$ in water, what is the specific gravity of the block of wood?
2. What is the weight of 8.17^{hl} of alcohol, specific gravity .83?
3. What will 97^{l} alcohol weigh, of specific gravity .817? of specific gravity .819? of specific gravity .823? .838? .847?



Hektoliter.

4. A bar of aluminum 113^{mm} long, 17^{mm} wide, and 13^{mm} thick, is said to be of specific gravity 2.57. What does it weigh? If it really is of specific gravity 2.67, what does it weigh?

13. How many square centimeters of surface on a ball 7^{cm} in diameter?

14. How many square centimeters of surface on a ball 18^{cm} in diameter?

15. How many square meters of surface on a hemispherical dome 11.27^{m} in diameter?

16. What is the interior surface of a hemispherical basin 12^{cm} in diameter?

17. What is the interior surface of a hemispherical vase 70^{cm} in diameter?

206. *Half the diameter is the radius; and* 3.1416 *times the square of the radius is the area of the circle.*

18. Find, by this rule, the area of example 9.

19. How many square centimeters are inclosed in a circle struck with a radius of 7^{cm}?

20. In a sheet of zinc 1.76^{m} long and 89^{cm} wide are two circular openings, one of which has a radius 10.5^{cm}, the other a radius 9.2^{cm}. What is the area of the zinc left?

21. What is the area of a circle of which the radius is 24^{m}?

22. A piece of land in the form of a circle has a radius of 40^{m}; in the middle of it is a pond forming a circle of 15^{m} radius. What is the total surface? the surface of the pond? the surface of the land to cultivate?

Carpeting Rooms.

207. Carpeting is made of various widths and is sold by the length.

In determining the number of meters required for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed. The number of meters in a strip multiplied by

the number of strips will give the required number of meters, without allowance for waste in matching patterns.

23. How many meters of carpet 60^{cm} wide will be required for a room 6^m long and 5.4^m wide, the strips running lengthwise? how many meters would be required if the carpet were 80^{cm} wide?

Since the room is 540^{cm} wide, it will take $\frac{540}{60}$, or 9 widths of carpet 60^{cm} wide; that is, the amount required will be $9 \times 6^m = 54^m$. If the carpet were 80^{cm} wide, it would take $\frac{540}{80}$, or 7 widths;* that is, the amount required will be $7 \times 6^m = 42^m$.

24. How many meters of carpet 56^{cm} wide will be required for a room 8.32^m long and 6.6^m wide, strips running lengthwise?

25. How many meters of carpet 70^{cm} wide will be required for a room 7^m long and 5.4^m wide, strips running across the room?

26. How many meters of carpet 80^{cm} wide will be required for a room 6^m long and 5.47^m wide, strips running across the room?

27. How many meters of carpet 90^{cm} wide will be required for a room 5^m long and 4.5^m wide, strips running lengthwise? How much will it cost, at $1.875 a meter?

28. How many meters of carpet 75^{cm} wide will be required for a room 5.25^m long and 4.75^m wide, strips running across the room? How much will it cost, at $2.125 a meter?

29. How many meters of carpet 75^{cm} wide will be required for a room 5.6^m square? How wide a strip will have to be turned under? How much will the carpet cost, at $1.25 a meter?

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* Six widths would leave a surface 60^{cm} wide to be covered. This surface would require another strip, of which a width of 20^{cm} would be "turned under." It is economy to select carpeting of the right width for the room.

Papering and Plastering.

208. The area of the **four walls** of a room is equal to that of a rectangle whose length is the **perimeter** of the room, and whose breadth is the **height** of the room.

Perimeter = twice the length + twice the breadth.
Area = **perimeter × height.**

30. Find the area of the walls of a room whose length is 6.12^{m}, breadth 5.05^{m}, and height 3.5^{m}?

$$\text{Perimeter} = 2\,(6.12^{m} + 5.05^{m}) = 22.34^{m},$$
$$\text{Area} = 22.34^{m} \times 3.5^{m} = 78.19^{qm}.$$

31. How many rolls of paper 45^{cm} wide and 8^{m} long, allowing 11.19^{qm} for doors and windows, will be required to paper this room?

32. Find the cost of papering a room 8^{m} long, 5.5^{m} wide, and 4.5^{m} high, with paper 50^{cm} wide and 7.5^{m} in a roll, at $1.25 a roll, put on? There is a base-board 25^{cm} wide running round the room, and an allowance of 11^{qm} is made for doors and windows.

33. Find the cost of plastering this room, at $.50 a square meter.

34. Find the cost of papering a room 5.5^{m} long, 4.8^{m} wide, and 3.2^{m} high, with paper 45^{cm} wide, 7.5^{m} in a roll, at $.875 a roll, put on, allowing 12^{qm} for base-board, doors, etc.

35. Find the cost of plastering this room, at $.45 a square meter.

36. Find the cost of papering a room 6^{m} square and 3.5^{m} high, with paper 45^{cm} wide and 7.5^{m} in a roll, at $.75 a roll, put on; and of putting on a border, at 5 cents per running meter.

37. Find the cost of plastering this room, at $.36 a square meter.

38. Find the cost of papering a room 13^m long, 12^m wide, and 7^m high, with paper 45^{cm} wide and 7.5^m in a roll, at $1.50 a roll, put on; and of putting on a border, at $.30 a running meter, allowing 115^{qm} for baseboards, doors, etc.

39. Find the cost of plastering this room, at $.60 a square meter.

Board Measure.

209. When boards are 25^{mm} or less in thickness, they are sold by the square meter. Likewise, boards more than 25^{mm} thick, plank and "dimension" lumber are sold by board measure. That is, by calculating the number of square meters of boards 25^{mm} thick to which each piece is equivalent.

Thus, a board 4^m long, 25^{cm} wide, and 25^{mm} or less in thickness, is a meter, board measure; but a plank 4^m long, 25^{cm} wide, and 75^{mm} thick, is three meters, board measure. For, since the plank is three times as thick as an ordinary board, it is equivalent to three boards, each 4^m long and 25^{cm} wide.

If a board tapers regularly, its average width is found by taking one-half the sum of its end widths.

How many meters, board measure:

40. In a board 8^m long and 20^{cm} wide?

41. In a joist 5^m long, 25^{cm} wide, and 75^{mm} thick?

42. In a stick of timber 15^m long and 40^{cm} square?

43. In 2 joists 5^m long, 27.5^{cm} wide, and 50^{mm} thick?

44. In 10 planks, each 4^m long, 45^{cm} wide, and 10^{cm} thick? and what is the value of these planks, at $25 a hundred meters?

45. In 25 box boards, each 4^m long, 42^{cm} wide, and 20^{mm} thick? and what is their value, at $14 a hundred meters?

Find the cost of:

46. Ten joists 4.5^m long, 10^{cm} wide, and 7.5^{cm} thick, at \$11 a hundred meters.

47. Thirty-six planks, each 4^m long, 27.8^{cm} wide, and 75^{mm} thick, at \$16 a hundred meters.

48. Three sticks of timber, each 8^m long, 22.5^{cm} wide, and 20^{cm} thick, at \$17.50 a hundred meters.

49. A board 8.25^m long, 28^{cm} wide at one end and 35^{cm} at the other, and 31.25^{mm} thick, at \$.30 a meter?

50. A stick of timber 10^m long, 25^{cm} thick, 30^{cm} wide at one end and 25^{cm} wide at the other, at \$14 a hundred meters.

51. The flooring for a two-story building 16^m by 10.5^m; the flooring being 32^{mm} thick, and worth \$30 a hundred meters.

52. The flooring timbers for this building, the timbers being 25^{cm} by 50^{mm}, and placed on edge 30^{cm} apart, and worth \$11.50 a hundred meters.

53. The fencing to enclose a field 150^m long and 75^m wide; the posts are set 2.5^m apart, and cost \$.25 apiece; the fence is 5 boards high; the bottom board is 30^{cm}, the top board 25^{cm}, and the other three each 22.5^{cm} wide, and the boards cost \$13.25 a hundred meters.

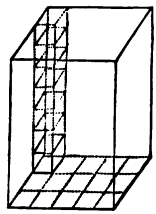
NOTE. Round logs are sold by board measure; .25 is deducted for slabs. A scale is used, the calipers of which are applied to the middle of the log, and the amount of boards for a given length is read off the scale.

Large and heavy timber is sold by the ton; only .20 is allowed for slabs, and the amount is reckoned in cubic measure instead of board measure.

Measures of Volume.

To find the volume of a rectangular solid.

210. In the figure represented in the margin, let the length contain 5, the breadth 3, and the height 7 *linear centimeters.*

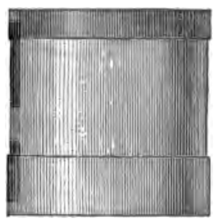


The base may be divided into square centimeters: there will be three rows of 5 square centimeters each; in all, 15 square centimeters. Upon each square centimeter may be placed a pile of 7 cubic centimeters; so that the solid will contain 15 × 7 cubic centimeters, that is, 3 × 5 × 7 *cubic centimeters.* Hence,

Express the length, breadth, and height of a rectangular solid in the **same linear unit**; *the product of these numbers will be its volume in* **cubic units** *of the same name as the linear unit of the edges.*

Exercise VI.

1. How many cubic centimeters in a block 9^{cm} long by 7^{cm} wide, and 6^{cm} deep?



Hektoliter.

2. If wood is cut into 120^{cm} lengths, and a pile is 43.7^{m} long and 1.4^{m} high, how many sters of wood are there in it?
3. In a grain elevator is a bin 11.2^{m} long, 4.34^{m} wide, and 2.83^{m} deep. How many hektoliters of grain will it hold?
4. If a liter of grain weigh .81 of the weight of a liter of water, how much will the grain in that bin weigh?

5. A bin measuring 16^{m} by 9.7^{m}, and 2.8^{m} deep, is full of oats, worth \$.98 a hektoliter. What is the whole worth?
6. A vat 197^{cm} long, 87^{cm} wide, and 63^{cm} deep, holds how many liters? What would be the weight of water required to fill it?
7. Add 1341^{ccm}, 231^{l}, and 2.13^{hl}, and give the sum in terms of each of the three units.
8. If a spring pours out 467.8^{l} each minute, how many hektoliters will it deliver in 60 minutes? in 37 minutes? in 78 minutes?
9. If 67.3^{l} of oil in a vat with perpendicular sides fill it to a depth of 173^{mm}, how deep will 13.7 times that quantity fill it? and how many hektoliters will there be?
10. Into a round cup 10^{cm} across, with perpendicular sides, pour oil until it is 1^{cm} deep; then there are 78.54^{ccm} of oil in the cup. (See § 204.) How many cubic centimeters will there be when the oil is 38^{mm} deep?
11. What is the capacity of a tin cup 95^{mm} across and 11.08^{cm} deep?
12. What are the capacities of two cylindrical vessels, one being 16.24^{cm} across and 19.95^{cm} deep, the other 75.4^{mm} across and 87.9^{mm} deep?

211. *The volume of a sphere is found by multiplying the cube of the diameter by* .5236.

13. How many cubic centimeters in a ball 10^{cm} in diameter? How much less if you take the more exact multiplier? § 201.
14. Into a cubical box 20^{cm} on a side, and full of water, an iron ball 20^{cm} in diameter is gently lowered until it touches bottom. How much water is left in the box? Answer in liters and in cubic centimeters.

15. One cask contains 171.4^{l} of oil; another, 209.3^{l}; a third, 73.8^{l}; while a square vat, 137^{cm} each way, is filled to a depth of 69^{cm}. How much oil in all the vessels? in liters and in hektoliters.
16. How many liters of air in a room 7.8^{m} long, 6.23^{m} wide, and 3^{m} high?
17. If a person's breathing spoil the air at the rate of $.2175^{cbm}$ a minute, how long will it take 3 persons sitting in the room, closed, to spoil the air?
18. How long, at the same rate, would the air in a hall 22^{m} long, 16^{m} wide, and 7^{m} high, last an audience of 280 persons?
19. How many cubic meters of wood in a round stick of equal size throughout, 37^{cm} in diameter and 8.4^{m} long?

EXERCISE VII.

1. What is the weight, in kilograms, of a hektoliter of water? of 73.8^{l} of water? of a cubic meter of water? of a cubic centimeter?
2. If a man buys half a ton of potatoes for $20, and retails them all, without waste, at 5 cents a kilogram, what profit does he make on the whole?
3. What is the weight of water required to fill a vat 98^{cm} long, 71^{cm} wide, and 38^{cm} deep?
4. If the vat of the last example were filled with brine weighing 1.04^{kg} to the liter, what would be the weight of the brine?
5. If the vat of Example 3 were filled with wine weighing $.981^{kg}$ to the liter, what would be its weight?
6. What is the total weight of 13 men averaging 73.48^{kg} each?

7. How many kilograms, and how many tons, would 3.6175^{cbm} of brick weigh, at 2 tons to a cubic meter? at 2.34 tons?
8. From a barrel containing 67^{kg} of granulated sugar were taken three parcels of 2.75^{kg} each, and four parcels of 7.50^{kg} each. How much is left in the barrel?
9. Into how many pills of 325^{mg} each can a mass of 7.8^{g} be divided?
10. A mass of 21.8^{g} is divided into 60 pills. What is the weight of each pill?

Specific Gravity.

212. *The* **specific gravity** *of any substance is* **the number of times** *the weight of the substance contains the weight of an equal bulk of water.*

Thus, if a sample of quicksilver has a specific gravity of 13.6, it is 13.6 times as heavy as water; a cubic centimeter of it would weigh 13.6^{g}; a liter of it would weigh 13.6^{kg}; and a cubic meter of it would weigh 13.6^{t}.

Again, if the specific gravity of a certain alcohol is .827, that is, if the alcohol weighs .827 as much as an equal bulk of water, then a cubic centimeter of it would weigh $.827^{g}$; a liter, $.827^{kg}$; and a cubic meter, $.827^{t}$.

213. Therefore, the **specific gravity** of a substance is the **number** that expresses the **weight** of a **cubic centimeter** of it in **grams**; or of a **liter** in **kilograms**; or of a **cubic meter** in **tons.**

214. When a substance heavier than water is under water, the water buoys it up just the amount of the weight of the water displaced by it. I weigh a lump of coal suspended by a thread; it weighs 1017^{g}. I then hang it, still suspended to the balance, in a pail of water, and lower it

until the coal is covered with water; it now weighs 531^{g}, and has lost 486^{g}. The first information I get is, that the lump contains 486^{ccm} of coal, for it displaces 486^{ccm} of water. Then to obtain its specific gravity, that is, its weight compared with that of water as unity (§ 166), we have $1017 \div 486 = 2.092$ as the specific gravity of the piece of coal.

A piece of stone weighing 1.1^{kg} in air and $.6^{kg}$ in water, is bound by a thread to a block of wood; the two together weigh 1.28^{kg} in air and $.54^{kg}$ in water. What is the specific gravity of the wood?

The weight of the wood in air $= 1.28^{kg} - 1.1^{kg} = .18^{kg}$.

The weight of water displaced by stone and wood $= 1.28^{kg} - .54^{kg} = .74^{kg}$.

The weight of water displaced by stone alone $= 1.1^{kg} - .6^{kg} = .5^{kg}$.

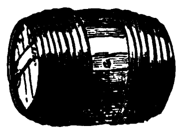
The weight of water displaced by wood, therefore, $= .74^{kg} - .5^{kg} = .24^{kg}$.

Hence, the specific gravity of the wood $= .18^{kg} \div .24^{kg} = .75$.

Ans. 0.75.

Exercise VIII.

1. If a stone weighs 1.3^{kg} in air and $.68^{kg}$ in water, and the the stone and a block of wood together weigh 1.55^{kg} in air and $.63^{kg}$ in water, what is the specific gravity of the block of wood?
2. What is the weight of 8.17^{hl} of alcohol, specific gravity .83?
3. What will 97^{l} alcohol weigh, of specific gravity .817? of specific gravity .819? of specific gravity .823? .838? .847?



Hektoliter.

4. A bar of aluminum 113^{mm} long, 17^{mm} wide, and 13^{mm} thick, is said to be of specific gravity 2.57. What does it weigh? If it really is of specific gravity 2.67, what does it weigh?

5. **What would be the specific gravity of the bar of the last example if it weighed 65.137^{g}?**
6. **What is the weight of a bar of aluminum 371^{mm} by 63^{mm} by 84^{mm}, specific gravity being 2.63?**
7. **An irregular mass of copper, gently lowered into a pail brimful of water, caused 1.374^{l} to run over. What did it weigh if of specific gravity 8.91? if 8.89?**
8. **What was the specific gravity of that copper if the mass weighed 12.3016^{kg}?**

215. The pupil will have seen, from the preceding examples, that to find the specific gravity of a body he may

Divide the **weight in grams** by the **bulk in cubic centimeters**,
the **weight in kilograms** by the **bulk in liters**, or
the **weight in tons** by the **bulk in cubic meters.**

9. A plate of iron 137^{cm} long, 64.3^{cm} wide, and 4.31^{cm} thick, weighs 277.54^{kg}. What is its specific gravity? What would the same mass weigh at specific gravity 7.47? at 7.79?
10. What is the specific gravity of sea-water when a hektoliter weighs 102.58^{kg}? what when 3^{l} weigh 3077^{g}?
11. What is the specific gravity of a substance of which 7.3^{ccm} weighs 31.5^{g}?
12. If a cubic meter of sand weighs 1723^{kg}, what is its specific gravity? If 3.4^{cbm} of gravel weigh 7.134 tons, what is the specific gravity?
13. If a cubic centimeter of metal weighs 7.3^{g}, what is its specific gravity?
14. What is the specific gravity of a fluid weighing 2.317^{kg} to a liter?
15. If a body weigh 3.71^{kg} in air and 2.38^{kg} in water, what is its specific gravity?
16. A piece of ore weighing 3.77^{kg} weighs in water only 2.53^{kg}. What is its specific gravity?

17. How many cubic centimeters in a stone which loses 17.8^{g} of its weight when weighed in water? What is its specific gravity if weighed in air it weighs 33.7^{g}?

18. In a wrought-iron bottle I find 2.63^{l} of quicksilver, weighing 35.81^{kg}; in another 2.59^{l}, weighing 35.193^{kg}; in a third, 2.617^{l}, weighing 35.571^{kg}. What is the specific gravity of each? What would be the specific gravity if the three were emptied into one vessel and mixed?

19. A plate of iron 89^{cm} by 17^{cm} by 7^{cm} weighs 79.43^{kg}. What is its specific gravity?

EXERCISE IX.

1. If three men eat 8^{kg} a week, how much would one man eat at the same rate? How much would seven men? At the same rate, how much do the three men eat in one day? and how much each man? At the same rate, how much would seven men eat each day? each week? in 5 weeks?

2. At the same rate, how much would 17 men eat in 3 weeks and 4 days?

3. If one hektoliter of oats is enough for 5 horses one week, how much is enough for 1 horse one week? for 1 horse 7 weeks? for 11 horses 17 weeks?

4. If two hektoliters of grain are enough for 3 horses 5 days, how much is enough for 3 horses 1 day? for 1 horse 1 day? for 7 horses 6 days?

5. Mix 17 liters of vinegar, costing 6 cents a liter, with 39^{l} at 5 cents, 21^{l} at 7 cents, and 13^{l} of water costing nothing. Find the amount of the mixture, and its cost?

6. For how much a liter must I sell that mixture, in order to gain 96 cents? for how much to clear $1.41?

7. A grocer sold 421 kegs of butter for \$4995.25; 56 kegs brought \$12.50 a keg; 91 brought \$11.75 a keg; and 100 kegs brought \$12.25 a keg. For how much a keg were the other kegs sold?

8. If 3 tons of coal cost \$15.75, how many tons will \$36.75 buy?

9. If 5^m of cloth cost \$18.75, what should 7^m cost?

10. If a tap running 3.5^l a minute fills a tub in 16 minutes, how long should a tap delivering 5^l a minute be in filling the same tub?

11. If both taps of the last example be opened at once, how soon will they fill the tub?

12. If 3 men can dig 378^m of ditch in 2 days, how long will it take 5 men, at the same rate, to dig 787^m?

13. Into a tub which will hold 48^l, one tap is delivering water at the rate of 3.7^l a minute; while out of it, by another tap, the water is running at 2.5^l a minute. How long will it take to fill the tub, beginning with it empty?

14. A tap discharges into a tub 4.2^l a minute; from the tub water is also running, by a second tap; the water in the tub gains 30^l in 18 minutes. How fast is the second tap discharging?

15. If a wheel is 1.2^m across, how often will it turn in going one kilometer?

16. How many times in a minute does the wheel of the last example turn, when the carriage is driven 14^{km} an hour?

17. What is the weight of the water in a tank if it would take a flow of 8.7^l a minute 1 hour and 38 minutes to empty it?

18. Replace that bulk of water with oil worth \$18.75 a hektoliter, and what will the contents of the tank be worth?

Exercise X.

1. A train leaves Paris at 11 o'clock A.M., and reaches Lyons at 10 o'clock P.M. How many meters does it travel in an hour, the distance from Paris to Lyons being 512.7^{km}?
2. A railroad has a single track 11.450^{km} long. How many rails 4.569^{m} in length did it require to lay the track?
3. A book is 2.1^{cm} in thickness; each leaf is $.05^{mm}$ thick. Find the number of pages in the book.
4. The cost of opening a canal amounts to \$25,400 a kilometer. How much would a canal cost which was 113.253^{km} in length?
5. The expense of laying out a paved road is \$12,500 a kilometer. How much would a road cost which was 72.053^{km} long?
6. The cost of building a railroad is about \$78,000 a kilometer in France, and only \$25,000 in the United States. How much would it cost in each country to make a road 295.671^{km} long?
7. If you must go up 211 steps to reach the top of a tower, and each step is 195^{mm} high, what is the height of the tower?
8. A house has 5 stories, each story has 19 stairs, each stair is 16^{cm} in height. Calculate how high the floor of the fifth story is from the ground.
9. A ream of paper contains 20 quires, each quire has 24 sheets, the ream is 13.5^{cm} in thickness. Find the thickness of each sheet.
10. The equator on a terrestrial globe measures $.80^{m}$ in circumference. By the aid of a tape-measure we find that the distance between two cities on this globe is $.046^{m}$. What is really the distance in kilometers between the two cities? (The earth's equator is $40,075.45^{km}$.)

11. Upon a military map we find that the distance from Paris to St. Denis is 78^{mm}. What is the distance in kilometers from Paris to St. Denis? The map is made on the scale of 1 to 80,000; that is, 1^{m} on the map represents $80{,}000^{m}$ of actual measurement upon the ground.
12. Give the number of revolutions made by the wheels of a carriage in travelling 82^{km}. The wheels are 1354^{mm} in diameter.
13. How many hektars in a square kilometer? how many ars? how many square meters?
14. France has about $542{,}000^{qkm}$. How many hektars does it measure?
15. A piece of land 1224.5^{m} square is sold at $140 a hektar. How much does the land bring?
16. The total surface measurement of the glass in the windows of a house is 182^{qm}. How many panes of 53^{cm} by 48^{cm} will it take to supply the windows?
17. How many square slabs of marble 150^{qcm} on he surface will it require to pave a court whose area is 25.35^{qm}?
18. A speculator bought 31.0728^{ha} of land for $1,296 a hektar. For how much a square meter must he sell it to realize a profit of $1,937?
19. A man is offered $6,000 for 2.5 ars of land. He declines to sell; and soon after, the town gives him $25.20 a square meter. How much did he make by refusing the first offer?
20. A man surveys a piece of land and finds that it measures 14.0715^{ha}. He afterwards discovers that his chain was too short by $.03^{m}$. How can he calculate the real superficial measurement of his land without surveying it again? (A surveyor's chain is 10^{m} long.)

21. The railroad from Paris to Orleans has a double track; each rail is 4^{m} long, and the distance from Paris to Orleans is 121^{km}. What is the number of rails used in laying the track? The width of the road is 15^{m}; how many hektars of land does the road include?

22. Calculate the number of ars in a surface which a ream of paper (480 sheets) will cover. The sheets are 30.3^{cm} long and 195^{mm} wide.

23. A pile of wood is 4.25^{m} long, 1.33^{m} thick, and 2.60^{m} high. How many sters are there in it?

24. A beam is 7.070^{m} long; its two other dimensions are $.258^{m}$ and 87^{mm}. Find its volume.

25. A bar of iron 3^{m} long measures 45^{mm} square on the end where it has been evenly cut. The bar is heated and drawn out to a greater length by being passed through an orifice 24^{mm} square. What is the length of the bar after the operation?

26. A reservoir is 1.50^{m} wide, 2.80^{m} long, and 1.25^{m} deep. Find how many liters it contains when full, and to what height it would be necessary to raise it that it might contain 10^{cbm}?

27. Suppose a box to be 3.75^{m} long, 3.50^{m} wide, and $.50^{m}$ high. How much lime would it take to fill it with mortar, reckoning that 1^{cbm} of lime after being slaked becomes 1.80^{cbm} of mortar?

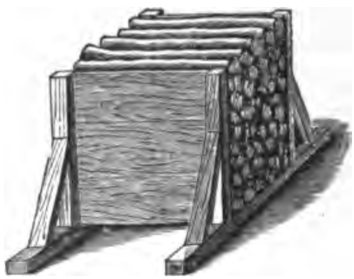
28. A chest has the following dimensions: 1.17^{m}, $.90^{m}$, 1.04^{m}. How many cakes of soap 13^{cm} square on the bottom and 29^{cm} thick could be put in it? .12 of the volume of the chest must be deducted for packing.

29. A cubic meter of dry plaster makes 1.18^{cbm} when tempered; tempered plaster increases 1 in every 100, twenty-four hours after it is mixed. What volume of tempered plaster would be obtained from 55 sacks of 25^{l} each of dry plaster?

30. A reservoir is 2.80^{m} long, 1.50^{m} wide, and 1.25^{m} deep. How many liters will be required to fill .80 of it?

31. A man buys 1415^{hl} of wheat for $3.50 a hektoliter; but the measure used proves too small, the mistake amounted to 3^{l} in every hektoliter. What was the quantity of wheat delivered to the purchaser, the cost, and the reduction which ought to be made to him on account of the error?

32. The dimensions of a tile are as follows: length 22^{cm} width 11^{cm}, thickness 55^{mm}. Find the volume of the tile, and the number of tiles in a pile of 25^{cbm}.



Ster of Wood.

33. The measurement of a pile of wood shows that a ster could be filled from it 25.68 times. Give the volume of the pile in cubic meters, reckoning the length of the logs to be 1.15^{m}.

NOTE. A ster is a frame, as represented in the diagram, one meter high and one meter between the upright posts. The ster may be filled with wood of any length, and the volume will be as many cubic meters as the sticks of wood are meters long.

34. A liter of air weighs 1.273^{g}. How much does a cubic meter of air weigh?

35. A package of candles which weighs 465^g is sold at 28 cents. What is the price of a kilogram of candles?
36. How many times would 3.243^l of water fill a liter?
37. Give the weight in kilograms of 43.4578^{ccm} of pure water.
38. The volume of an engine's axletree is $.245^{cbm}$. Find its weight, the specific gravity of the iron being 7.8.
39. Calculate the volume of a gram of the following substances: proof spirit, specific gravity .865; tin, specific gravity 7.291; lead, specific gravity 11.35; copper, specific gravity 8.85; silver, specific gravity 10.47; cork, specific gravity .240.
40. Olive oil costs 60 cents a kilogram. What is the price of a liter? The specific gravity of olive oil is .914.
41. Pure alcohol costs $1.87 a kilogram. What is the price of a liter? The specific gravity of alcohol is .792.
42. A man wants to build a shed large enough to hold 135^{st} of wood; if the shed is to be 3^m high and 5^m wide, how long must it be?
43. In a country where fire-wood is cut 1.16^m long, what height must the sides of the ster be to hold a cubic meter?
44. If a ster of cork costs $20.00, how much would 100^{kg} cost, the cork weighing one quarter as much as water?
45. A liter of powder weighs 825^g. What would be the volume of a charge for a gun if the charge weighed 5^g? Calculate the volume in cubic centimeters.
46. Out of gold which weighs 19.362 times as much as water, sheets of gold-foil are made which are $.010^{mm}$ in thickness. What surface would 3^g of gold cover?
47. Find the weight of an oak board 3.25^m long, $.31^m$ wide, and $.04^m$ thick; the specific gravity of the oak being .808.

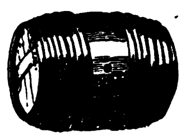
48. Find the weight of a bar of iron having the following dimensions: length 3.6^{m}, width 6^{cm}, thickness 2^{cm}; the specific gravity of the iron being 7.8.

49. How many lead balls each weighing 27^{g} could be obtained by melting a mass of lead, cubic in form, the edge measuring $.356^{m}$, the specific gravity of the lead being 11.35?

50. Marble costs $30.95 a cubic meter, and the specific gravity of marble is 2.73. If a block of marble weighs 1260^{kg}, what is its volume, and what is it worth?

51. Sea-water contains 28 parts, by weight, of salt in 1000. A liter of sea-water weighs 1.025^{kg}. How many kilograms of salt could be obtained from 126.276842^{cbm} of sea-water?

52. An empty cask weighs 17.06^{kg}; when filled with water it weighs 275.8^{kg}. How many liters does it hold? How many casks of this size would it require to receive the wine from a vat containing 3.008^{cbm}?


Cask.

53. It takes about 204.8^{l} of wheat to sow a hektar. How many cubic meters would it take to sow a square kilometer?

54. A piece of road 1^{km} long and 7^{m} wide is to be macadamized; the macadamizing is to be 33^{cm} deep; it costs 43 cents a cubic meter to prepare the stones. What will enough for the road cost?

55. A gasometer holds $28{,}000^{cbm}$ of gas. How many jets would this gasometer feed, when each jet burns 125^{l} an hour, and is used 4 hours every evening?

56. The city of Venice is situated in the midst of a great lake of salt water, communicating with the sea, and all the rain-water is caught for the cisterns. Ordi-

nary years the fall of rain in Venice is 82^{cm}; the surface of the city, after the canals have been deducted, is 520^{ha}; reckoning the population at 115,530, how many liters a day of rain-water could each inhabitant have?

57. Find the weight of a bar of iron 5.35^{m} long, 4.56^{cm} thick, and 3.54^{cm} wide. Find, also, the width of an oak beam 4.30^{m} long, 9.12^{cm} thick, which has the same weight. The specific gravity of the oak to be reckoned at 1.026, that of the iron 7.788.

58. Give the specific gravity and volume of a body weighing 35^{kg} in air and 30^{kg} in water.

59. A ster of piled oak wood weighs 425^{kg}; the specific gravity of the wood is .74. What is the volume occupied by the spaces between the logs? For how much must 100^{kg} of separate sticks be sold in order to bring the same amount as when sold by the ster; a ster selling for $2.20?

60. Wrought iron sells for $7.00 per 100^{kg}. A bar of iron 4.5^{cm} wide, 3.3^{cm} thick costs $5.08; what is its length, reckoning the specific gravity of the iron at 7.4?

61. Experiment shows that water weighs 770 times as much as air; and the specific gravity of mercury, in comparison with water, is 13.6. How many liters of air will it take to weigh as much as a liter of mercury?

62. A mass of lead weighing 753^{kg} is made into sheets $.1^{mm}$ thick. Calculate, in square meters, the surface which can be covered by the sheets thus obtained. The specific gravity of the lead is 11.3.

63. A rectangular sheet of tin of uniform thickness is 85^{cm} wide, 1.35^{m} long; it weighs 268^{g}. What is its thickness, reckoning the specific gravity of tin at 7.3?

64. The fine coal which collects about the shafts of the mines and in the coal-yards, was for a long time

7. How many kilograms, and how many tons, would 3.6175^{cbm} of brick weigh, at 2 tons to a cubic meter? at 2.34 tons?
8. From a barrel containing 67^{kg} of granulated sugar were taken three parcels of 2.75^{kg} each, and four parcels of 7.50^{kg} each. How much is left in the barrel?
9. Into how many pills of 325^{mg} each can a mass of 7.8^{g} be divided?
10. A mass of 21.8^{g} is divided into 60 pills. What is the weight of each pill?

Specific Gravity.

212. *The* **specific gravity** *of any substance is* **the number of times** *the weight of the substance contains the weight of an equal bulk of water.*

Thus, if a sample of quicksilver has a specific gravity of 13.6, it is 13.6 times as heavy as water; a cubic centimeter of it would weigh 13.6^{g}; a liter of it would weigh 13.6^{kg}; and a cubic meter of it would weigh 13.6^{t}.

Again, if the specific gravity of a certain alcohol is .827, that is, if the alcohol weighs .827 as much as an equal bulk of water, then a cubic centimeter of it would weigh $.827^{g}$; a liter, $.827^{kg}$; and a cubic meter, $.827^{t}$.

213. Therefore, the **specific gravity** of a substance is the **number** that expresses the **weight** of a **cubic centimeter** of it in **grams**; or of a **liter** in **kilograms**; or of a **cubic meter** in **tons.**

214. When a substance heavier than water is under water, the water buoys it up just the amount of the weight of the water displaced by it. I weigh a lump of coal suspended by a thread; it weighs 1017^{g}. I then hang it, still suspended to the balance, in a pail of water, and lower it

until the coal is covered with water; it now weighs 531^g, and has lost 486^g. The first information I get is, that the lump contains 486^{ccm} of coal, for it displaces 486^{ccm} of water. Then to obtain its specific gravity, that is, its weight compared with that of water as unity (§ 166), we have $1017 \div 486 = 2.092$ as the specific gravity of the piece of coal.

A piece of stone weighing 1.1^{kg} in air and $.6^{kg}$ in water, is bound by a thread to a block of wood; the two together weigh 1.28^{kg} in air and $.54^{kg}$ in water. What is the specific gravity of the wood?

The weight of the wood in air $= 1.28^{kg} - 1.1^{kg} = .18^{kg}$.

The weight of water displaced by stone and wood $= 1.28^{kg} - .54^{kg} = .74^{kg}$.

The weight of water displaced by stone alone $= 1.1^{kg} - .6^{kg} = .5^{kg}$.

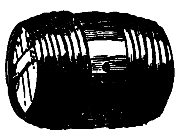
The weight of water displaced by wood, therefore, $= .74^{kg} - .5^{kg} = .24^{kg}$.

Hence, the specific gravity of the wood $= .18^{kg} \div .24^{kg} = .75$.

Ans. 0.75.

Exercise VIII.

1. If a stone weighs 1.3^{kg} in air and $.68^{kg}$ in water, and the the stone and a block of wood together weigh 1.55^{kg} in air and $.63^{kg}$ in water, what is the specific gravity of the block of wood?
2. What is the weight of 8.17^{hl} of alcohol, specific gravity .83?
3. What will 97^l alcohol weigh, of specific gravity .817? of specific gravity .819? of specific gravity .823? .838? .847?



Hektoliter.

4. A bar of aluminum 113^{mm} long, 17^{mm} wide, and 13^{mm} thick, is said to be of specific gravity 2.57. What does it weigh? If it really is of specific gravity 2.67, what does it weigh?

5. What would be the specific gravity of the bar of the last example if it weighed 65.137^{g}?
6. What is the weight of a bar of aluminum 371^{mm} by 63^{mm} by 84^{mm}, specific gravity being 2.63?
7. An irregular mass of copper, gently lowered into a pail brimful of water, caused 1.374^{l} to run over. What did it weigh if of specific gravity 8.91? if 8.89?
8. What was the specific gravity of that copper if the mass weighed 12.3016^{kg}?

215. The pupil will have seen, from the preceding examples, that to find the specific gravity of a body he may

Divide the **weight in grams** by the **bulk in cubic centimeters,**
the **weight in kilograms** by the **bulk in liters,** or
the **weight in tons** by the **bulk in cubic meters.**

9. A plate of iron 137^{cm} long, 64.3^{cm} wide, and 4.31^{cm} thick, weighs 277.54^{kg}. What is its specific gravity? What would the same mass weigh at specific gravity 7.47? at 7.79?
10. What is the specific gravity of sea-water when a hektoliter weighs 102.58^{kg}? what when 3^{l} weigh 3077^{g}?
11. What is the specific gravity of a substance of which 7.3^{ccm} weighs 31.5^{g}?
12. If a cubic meter of sand weighs 1723^{kg}, what is its specific gravity? If 3.4^{cbm} of gravel weigh 7.134 tons, what is the specific gravity?
13. If a cubic centimeter of metal weighs 7.3^{g}, what is its specific gravity?
14. What is the specific gravity of a fluid weighing 2.317^{kg} to a liter?
15. If a body weigh 3.71^{kg} in air and 2.38^{kg} in water, what is its specific gravity?
16. A piece of ore weighing 3.77^{kg} weighs in water only 2.53^{kg}. What is its specific gravity?

17. How many cubic centimeters in a stone which loses 17.8^g of its weight when weighed in water? What is its specific gravity if weighed in air it weighs 33.7^g?
18. In a wrought-iron bottle I find 2.63^l of quicksilver, weighing 35.81^{kg}; in another 2.59^l, weighing 35.193^{kg}; in a third, 2.617^l, weighing 35.571^{kg}. What is the specific gravity of each? What would be the specific gravity if the three were emptied into one vessel and mixed?
19. A plate of iron 89^{cm} by 17^{cm} by 7^{cm} weighs 79.43^{kg}. What is its specific gravity?

Exercise IX.

1. If three men eat 8^{kg} a week, how much would one man eat at the same rate? How much would seven men? At the same rate, how much do the three men eat in one day? and how much each man? At the same rate, how much would seven men eat each day? each week? in 5 weeks?
2. At the same rate, how much would 17 men eat in 3 weeks and 4 days?
3. If one hektoliter of oats is enough for 5 horses one week, how much is enough for 1 horse one week? for 1 horse 7 weeks? for 11 horses 17 weeks?
4. If two hektoliters of grain are enough for 3 horses 5 days, how much is enough for 3 horses 1 day? for 1 horse 1 day? for 7 horses 6 days?
5. Mix 17 liters of vinegar, costing 6 cents a liter, with 39^l at 5 cents, 21^l at 7 cents, and 13^l of water costing nothing. Find the amount of the mixture, and its cost?
6. For how much a liter must I sell that mixture, in order to gain 96 cents? for how much to clear \$1.41?

7. A grocer sold 421 kegs of butter for \$4995.25; 56 kegs brought \$12.50 a keg; 91 brought \$11.75 a keg; and 100 kegs brought \$12.25 a keg. For how much a keg were the other kegs sold?
8. If 3 tons of coal cost \$15.75, how many tons will \$36.75 buy?
9. If 5^m of cloth cost \$18.75, what should 7^m cost?
10. If a tap running 3.5^l a minute fills a tub in 16 minutes, how long should a tap delivering 5^l a minute be in filling the same tub?
11. If both taps of the last example be opened at once, how soon will they fill the tub?
12. If 3 men can dig 378^m of ditch in 2 days, how long will it take 5 men, at the same rate, to dig 787^m?
13. Into a tub which will hold 48^l, one tap is delivering water at the rate of 3.7^l a minute; while out of it, by another tap, the water is running at 2.5^l a minute. How long will it take to fill the tub, beginning with it empty?
14. A tap discharges into a tub 4.2^l a minute; from the tub water is also running, by a second tap; the water in the tub gains 30^l in 18 minutes. How fast is the second tap discharging?
15. If a wheel is 1.2^m across, how often will it turn in going one kilometer?
16. How many times in a minute does the wheel of the last example turn, when the carriage is driven 14^{km} an hour?
17. What is the weight of the water in a tank if it would take a flow of 8.7^l a minute 1 hour and 38 minutes to empty it?
18. Replace that bulk of water with oil worth \$18.75 a hektoliter, and what will the contents of the tank be worth?

EXERCISE X.

1. A train leaves Paris at 11 o'clock A.M., and reaches Lyons at 10 o'clock P.M. How many meters does it travel in an hour, the distance from Paris to Lyons being 512.7^{km}?
2. A railroad has a single track 11.450^{km} long. How many rails 4.569^{m} in length did it require to lay the track?
3. A book is 2.1^{cm} in thickness; each leaf is $.05^{mm}$ thick. Find the number of pages in the book.
4. The cost of opening a canal amounts to \$25,400 a kilometer. How much would a canal cost which was 113.253^{km} in length?
5. The expense of laying out a paved road is \$12,500 a kilometer. How much would a road cost which was 72.053^{km} long?
6. The cost of building a railroad is about \$78,000 a kilometer in France, and only \$25,000 in the United States. How much would it cost in each country to make a road 295.671^{km} long?
7. If you must go up 211 steps to reach the top of a tower, and each step is 195^{mm} high, what is the height of the tower?
8. A house has 5 stories, each story has 19 stairs, each stair is 16^{cm} in height. Calculate how high the floor of the fifth story is from the ground.
9. A ream of paper contains 20 quires, each quire has 24 sheets, the ream is 13.5^{cm} in thickness. Find the thickness of each sheet.
10. The equator on a terrestrial globe measures $.80^{m}$ in circumference. By the aid of a tape-measure we find that the distance between two cities on this globe is $.046^{m}$. What is really the distance in kilometers between the two cities? (The earth's equator is $40{,}075.45^{km}$.)

11. Upon a military map we find that the distance from Paris to St. Denis is 78^{mm}. What is the distance in kilometers from Paris to St. Denis? The map is made on the scale of 1 to 80,000; that is, 1^{m} on the map represents $80{,}000^{m}$ of actual measurement upon the ground.
12. Give the number of revolutions made by the wheels of a carriage in travelling 82^{km}. The wheels are 1354^{mm} in diameter.
13. How many hektars in a square kilometer? how many ars? how many square meters?
14. France has about $542{,}000^{qkm}$. How many hektars does it measure?
15. A piece of land 1224.5^{m} square is sold at $140 a hektar. How much does the land bring?
16. The total surface measurement of the glass in the windows of a house is 182^{qm}. How many panes of 53^{cm} by 48^{cm} will it take to supply the windows?
17. How many square slabs of marble 150^{qcm} on the surface will it require to pave a court whose area is 25.35^{qm}?
18. A speculator bought 31.0728^{ha} of land for $1,296 a hektar. For how much a square meter must he sell it to realize a profit of $1,937?
19. A man is offered $6,000 for 2.5 ars of land. He declines to sell; and soon after, the town gives him $25.20 a square meter. How much did he make by refusing the first offer?
20. A man surveys a piece of land and finds that it measures 14.0715^{ha}. He afterwards discovers that his chain was too short by $.03^{m}$. How can he calculate the real superficial measurement of his land without surveying it again? (A surveyor's chain is 10^{m} long.)

21. The railroad from Paris to Orleans has a double track; each rail is 4^{m} long, and the distance from Paris to Orleans is 121^{km}. What is the number of rails used in laying the track? The width of the road is 15^{m}; how many hektars of land does the road include?

22. Calculate the number of ars in a surface which a ream of paper (480 sheets) will cover. The sheets are 30.3^{cm} long and 195^{mm} wide.

23. A pile of wood is 4.25^{m} long, 1.33^{m} thick, and 2.60^{m} high. How many sters are there in it?

24. A beam is 7.070^{m} long; its two other dimensions are $.258^{m}$ and 87^{mm}. Find its volume.

25. A bar of iron 3^{m} long measures 45^{mm} square on the end where it has been evenly cut. The bar is heated and drawn out to a greater length by being passed through an orifice 24^{mm} square. What is the length of the bar after the operation?

26. A reservoir is 1.50^{m} wide, 2.80^{m} long, and 1.25^{m} deep. Find how many liters it contains when full, and to what height it would be necessary to raise it that it might contain 10^{cbm}?

27. Suppose a box to be 3.75^{m} long, 3.50^{m} wide, and $.50^{m}$ high. How much lime would it take to fill it with mortar, reckoning that 1^{cbm} of lime after being slaked becomes 1.80^{cbm} of mortar?

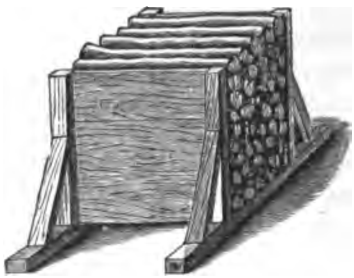
28. A chest has the following dimensions: 1.17^{m}, $.90^{m}$, 1.04^{m}. How many cakes of soap 13^{cm} square on the bottom and 29^{cm} thick could be put in it? .12 of the volume of the chest must be deducted for packing.

29. A cubic meter of dry plaster makes 1.18^{cbm} when tempered; tempered plaster increases 1 in every 100, twenty-four hours after it is mixed. What volume of tempered plaster would be obtained from 55 sacks of 25^{l} each of dry plaster?

30. A reservoir is 2.80^{m} long, 1.50^{m} wide, and 1.25^{m} deep. How many liters will be required to fill .80 of it?

31. A man buys 1415^{hl} of wheat for $3.50 a hektoliter; but the measure used proves too small, the mistake amounted to 3^{l} in every hektoliter. What was the quantity of wheat delivered to the purchaser, the cost, and the reduction which ought to be made to him on account of the error?

32. The dimensions of a tile are as follows: length 22^{cm} width 11^{cm}, thickness 55^{mm}. Find the volume of the tile, and the number of tiles in a pile of 25^{cbm}.



Ster of Wood.

33. The measurement of a pile of wood shows that a ster could be filled from it 25.68 times. Give the volume of the pile in cubic meters, reckoning the length of the logs to be 1.15^{m}.

NOTE. A ster is a frame, as represented in the diagram, one meter high and one meter between the upright posts. The ster may be filled with wood of any length, and the volume will be as many cubic meters as the sticks of wood are meters long.

34. A liter of air weighs 1.273^{g}. How much does a cubic meter of air weigh?

35. A package of candles which weighs 465^g is sold at 28 cents. What is the price of a kilogram of candles?

36. How many times would 3.243^t of water fill a liter?

37. Give the weight in kilograms of 43.4578^{ccm} of pure water.

38. The volume of an engine's axletree is $.245^{cbm}$. Find its weight, the specific gravity of the iron being 7.8.

39. Calculate the volume of a gram of the following substances: proof spirit, specific gravity .865; tin, specific gravity 7.291; lead, specific gravity 11.35; copper, specific gravity 8.85; silver, specific gravity 10.47; cork, specific gravity .240.

40. Olive oil costs 60 cents a kilogram. What is the price of a liter? The specific gravity of olive oil is .914.

41. Pure alcohol costs $1.87 a kilogram. What is the price of a liter? The specific gravity of alcohol is .792.

42. A man wants to build a shed large enough to hold 135^{st} of wood; if the shed is to be 3^m high and 5^m wide, how long must it be?

43. In a country where fire-wood is cut 1.16^m long, what height must the sides of the ster be to hold a cubic meter?

44. If a ster of cork costs $20.00, how much would 100^{kg} cost, the cork weighing one quarter as much as water?

45. A liter of powder weighs 825^g. What would be the volume of a charge for a gun if the charge weighed 5^g? Calculate the volume in cubic centimeters.

46. Out of gold which weighs 19.362 times as much as water, sheets of gold-foil are made which are $.010^{mm}$ in thickness. What surface would 3^g of gold cover?

47. Find the weight of an oak board 3.25^m long, $.31^m$ wide, and $.04^m$ thick; the specific gravity of the oak being .808.

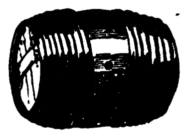
48. Find the weight of a bar of iron having the following dimensions: length 3.6^{m}, width 6^{cm}, thickness 2^{cm}; the specific gravity of the iron being 7.8.

49. How many lead balls each weighing 27^{g} could be obtained by melting a mass of lead, cubic in form, the edge measuring .356^{m}, the specific gravity of the lead being 11.35?

50. Marble costs $30.95 a cubic meter, and the specific gravity of marble is 2.73. If a block of marble weighs 1260^{kg}, what is its volume, and what is it worth?

51. Sea-water contains 28 parts, by weight, of salt in 1000. A liter of sea-water weighs 1.025^{kg}. How many kilograms of salt could be obtained from 126.276842^{cbm} of sea-water?

52. An empty cask weighs 17.06^{kg}; when filled with water it weighs 275.8^{kg}. How many liters does it hold? How many casks of this size would it require to receive the wine from a vat containing 3.008^{cbm}?



Cask.

53. It takes about 204.8^{l} of wheat to sow a hektar. How many cubic meters would it take to sow a square kilometer?

54. A piece of road 1^{km} long and 7^{m} wide is to be macadamized; the macadamizing is to be 33^{cm} deep; it costs 43 cents a cubic meter to prepare the stones. What will enough for the road cost?

55. A gasometer holds 28,000^{cbm} of gas. How many jets would this gasometer feed, when each jet burns 125^{l} an hour, and is used 4 hours every evening?

56. The city of Venice is situated in the midst of a great lake of salt water, communicating with the sea, and all the rain-water is caught for the cisterns. Ordi-

nary years the fall of rain in Venice is 82^{cm}; the surface of the city, after the canals have been deducted, is 520^{ha}; reckoning the population at 115,530, how many liters a day of rain-water could each inhabitant have?

57. Find the weight of a bar of iron 5.35^{m} long, 4.56^{cm} thick, and 3.54^{cm} wide. Find, also, the width of an oak beam 4.30^{m} long, 9.12^{cm} thick, which has the same weight. The specific gravity of the oak to be reckoned at 1.026, that of the iron 7.788.

58. Give the specific gravity and volume of a body weighing 35^{kg} in air and 30^{kg} in water.

59. A ster of piled oak wood weighs 425^{kg}; the specific gravity of the wood is .74. What is the volume occupied by the spaces between the logs? For how much must 100^{kg} of separate sticks be sold in order to bring the same amount as when sold by the ster; a ster selling for $2.20?

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63. A rectangular sheet of tin of uniform thickness is 85^{cm} wide, 1.35^{m} long; it weighs 268^{g}. What is its thickness, reckoning the specific gravity of tin at 7.3?

64. The fine coal which collects about the shafts of the mines and in the coal-yards, was for a long time

wasted, because it could not be burned in stoves and grates. Now, this dust is mixed with tar in the proportion of 92^{kg} of dust and 8^{kg} of tar; the mixture is heated, and afterwards pressed in rectangular moulds of 14.75^{cm}, 18.5^{cm}, and 29^{cm}; each one of these blocks weighs 10^{kg}, they are sold at $3.00 a ton, and make excellent fuel for heating steam boilers. Give the specific gravity of this fuel; also, the sum which would be realized in thus utilizing $800{,}000^{t}$ of coal dust, the cost of tar, mixing, etc., being $.50 a ton?

65. A bar of iron a millimeter square on the end will break under a tension of 30^{kg}. Find the length at which a suspended bar of iron will break from its own weight, the specific gravity of the iron being 7.8?

66. Fifty-three kilograms of starch are obtained from 100^{kg} of wheat. A hektar of land produces 1363^{l} of wheat; a hektoliter of wheat weighs 78^{kg}. If the wheat harvested from a field measuring 2^{ha} and 33^{qm} is taken to a starch factory, how much starch will be made from it?

67. A gardener wishes to provide glass for his hot-beds. The beds cover 2.65^{a}; the panes will cover .75 of the whole surface, the rest being taken up by the frames and alleys. First, find how many panes measuring 45^{cm} by 37^{cm} it will take to cover the beds; then find the price of the glass, at a cost of 95 cents a square meter.

68. A jar full of water weighs 1.325^{kg}; filled with mercury it weighs 12.540^{kg}. What is the capacity of the jar, and its weight? The specific gravity of the mercury is 13.59.

69. A hektoliter of rape-seed weighs 63^{kg}, and 32^{l} of oil can be extracted from it. How many kilograms of the seed will it take to make a hektoliter of oil?

70. Common burning gas is .97 of the weight of air, and a liter of air weighs 1.293^g. In a shop there are 65 jets, each one of which burns 123^l an hour, and is used 5 hours in the winter evenings. Calculate the weight of the gas used in a month, and the expense of lighting the shop, when gas costs 6 cents a cubic meter.

71. A merchant buys one kind of wine at 30 cents a liter, another kind at 21 cents a liter; he mixes the two kinds by putting 5^l of the first with 8^l of the second. For how much a liter must he sell the mixture in order to gain $3.75 a hektoliter?

72. If it requires 360 tiles to drain an ar of land, what will it cost to drain 17.784^{ha}, when the tiles cost $20 a thousand, and the expense of laying is the same as the cost of the tiles?

73. It is found in building that hewn stone of medium durability ought not to support, as a permanent weight, more than .07 of the weight that it would require to crush it. A certain kind of stone used for building will be crushed under a weight of 250^{kg} a square centimeter. What is the greatest height to which a wall constructed of this material can be safely carried, the specific gravity of the stone being 2.1?

74. Several different kinds of wine are mixed as follows: 245^l at 20 cents a liter, 547^l at 15 cents a liter, 344^l at 25 cents a liter. How much does the mixture cost a liter?

75. A farmer wishes to drain a field of 8.75^{ha}. Each hektar requires 750^m of ditches. The opening of these ditches costs 10 cents a running meter; the tiles are 30^{cm} long and cost $15 a thousand. He pays 2 cents a meter for laying the tiles, and 4 cents a meter for filling the ditches. What is the cost of draining the field?

CHAPTER X.

MULTIPLES AND MEASURES OF NUMBERS.

216. If one number can be divided by another number, *without remainder*, the divisor is called a **factor**, or **measure**, of the dividend, and the dividend a **multiple** of the divisor.

Thus, 35 can be divided by 5 without remainder; therefore 5 is called a factor or measure, of 35, and 35 a multiple of 5.

217. Numbers which can be divided, without remainder, only by themselves and 1, are called **prime numbers.**

The smaller ones are easily found by trial, such as 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, etc.

218. Other numbers are each the product of a **fixed set** of prime factors, and are called **composite numbers.**

219. Numbers divisible by 2 are called **even numbers.** All other numbers are **odd numbers.** All primes (except 2) are odd.

220. Write a series of natural numbers in order; cancel the even ones; then place a dot over the multiples of 3: you produce this result:

$$1,\ \cancel{2},\ \dot{3},\ \cancel{4},\ 5,\ \dot{\cancel{6}},\ 7,\ \cancel{8},\ \dot{9},\ \cancel{10},\ 11,\ \dot{\cancel{12}},\ 13,\ \cancel{14},\ \dot{15},\ \text{etc.}$$

Each multiple of 6 has both the dot and the cancelling line, and the only numbers without the dot or line come just before or just after the multiples of 6. Therefore,

If a prime number be divided by 6 *the remainder must be either* 1 *or* 5.

221. Find the prime factors of 72.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

That is, $$72 = 2 \times 2 \times 2 \times 3 \times 3,$$
$$= 2^3 \times 3^2.$$

Hence, to separate a composite number into its prime factors,

Divide the given number by any prime number that is contained in it without remainder, then the quotient by any prime number that is contained in it without remainder, and so on until the quotient is itself a prime number. The several divisors and the last quotient are the prime factors required.

222. The following tests are very useful for determining without actual division whether a number contains certain factors:

1. A number is divisible by 2 if its *last digit is even.*
2. A number is divisible by 4 (2^2) if the number denoted by the *last two digits* is divisible by 4.
3. A number is divisible by 8 (2^3) if the number denoted by the *last three digits* is divisible by 8.
4. A number is divisible by 3 if the *sum of its digits* is divisible by 3.
5. A number is divisible by 9 (3^2) if the *sum of its digits* is divisible by 9.
6. A number is divisible by 5 if its *last digit* is either 5 or 0.

7. A number is divisible by 25 (5^2) if the number denoted by the *last two digits* is divisible by 25.

8. A number is divisible by 125 (5^3) if the number denoted by the *last three digits* is divisible by 125.

9. A number is divisible by 6 if its *last digit is even* and *the sum of its digits* is divisible by 3.

10. A number is divisible by 11 if the *difference between the sum of the digits in the even places and the sum of the digits in the odd places* is either 0 or a multiple of 11.

NOTE. The shortest method of dividing by 25 is to multiply by 4 and divide by 100; by 125, is to multiply by 8 and divide by 1000.

In adding the digits of a number to determine whether their sum is a multiple of a certain number, omit those digits which are seen at a glance to be multiples of the number. Thus, to discover whether 8,983,167 is divisible by 3, omit 9, 3, 6 (8, 1), which are manifestly multiples of 3, and simply add 8 and 7.

223. Other prime factors, 7, 13, 17, 19, sometimes betray their presence to one familiar with the subject; but, practically, the best way to detect them is to attempt to divide by them.

224. If we divide any number less than 121 (11^2) by 11, or by a number greater than 11, it is plain that the quotient is less than 11.

If we divide any number between 121 and 143 (11×13) by 11, the quotient will evidently lie between 11 and 13; and, since there are no prime numbers between 11 and 13, the quotient, if a whole number, must be composite, and contain factors smaller than 11.

What is thus proved of 11 and 13 is evidently true of any two *adjacent* prime numbers; namely, that, excepting the *second power* of the smaller prime number, *every composite number less than the product of two adjacent prime numbers, contains prime factors less than the smaller of these two numbers.*

Thus, every composite number less than 4087 (61×67), except 3721 (61^2), contains prime factors less than 61.

225. From the preceding article, the value of the following table, in discovering the prime factors of a given number, will be apparent.

Primes . .	7	11	13	17	19	23	29	31	37
Powers . .	49	121	169	289	361	529	841	961	1369
Products .	77	143	221	323	437	667	899	1147	1517
Primes . .	41	43	47	53	59	61	67	71	73
Powers . .	1681	1849	2209	2809	3481	3721	4489	5041	5329
Products .	1763	2021	2491	3127	3599	4087	4757	5183	5767
Primes . .	79	83	89	97	101	103	107	109	113
Powers . .	6241	6889	7921	9409	10201	10609	11449	11881	12769
Products .	6557	7387	8633	9797	10403	11021	11663	12317	14351

Opposite to "Powers" are placed the squares of the primes from 7 to 109; and opposite to "Products" are placed the products of the successive pairs of adjacent primes from 7 to 113.

226. Find the prime factors of 610,764.

2^2	610,764
3	152,691
7	50,897
11	7,271
	661

As 64 is divisible by 4, but 764 is not divisible by 8, 2^2 is the highest power of 2 contained in 610,764.

As the sum of the digits 152,691 is divisible by 3 but not by 9, 3^1 is the highest power of 3 contained in 152,691.

The next quotient, 50,897, does not contain 5; but divided by 7 gives 7271. 7271 does not contain 7; but, since $7 + 7 - (2 + 1) = 11$, it is divisible by 11.

The quotient 661 when divided by 6 gives a remainder of 1, which shows that it *may be* a prime number. It cannot be divided by 11, 13, 17, or 19, and is seen by the table to be less than $667 = (23 \times 29)$, and not equal to 529 (23^2); therefore it is a prime number.

Thus, $610{,}764 = 2^2 \times 3 \times 7 \times 11 \times 661$.

Exercise XI.

Find the prime factors of:

1. 148; 264; 178; 183; 173; 187; 346; 343;
2. 210; 353; 5280; 231; 31,416; 1369; 1368;
3. 247; 327; 179; 83; 2125; 2353; 2333;
4. 165; 168; 2148; 16,662; 321; 1551; 38;
5. 82; 129; 72; 66; 68; 65; 76; 86; 88; 142;
6. 326; 368; 464; 292; 362; 365; 730; 42;
7. 868; 999; 822; 1346; 7641; 6234; 234;
8. 579; 577; 212; 126; 128; 8192; 8190;
9. 8197; 3125; 2401; 1331; 78,309; 25,179.

227. A number is not only divisible by each of its prime factors, but by every possible combination of them. For example, 120 is $2^3 \times 3 \times 5$, and is divisible either by 2, 4, 8, 6, 12, 24, 30, 60, 10, 20, 40, or 15.

228. The number 14.21 may be put in the form of $1421 \times .01$; and be thus resolved into $7^2 \times 29 \times .01$. But .01 is not properly a factor, it is a divisor; it is the reciprocal of $2^2 \times 5^2$. Nevertheless, it is frequently of great practical advantage to separate mixed decimals, in this way, by first taking out the apparent factors .1, .01, .001, etc. Thus, the factors of 142.1 may be said to be 7, 7, 29, and .1; of 1.421, 7, 7, 29, and .001.

Exercise XII.

Find the prime factors of:

1. 8.4; 7.6; 1.08; .144; .036; .037; 21.45;
2. 14.6; 2.61; 21.2; 78.54; .5236; .00052;
3. 8.67; 48.3; 99.99; 5.04; 1.485; .216;
4. 34.87; 32.4; 5.115; 71.2; 2.993.

Greatest Common Measure.

229. The measures of 30 and 50 respectively are: 1, 2, 3, 5, 6, 10, 15, 30; 1, 2, 5, 10, 25, 50. It will be seen that these two numbers have the measures 1, 2, 5, 10 in common, and of these measures 10 is the greatest.

230. The measures that two or more numbers have in common are called their *common measures*, and the greatest of these is called their **Greatest Common Measure,** which for the sake of brevity is indicated by the letters G. C. M.

231. If two or more numbers have no common measure except unity, they are said to be *prime to each other*. Thus 8 and 27 are prime to each other.

232. The prime factors of 30 are 2, 3, 5.
The prime factors of 50 are 2, 5^2.
The prime factors *common* to 30 and 50 are 2, 5.
The G. C. M. of 20 and 30, namely 10, is 2×5.

That is,

The G. C. M. *of two or more numbers consists of the* **prime factors common to the numbers,** *each prime factor having the* **lowest exponent** *that it has in any one of the numbers.*

233. Hence, to find the G. C. M. of two or more numbers,

Separate the numbers into their prime factors.

Select the **lowest power** *of each factor that is common to the given numbers, and find the product of these powers.*

Find the G. C. M. of 108, 396, 1440.

$$\begin{array}{r|r} 2^2 & 108 \\ \hline 3^2 & 27 \\ \hline & 3 \end{array} \qquad \begin{array}{r|r} 2^2 & 396 \\ \hline 3^2 & 99 \\ \hline & 11 \end{array} \qquad \begin{array}{r|r} 2^3 & 1440 \\ \hline 2^2 & 180 \\ \hline 3^2 & 45 \\ \hline & 5 \end{array}$$

Hence, the G. C. M. $= 2^2 \times 3^2$, or 36.

234. The factors that are common to two or more numbers may be taken out of the numbers simultaneously, as follows:

$$\begin{array}{r|rrr} 2^2 & 108 & 396 & 1440 \\ \hline 3^2 & 27 & 99 & 360 \\ \hline & 3 & 11 & 40 \end{array}$$

By the tests given in § 222, 4 (2^2) is seen to be common to the numbers, and 9 (3^2) common to the resulting quotients; the quotients 3, 11, 40, have no common factor, therefore 2^2 and 3^2 are the only factors common to the numbers. Hence, their G. C. M. is $2^2 \times 3^2$, or 36.

Exercise XIII.

Find the G. C. M. of

1. 27 and 33.
2. 13 and 39.
3. 8 and 28.
4. 27 and 45.
5. 81 and 108.
6. 4, 10, 12.
7. 4, 6, 10.
8. 9, 12, 21.
9. 10, 15, 25.
10. 14, 98, 42.
11. 30, 18, 54.
12. 14, 56, 42.
13. 96, 36, 48.
14. 84, 105, 63.
15. 24, 60, 84, 128.
16. 45, 81, 27, 90.
17. 78, 18, 54, 42.
18. 98, 28, 70, 42.
19. 96, 112, 80, 32.
20. 24, 96, 48, 120.
21. 84, 252, 168, 210.
22. 33, 88, 77, 55.
23. 252, 315, 420, 504.
24. 128, 192, 320, 368, 432.
25. 136, 204, 357, 459.
26. 909, 1414, 2323, 4242.

235. When it is required to find the G. C. M. of two or more numbers which cannot readily be resolved into factors, the method to be employed is as follows:

Find the G. C. M. of 18 and 48.

$$
\begin{array}{l}
18)48(2 \\
\quad 36 \\
\quad \overline{12})18(1 \\
\qquad 12 \\
\qquad \overline{\ 6})12(2 \\
\qquad\quad 12 \\
\qquad\quad \overline{\quad}
\end{array}
$$

6 is the G. C. M. required.

This method depends upon two principles:

1. That every factor of a number is also a factor of every multiple of that number. Thus 4, which is a factor of 12, is a factor also of 24, 36, etc.

2. That any common factor of two numbers is also a factor of their sum and of their difference. Thus 4, which is a common factor of 24 and 36, is also a factor of 60 and 12.

Apply these principles to this example:

Since 6 is a factor of itself and of 12, it is, by (2), a factor of 18.

Since 6 is a factor of 18, it is, by (1), a factor of 2×18, or 36; and therefore, by (2), it is a factor of $36 + 12$, or 48.

Hence, 6 is a common factor of 18 and 48.

Again, every common factor of 18 and 48 is, by (1), a factor of 2×18, or 36; and, by (2), a factor of $48 - 36$, or 12.

Every such factor, being now a common factor of 18 and 12 is, by (2), a factor of $18 - 12$, or 6.

Therefore, the **greatest** common factor of 18 and 48 is contained in 6, and cannot be greater than 6. And 6, which has been shown to be a common factor of 18 and 48, must be their G. C. M.

236. It will be seen that every remainder in the course of the operation contains, as a factor of itself, the G. C. M. sought; and that this is the greatest factor common to that remainder and the preceding divisor.

Therefore, a factor which is discovered, at any stage of the process, to belong to one of the numbers and not to the

other, may be ejected; and a factor which is discovered, at any stage of the process, to belong to both numbers, may be taken out and reserved as a factor of the G. C. M.

(1) Find the G. C. M. of 11,237 and 12,559.

```
11237)12559(1
      11237
    2)1322
       661)11237(17
           661
           4627
           4627
```

Hence, the G. C. M. is 661.

The factor 2 is thrown out of the first remainder 1322, for it is not contained in 11,237, and therefore is not a factor of the G. C. M. sought.

(2) Find the G. C. M. of 269,178 and 352,002.

```
6 | 269178   352002
    44863 )  58667(1
             44863
           4)13804
              3451)44863(13
                   3451
                   10353
                   10353
```

Hence, the G.C.M. is $6 \times 3451 = 20{,}706$.

The common factor 6 is first taken out from both numbers. From the remainder 13,804 the factor 4, which is prime to 44,863, is ejected. The resulting number 3451 is contained 13 times in 44,863, and therefore the G.C.M. is $6 \times 3451 = 20{,}706$.

Exercise XIV.

Find the G. C. M. of

1. 2,479 and 3,589.
2. 3,045 and 6,195.
3. 568 and 712.
4. 11,023 and 6,493.
5. 1,485 and 2,160.
6. 7,040 and 7,392.
7. 2,760 and 4,485.
8. 1,177 and 2,675.
9. 78,473 and 94,653.
10. 35,143 and 10,283.
11. 44,323 and 61,087.
12. 232,353 and 39,699.
13. 33,853 and 35,017.
14. 5,115 and 7,254.
15. 2,268 and 3,348.
16. 1,003 and 2,419.
17. 419 and 52,301.
18. 30,072 and 133,784.
19. 4,257 and 10,836.
20. 17,104 and 27,794.

237. To find the G. C. M. of several large numbers:

Find the G. C. M. of two of the numbers; then of that result and a third number; then of that result and a fourth; and so on. The last G. C. M. is the one required.

The work can generally be very much shortened by removing from each of the numbers all factors less than 13. Of these, the factors common to the numbers must be retained as factors of the G. C. M.

Find the G. C. M. of 3555, 4977, and 6636.

3 | 3555	4977	6636
5)1185	3)1659	4)2212
3)237	7)553	7)553
79	79	79

Hence, the G. C. M. is 3×79, or 237.

The common factor 3 is first taken out and reserved as a factor of the G. C. M. From the resulting quotients the factors less than 13 are removed, and 79 is found to be common to the numbers. Hence, their G. C. M. is 3×79.

EXERCISE XV.

Find the G. C. M. of

1. 855, 1,197, 1,596.
2. 3,864, 3,404, 3,657.
3. 15,561, 11,115, 13,585.
4. 2,943, 2,616, 4,578.
5. 1,177, 1,391, 1,819.
6. 4,939, 1,347, 3,143.
7. 740, 333, 296.
8. 833, 1,785, 1,309.
9. 7,326, 8,547, 9,768, 22,755.
10. 4,994, 7,491, 9,988, 12,485, 16,571.

LEAST COMMON MULTIPLE.

238. The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, etc.

The multiples of 3 are 3, 6, 9, 12, 15, 18, etc.

The multiples common to 2 and 3 are seen to be 6, 12, 18, etc.; and of these, 6 is the least.

239. The multiples that two or more numbers have in common are called their *common multiples*, and the least of these is called their **Least Common Multiple**, which is indicated by the letters L. C. M.

Find the L. C. M. of 7, 8, 9, 10.

The L. C. M. of 7, 8, 9, 10 must contain the factor 7, else it would not be a multiple of 7. It must also contain 2^3 to be a multiple of 8, and 3^2 to be a multiple of 9. It must contain the factors 2 and 5 to be a multiple of 10. That is, the L. C. M. of 7, 8, 9, 10, must contain the factors 7, 2^3, 3^2, and 5; therefore it is $7 \times 2^3 \times 3^2 \times 5$, or 2520. Hence,

240. To find the L. C. M. of two or more numbers:

Separate each number into its prime factors.

Select from these the **highest power** *of each.*

Find the product of these powers.

Find the L. C. M. of 14, 18, 21, 27, 28, 126.

$$14 = 2 \times 7,$$
$$18 = 2 \times 3^2,$$
$$21 = 3 \times 7,$$
$$27 = 3^3,$$
$$28 = 2^2 \times 7,$$
$$126 = 2 \times 3^2 \times 7.$$

Hence, the L. C. M. $= 2^2 \times 3^3 \times 7 = 756$.

241. The L. C. M. of 14, 18, 21, 27, 28, 126, may be found as follows:

2	~~14~~	~~18~~	~~21~~	27	28	126
2				27	14	63
3				27	~~7~~	63
3				9		21
				3		7

Hence, the L. C. M. $= 2^2 \times 3^3 \times 7$, or 756.

Since 14 is contained in 28, it is omitted, for any multiple of 28 is also a multiple of 14. Likewise 18, which is contained in 126, and 21, which is also contained in 126, are omitted.

The even numbers are divided by 2; the quotients and the odd numbers are written below the horizontal line.

The operation of dividing by 2 is repeated; the quotients and the odd numbers are written below.

Of the resulting numbers (27, 7, 63) 7, which is contained in 63, is omitted, and division by 3 gives the quotients 9 and 21.

The second division by 3 gives the quotients 3 and 7, which are seen to be prime to each other.

By this process the prime factors of each of the given numbers are obtained as divisors or as last quotients; therefore, the product of the divisors and last quotients is the L. C. M. required.

Exercise XVI.

Find the L. C. M. of

1. 6, 14, 21.
2. 8, 12, 3, 24.
3. 6, 10, 15,
4. 9, 12, 18, 4.
5. 15, 21, 35.
6. 12, 20, 24.
7. 14, 24, 28.
8. 12, 15, 20.
9. 16, 24, 32.
10. 21, 33, 77.
11. 27, 33, 99.
12. 7, 11, 13.
13. 77, 55, 35.
14. 16, 18, 27, 72.
15. 10, 12, 22, 33, 60.
16. 15, 16, 18, 20, 22, 24.
17. 56, 64, 70, 84, 112.
18. 48, 54, 81, 144, 162.
19. 75, 100, 120, 150, 180.
20. 112, 168, 196, 224.
21. 7, 14, 15, 21, 45.
22. 16, 25, 81.
23. 26, 39, 52, 65.
24. 80, 72, 225, 48.
25. 10, 20, 30, 40, 50, 60.
26. 30, 42, 105, 70.
27. 36, 24, 35, 20.
28. 7, 11, 14, 15.
29. 12, 18, 27, 63, 28.
30. 34, 26, 65, 85, 51, 39.
31. 12, 18, 96, 144.
32. 84, 156, 63, 99.
33. 17, 51, 119, 210.
34. 16, 30, 48, 56, 72.
35. 27, 33, 54, 69, 132.
36. 15, 26, 39, 65, 180.
37. 44, 126, 198, 280, 330.
38. 50, 338, 675, 975.
39. 552, 575, 920.
40. 228, 304, 342.
41. 1,080 and 1,260.
42. 600 and 480.
43. 1,564 and 1,932.
44. 2,530 and 1,760.
45. 936 and 2,925.
46. 3,432 and 4,032.
47. 1,875 and 2,425.
48. 1,632 and 2,976.
49. 1,001 and 2,233.
50. 539 and 1,463.

242. If the given numbers are large and contain no prime factors that can readily be detected, it is best to obtain the common factors by the process for finding the G. C. M. under like circumstances.

Find the L. C. M. of 1247 and 1769.

```
1247)1769(1
     1247
   2| 522
   9| 261
       29)1247(43
          116
           87
           87
```

Hence, the G. C. M. $= 29$; and $1247 = 29 \times 43$,
also $1769 = 29 \times 61$.

$\therefore$ the L. C. M. $= 29 \times 43 \times 61 = 1247 \times 61 = 76{,}067$.

243. From this process it will be seen that:

The L. C. M. *of two numbers may be found by dividing one of the numbers by their* G. C. M. *and multiplying the quotient by the other number.*

The L. C. M. of two prime numbers, or of two numbers prime to each other, is their product.

EXERCISE XVII.

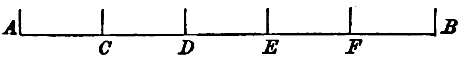
Find the L. C. M. of

1. 424 and 583.
2. 319 and 407.
3. 1,679 and 1,932.
4. 1,003 and 2,419.
5. 1,003 and 1,357.
6. 899 and 961.
7. 407, 703, 444.
8. 411, 959, 2,055.
9. 221 and 351.
10. 1,426 and 989.
11. 3,864, 3,404, 3,657.
12. 539 and 253.
13. 2,943, 2,616, 4,578.
14. 2,863 and 1,151.
15. 1,177, 1,391, 1,819.
16. 5,317 and 2,863.
17. 12,703 and 12,879.
18. 23,309 and 10,753.
19. 4,939 and 3,143.
20. 4,199 and 6,137.

CHAPTER XI.

COMMON FRACTIONS.

244. Instead of dividing the unit into 10 equal parts, 100 equal parts, and so on, it is often more convenient to divide it into halves, thirds, quarters, or other equal parts. These are called **common fractions.**



245. If a line AB be divided into 5 equal parts, at the points C, D, E, and F, the parts AC, AD, AE, and AF are fractions of the whole, being respectively *one-fifth*, *two-fifths*, *three-fifths*, *four-fifths* of the whole line, and are written $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

246. The expression $\frac{4}{5}$ means:

I. 4 of the parts when 1 unit has been divided into 5 equal parts.

II. $\frac{1}{5}$ of 4 units; for if *four* units be divided into 5 equal parts, one of these parts will be *four* times as great as one of the parts obtained by dividing *one* unit into 5 equal parts; that is, will be equal to $\frac{4}{5}$ of one unit.

III. The quotient of 4 divided by 5.

247. In the fraction $\frac{4}{5}$, the lower figure shows into how many equal parts the whole has been divided, and is therefore a **divisor.**

But since it shows the number of parts into which the whole has been divided, it shows the **name** of each part, and is therefore called the **denominator.**

The upper figure shows how many of these parts are taken, and is therefore called the **numerator.***

248. It will be observed that a figure written *above* the line serves a very different purpose from that of a figure written *below* the line. A figure written above the line denotes **number,** a figure below the line, **name.**

249. The expressions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{1}{6}$, $\frac{4}{7}$, $\frac{17}{21}$, are read one-half, two-thirds, three-fourths, three-fifths, one-sixth, four-sevenths, seventeen twenty-firsts.

In like manner, all other fractions are read by reading first the numerator, and then the denominator as an ordinal, with an *s* added if the numerator is more than one.

Read:

$\frac{5}{13}$, $\frac{7}{10}$, $\frac{18}{37}$, $\frac{99}{1000}$, $\frac{8}{21}$, $\frac{34}{89}$, $\frac{13}{32}$, $\frac{55}{144}$, $\frac{55}{56}$, $\frac{17}{55}$, $\frac{21}{23}$, $\frac{19}{38}$, $\frac{7}{12}$, $\frac{5}{11}$.

250. The numerator and denominator are called the **terms** of a fraction.

251. A **proper fraction** is one of which the numerator is less than the denominator; as $\frac{5}{7}$.

252. An **improper fraction** is one of which the numerator equals or exceeds the denominator; as $\frac{7}{7}$, $\frac{15}{2}$.

It is obvious that when the numerator is greater than the denominator, more than one unit must be regarded as divided into parts; thus, $\frac{29}{13}$ means that three units have been divided each into thirteen equal parts, and that all the parts of two units and three parts of the third are taken.

253. A **mixed number** is an expression consisting of a whole number and a fraction; as $5\frac{2}{7}$.

It means that some entire units are taken, and a fraction of another unit.

---

* Numerator and denominator are derived from the Latin *numerare,* to count, and *denominare,* to name.

254. An improper fraction represents a quantity which can also be represented by a whole number, or else by a mixed number. Thus, $\frac{17}{7} = 2\frac{3}{7}$.

For, if we suppose several units to be divided each into seven equal parts, and we take 17 of these parts, 14 (that is, 2×7) will make two units, and the three parts remaining will be three-sevenths of another unit.

255. To reduce an improper fraction to a whole or mixed number,

Divide the numerator by the denominator.

The quotient will be the whole number, and the remainder, if there be any, will be the numerator of the fractional part, of which the denominator is the same as the denominator of the improper fraction.

Exercise XVIII.

Reduce to whole or mixed numbers:

1. $\frac{18}{7}$.	4. $\frac{107}{11}$.	7. $\frac{874}{25}$.	10. $\frac{4684}{177}$.	13. $\frac{529}{29}$.
2. $\frac{21}{8}$.	5. $\frac{218}{16}$.	8. $\frac{481}{18}$.	11. $\frac{2829}{758}$.	14. $\frac{786}{42}$.
3. $\frac{25}{4}$.	6. $\frac{242}{22}$.	9. $\frac{3828}{75}$.	12. $\frac{299}{28}$.	15. $\frac{8975}{25}$.

256. A whole number, or a mixed number, represents a quantity which can also be represented by an improper fraction. Thus, $5\frac{7}{13} = \frac{72}{13}$.

For each unit contains 13 *thirteenths;* therefore 5 units contain 5×13, or 65, *thirteenths;* which, together with the 7 *thirteenths*, make 72 *thirteenths*. Therefore,

257. To reduce a mixed number to an improper fraction,

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; under this sum write the denominator.

258. A whole number may be expressed as a fraction with any given denominator. Thus, $7 = \frac{63}{9}$.

For, as each unit contains 9 *ninths*, 7 units contain 7×9 *ninths*, that is, 63 *ninths*.

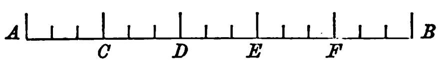
A whole number may be written in a fractional form with 1 for a denominator. Thus, $4 = \frac{4}{1}$.

Exercise XIX.

Reduce to improper fractions:

1. $3\frac{4}{9}$.
2. $5\frac{9}{10}$.
3. $12\frac{4}{11}$.
4. $10\frac{7}{19}$.
5. $8\frac{3}{7}$.
6. $12\frac{13}{15}$.
7. $84\frac{17}{20}$.
8. $864\frac{13}{97}$.
9. $41\frac{83}{1000}$.
10. $41\frac{3}{10000}$.
11. $400\frac{127}{400}$.
12. $5000\frac{83}{5000}$.
13. $10,000\frac{14}{15}$.
14. $3001\frac{83}{700}$.
15. $73\frac{72}{99}$.
16. Express 8, 7, 3, 5, 12, 13, 18, 20, 25 in the form of fractions, each having 5 for a denominator.
17. Express 21 in the form of fractions, having for denominators 3, 5, 7, 8, 12, 13, 20, 25, 30, 37.
18. Express 12, 15, 23 in the form of fractions each having for denominators 12, 15, 23, respectively.

259. If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the **value** of the fraction is not altered.



Thus, if the line AB be divided into 5 equal parts at the points C, D, E, and F, then AF is $\frac{4}{5}$ of AB. (1)

Now, if each of the parts be subdivided into 3 equal parts, AB will contain 15 of these subdivisions, and AF 12 of these subdivisions.

Therefore, AF is $\frac{12}{15}$ of AB. (2)

Comparing (1) and (2), $\frac{12}{15} = \frac{4}{5}$. Therefore,

260. To reduce a fraction to lower terms,

Divide the numerator and denominator by any common factor.

A fraction is expressed in its lowest terms when both the numerator and denominator are divided by the greatest common divisor.

(1) Reduce $\frac{324}{756}$ to its lowest terms.

$$\frac{324}{756} = \frac{36}{84} = \frac{9}{21} = \frac{3}{7};$$

the common divisors used being 9, 4, and 3.

(2) Reduce $\frac{1261}{1649}$ to its lowest terms.

Since no common factor can be readily detected, we find the G. C. M. thus:

$$\begin{array}{rl}
1261\,)\,1649\,(\,1 & \\
\underline{1261} & \\
4\,\boxed{388} & \\
97\,)\,1261\,(\,13 & \\
\underline{1261} &
\end{array}$$

Divide 1261 and 1649 each by 97, their G. C. M.

Then, $\frac{1261}{1649} = \frac{13}{17}$.

Exercise XX.

Reduce to lowest terms:

1. $\frac{120}{192}$.	7. $\frac{1848}{2352}$.	13. $\frac{6732}{9108}$.	19. $\frac{516}{2107}$.	25. $\frac{1177}{2675}$.
2. $\frac{105}{185}$.	8. $\frac{3960}{12672}$.	14. $\frac{6840}{27360}$.	20. $\frac{3872}{92807}$.	26. $\frac{11445}{15369}$.
3. $\frac{928}{1320}$.	9. $\frac{1848}{8003}$.	15. $\frac{5760}{7000}$.	21. $\frac{78473}{94653}$.	27. $\frac{14141}{16289}$.
4. $\frac{1728}{2448}$.	10. $\frac{924}{1092}$.	16. $\frac{875}{10000}$.	22. $\frac{17596}{26145}$.	28. $\frac{881496}{104763}$.
5. $\frac{1296}{6561}$.	11. $\frac{2640}{2970}$.	17. $\frac{2208}{4140}$.	23. $\frac{44323}{61087}$.	29. $\frac{65065}{999999}$.
6. $\frac{2310}{3080}$.	12. $\frac{324}{1092}$.	18. $\frac{1015}{1566}$.	24. $\frac{339}{1248}$.	30. $\frac{428571}{999999}$.

Multiplication of Fractions.

261. 4×2 *dollars* $= 8$ *dollars.*

4×2 *sevenths* $= 8$ *sevenths.*

If two like quantities are taken 4 times, the result will be 4 times 2 of the same quantities.

$\frac{2}{7} \times 14$ means $\frac{2}{7}$ of 14, which equals 4. Therefore,

262. To find the product of a whole number and a fraction,

Find the product of the numerator and whole number, and divide the result by the denominator.

A factor common to the whole number and the denominator of the fraction may be cancelled. For, cancelling a factor common to the whole number and the denominator of the fraction *before* the multiplication, is evidently equivalent to dividing the numerator and denominator of the resulting fraction by that factor *after* the multiplication. Which may be done by §§ 259, 260.

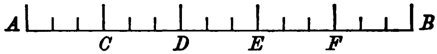
EXERCISE XXI.

Find the product of:

1. $\frac{3}{4} \times 2$.	6. $16 \times \frac{5}{8}$.	11. $\frac{13}{20} \times 3$.	16. $8 \times \frac{13}{20}$.
2. $\frac{3}{4} \times 9$.	7. $\frac{5}{8} \times 2$.	12. $\frac{13}{20} \times 4$.	17. $\frac{13}{20} \times 10$.
3. $10 \times \frac{3}{5}$.	8. $\frac{2}{15} \times 5$.	13. $5 \times \frac{13}{20}$.	18. $\frac{13}{20} \times 12$.
4. $15 \times \frac{3}{8}$.	9. $27 \times \frac{5}{9}$.	14. $6 \times \frac{13}{20}$.	19. $\frac{13}{20} \times 15$.
5. $\frac{9}{21} \times 7$.	10. $\frac{13}{20} \times 2$.	15. $7 \times \frac{13}{20}$.	20. $\frac{13}{20} \times 20$.

263. To find the value of a fraction of a fraction; or,
To multiply a fraction by a fraction:

Multiply $\frac{4}{5}$ by $\frac{2}{3}$.



$\frac{4}{5}$ multiplied by $\frac{2}{3}$ means $\frac{2}{3}$ of $\frac{4}{5}$.

If the line AB be divided into 5 equal parts at the points C, D, E, and F, AF will be $\frac{4}{5}$ of AB.

Now, if each part be subdivided into three equal parts, there will evidently be 15 such parts in the whole line, and each part will be $\frac{1}{15}$ of the line.

That is, $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$ of the whole.

$\frac{1}{3}$ of $\frac{4}{5}$ will be $\frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$, or $\frac{4}{15}$ of the whole.

And $\frac{2}{3}$ of $\frac{4}{5}$ will be *twice* $\frac{4}{15}$, that is, $\frac{8}{15}$ of the whole.

Therefore,

264. To multiply a fraction by a fraction,

Find the product of the numerators for the numerator of the product, and of the denominators for the denominator of the product.

Mixed numbers must first be reduced to improper fractions.

Any factor common to a numerator and denominator should be cancelled before the multiplication.

$$(1)\quad \frac{12}{25}\times 2\frac{13}{16}\times\frac{10}{21}=\frac{12}{25}\times\frac{45}{16}\times\frac{10}{21},$$

$$=\frac{\overset{\cancel{3}}{\cancel{12}}}{\underset{\cancel{5}}{\cancel{25}}}\times\frac{\overset{9}{\cancel{45}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{16}}}\times\frac{\overset{\cancel{2}}{\cancel{10}}}{\underset{7}{\cancel{21}}}=\frac{9}{14}.$$

12 and 16 contain a common factor, 4, which is cancelled;

3 (the quotient of 12 by 4) and 21 contain a common factor, 3;

45 and 25 the common factor 5;

5 and 10 the common factor 5;

finally, 2 and 4, the common factor 2; and the common factors are all cancelled.

There remains in the numerator 9; and in the denominator 2×7, from which is obtained the simple fraction $\frac{9}{14}$.

Exercise XXII.

Simplify:

1. $\frac{2}{3}$ of $\frac{7}{11}$.
3. $\frac{3}{7}$ of $\frac{5}{9}$.
5. $4\frac{4}{5}\times 2\frac{1}{7}$.
7. $\frac{1}{3}$ of $\frac{3}{5}$ of 10.
2. $\frac{3}{7}$ of $2\frac{1}{10}$.
4. $2\frac{2}{5}\times 2\frac{1}{2}$.
6. $4\frac{5}{6}\times 9\frac{1}{3}$.
8. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{3}$.
9. $\frac{4}{5}\times\frac{5}{6}\times\frac{3}{7}$ of $4\frac{1}{5}$.
14. $\frac{3}{5}$ of $\frac{10}{27}$ of $\frac{9}{20}$ of $8\frac{1}{3}$.
10. $\frac{5}{6}$ of $4\frac{1}{2}$.
15. $\frac{8}{11}\times\frac{20}{21}\times\frac{35}{48}\times 2\frac{10}{19}$.
11. $\frac{8}{9}$ of $\frac{9}{10}$ of $\frac{5}{7}$ of $\frac{3}{4}$ of $\frac{1}{5}$ of $15\frac{3}{4}$.
16. $\frac{42}{43}\times\frac{13}{105}\times 1\frac{7}{308}$.
12. $5\frac{3}{4}\times 8\frac{2}{5}$.
17. $\frac{5}{8}\times\frac{120}{121}\times\frac{66}{85}\times 17$.
13. $\frac{2}{3}\times\frac{4}{7}\times\frac{7}{15}\times 7\frac{1}{2}$.
18. $\frac{38}{39}\times\frac{52}{57}\times\frac{69}{86}\times 1\frac{20}{23}$.
19. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 10.

20. $\frac{7}{25}$ of $\frac{8}{11}$ of 30.

21. $\frac{113}{355} \times \frac{85}{226} \times \frac{12}{35} \times 1\frac{3}{4}$.

22. $\frac{7}{8} \times \frac{3}{4} \times \frac{8}{21} \times \frac{4}{5}$ of $\frac{5}{8}$ of $\frac{3}{4}$ of 8.

23. $\frac{2}{13}$ of $\frac{39}{40}$ of $\frac{52}{117}$.

24. $\frac{9}{11} \times \frac{7}{12} \times \frac{22}{63} \times 48$.

25. $\frac{39}{40}$ of $\frac{7}{65}$ of $\frac{25}{42}$ of 12.

26. $1\frac{2}{3}$ of $4\frac{1}{2}$ of $\frac{3}{8}$.

27. $2\frac{4}{9} \times 1\frac{3}{7} \times 1\frac{13}{15} \times 8$.

28. $3\frac{5}{7}$ of $2\frac{1}{2}$ of $1\frac{8}{13}$ of $1\frac{4}{11}$.

29. $\frac{11}{12} \times 5\frac{1}{3} \times 4\frac{1}{8} \times \frac{7}{22} \times 5$.

30. $\frac{2}{5}$ of $\frac{7}{15} \times 8\frac{2}{7} \times \frac{6}{29}$ of $1\frac{17}{18}$.

31. $\frac{17}{82} \times \frac{27}{38} \times \frac{123}{153}$.

32. $\frac{292}{465} \times \frac{573}{657} \times \frac{248}{1719}$.

33. $\frac{1205}{2763}$ of $\frac{712}{2169}$ of $\frac{1585}{1068}$.

DIVISION OF FRACTIONS.

265. The reciprocal of a number is 1 divided by the number; thus, the reciprocal of 4 is $\frac{1}{4}$, for $4 \times \frac{1}{4} = 1$.

The reciprocal of a fraction is the fraction with its terms interchanged; thus, $\frac{7}{8}$ is the reciprocal of $\frac{8}{7}$, for $\frac{7}{8} \times \frac{8}{7} = 1$.

266. Multiplying by the reciprocal of a number is the same as dividing by the number. See § 162. Therefore,

To divide by a whole number or a fraction,

Multiply by its reciprocal.

Thus:
$$\frac{3}{5} \div 4 = \frac{1}{4} \text{ of } \frac{3}{5} = \frac{1}{4} \times \frac{3}{5} = \frac{3}{20};$$
$$\frac{3}{5} \div \frac{7}{8} = \frac{8}{7} \text{ of } \frac{3}{5} = \frac{8}{7} \times \frac{3}{5} = \frac{24}{35}.$$

Mixed numbers must first be reduced to improper fractions.

EXERCISE XXIII.

Divide:

1. $\frac{24}{35}$ by 6.
2. $\frac{10}{11}$ by 5.
3. $\frac{3}{7}$ by 8.
4. $18\frac{2}{3}$ by 7.
5. $\frac{5}{8}$ by $\frac{3}{4}$.
6. $\frac{12}{16}$ by $\frac{3}{8}$
7. $1\frac{3}{4}$ by $3\frac{1}{3}$.
8. $5\frac{1}{5}$ by $4\frac{2}{3}$.
9. $8\frac{2}{9}$ by $4\frac{1}{9}$.
10. $7\frac{1}{5}$ by $4\frac{2}{7}$.
11. $6\frac{3}{4}$ by $9\frac{1}{2}$.
12. $8\frac{2}{3}$ by $4\frac{2}{3}$.
13. $3\frac{8}{9}$ by $\frac{14}{27}$.
14. $4\frac{3}{7}$ by $6\frac{8}{9}$.
15. 5 by $4\frac{2}{7}$.
16. $3\frac{4}{5}$ of $2\frac{1}{2}$ by $1\frac{1}{2}$ of $2\frac{1}{9}$.
17. $2\frac{2}{7}$ by $3\frac{1}{3}$ of $1\frac{1}{15}$.
18. $2\frac{9}{11}$ of $5\frac{1}{2}$ by $7\frac{3}{4}$.
19. $5\frac{3}{5}$ of $8\frac{1}{3}$ of $1\frac{4}{7}$ by $2\frac{1}{10}$ of $5\frac{5}{9}$.

Least Common Denominator.

267. To reduce fractions to equivalent fractions having the least common denominator:

Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to equivalent fractions having the least common denominator.

The L. C. M. of 3, 4, 6 = 12.

If both terms of $\frac{2}{3}$ be multiplied by 4, the value of the fraction will not be altered, but the form will be changed to $\frac{8}{12}$.

If both terms of $\frac{3}{4}$ be multiplied by 3, the equivalent fraction will be $\frac{9}{12}$.

And if both terms of $\frac{5}{6}$ be multiplied by 2, the equivalent fraction will be $\frac{10}{12}$.

The multipliers, 4, 3, and 2, are obtained by dividing the L. C. M. of the denominators, 12, by the respective denominators of the given fractions. Therefore,

268. To reduce fractions to equivalent fractions having the least common denominator:

Find the L. C. M. *of the denominators.*

Divide the L. C. M. *by the denominator of each fraction.*

Multiply the first numerator by the first quotient, the second numerator by the second quotient, and so on.

The products will be the numerators of the equivalent fractions.

The L. C. M. *of the given denominators will be the denominator of each of the equivalent fractions.*

Reduce $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{12}$ to equivalent fractions having the least common denominator (L. C. D.).

$$4 = 2^2,$$
$$8 = 2^3,$$
$$12 = 2^2 \times 3.$$

Hence, the L. C. D. $= 2^3 \times 3$, or 24.

$$\tfrac{3}{4} = \tfrac{18}{24},$$
$$\tfrac{7}{8} = \tfrac{21}{24},$$
$$\text{and } \tfrac{11}{12} = \tfrac{22}{24}.$$

The result may be written as follows:

$$\frac{18 \quad 21 \quad 22}{24}$$

By this operation the parts represented by the given fractions have been subdivided into smaller parts all of *one size*, and the numerators of the resulting fractions show the *number* of these smaller parts contained in the given fractions. Thus, the quantities denoted by $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{11}{12}$ are each subdivided into 24ths of the unit, and contain respectively 18, 21, and 22 of these subdivisions.

269. *Fractions may be compared by first reducing them to equivalent fractions having a common denominator.*

Determine the greater of the fractions $\frac{5}{7}$ and $\frac{11}{16}$.

In this case the least common denominator is 112.

$$\text{Hence, } \tfrac{5}{7} = \tfrac{80}{112},$$
$$\tfrac{11}{16} = \tfrac{77}{112}.$$
$$\therefore \tfrac{5}{7} \text{ is greater than } \tfrac{11}{16}.$$

Exercise XXIV.

Express with least common denominator:

1. $\frac{1}{2}$, $\frac{2}{5}$, $\frac{5}{6}$.
2. $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{8}$, $\frac{9}{10}$.
3. $\frac{5}{6}$, $\frac{1}{8}$, $\frac{5}{21}$, $\frac{19}{35}$.
4. $\frac{2}{15}$, $\frac{7}{20}$, $\frac{3}{25}$, $\frac{8}{45}$.
5. $\frac{12}{25}$, $\frac{17}{40}$, $\frac{13}{60}$, $\frac{19}{75}$.
6. $\frac{3}{8}$, $\frac{7}{30}$, $\frac{4}{35}$, $\frac{3}{28}$, $\frac{19}{24}$.
7. $\frac{11}{16}$, $\frac{7}{18}$, $\frac{13}{20}$, $\frac{23}{30}$, $\frac{17}{54}$.
8. Which is the greater, $\frac{13}{20}$ or $\frac{17}{25}$? $\frac{5}{6}$ or $\frac{7}{9}$? $\frac{3}{5}$ or $\frac{7}{12}$?
9. Arrange the fractions $\frac{7}{12}$, $\frac{11}{18}$, $\frac{13}{24}$ in order of magnitude.
10. Arrange the fractions $\frac{5}{12}$, $\frac{8}{15}$, $\frac{4}{11}$, $\frac{7}{18}$ in order of magnitude.

Addition of Fractions.

Find the sum of $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{5}{12}$.

$$4 = 2^2,$$
$$6 = 2 \times 3,$$
$$9 = 3^2,$$
$$12 = 2^2 \times 3.$$

Hence, the L. C. D. $= 2^2 \times 3^2 = 36$.

And $\frac{3}{4} + \frac{5}{6} + \frac{7}{9} + \frac{5}{12} = \frac{27+30+28+15}{36}$,
$= \frac{100}{36} = 2\frac{28}{36} = 2\frac{7}{9}$. *Ans.*

270. Therefore, to add fractions:

Reduce the fractions to equivalent fractions having the least common denominator.

Add the numerators of the equivalent fractions.

Write the result over the least common denominator.

Find the sum of $4\frac{11}{20}$, $2\frac{7}{15}$, $5\frac{5}{12}$.

$$20 = 2^2 \times 5,$$
$$15 = 3 \times 5,$$
$$12 = 2^2 \times 3.$$

Hence, the L. C. D. $= 2^2 \times 3 \times 5 = 60$.

And $4\frac{11}{20} + 2\frac{7}{15} + 5\frac{5}{12} = 11\frac{33+28+25}{60}$,
$= 11\frac{86}{60}$,
$= 12\frac{26}{60}$,
$= 12\frac{13}{30}$.

If any of the terms are integers, or mixed numbers, add together *separately* the integers and the fractions, and find the sum of the results.

EXERCISE XXV.

Find the sum of:

1. $\frac{1}{2}+\frac{3}{2}$.
2. $\frac{1}{3}+\frac{2}{3}+\frac{1}{3}$.
3. $\frac{1}{4}+\frac{1}{4}+\frac{3}{4}$.
4. $1\frac{1}{2}+2\frac{1}{2}$.
5. $1\frac{1}{3}+2\frac{2}{3}$.
6. $3\frac{1}{4}+\frac{3}{4}$.
7. $2\frac{3}{5}+3\frac{4}{5}$.
8. $1\frac{7}{8}+\frac{3}{8}$.
9. $\frac{9}{17}+\frac{8}{17}+\frac{14}{17}+\frac{11}{17}$.
10. $8\frac{9}{17}+6\frac{8}{17}+5\frac{14}{17}+\frac{11}{17}$.
11. $\frac{4}{5}+\frac{5}{6}$.
12. $\frac{3}{4}+\frac{7}{8}$.
13. $\frac{1}{2}+\frac{1}{6}$.
14. $\frac{4}{15}+\frac{11}{20}$.
15. $\frac{5}{16}+\frac{11}{24}$.
16. $12\frac{5}{8}+7\frac{3}{16}$.
17. $85\frac{7}{12}+27\frac{11}{18}$.
18. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$.
19. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$.
20. $\frac{5}{6}+\frac{11}{12}+\frac{8}{15}+\frac{7}{20}+\frac{13}{30}$.
21. $5\frac{17}{20}+11\frac{19}{30}+24\frac{21}{40}+\frac{9}{50}+17\frac{8}{15}+14+11\frac{5}{12}$.
22. $9\frac{4}{7}+15\frac{11}{28}+163\frac{17}{63}+1\frac{11}{42}+10\frac{1}{4}$.
23. $3\frac{3}{5}+4\frac{2}{3}+1\frac{5}{6}+2$.
24. $1\frac{3}{20}+2\frac{2}{25}+5\frac{7}{30}+\frac{4}{15}$.
25. $\frac{2}{7}+1\frac{4}{9}+2+3\frac{3}{8}+4\frac{5}{12}$.
26. $4\frac{4}{9}+3\frac{3}{8}+2\frac{2}{7}+1\frac{1}{6}+\frac{9}{14}$.
27. $\frac{11}{35}+\frac{7}{40}+10+\frac{23}{60}$.
28. $\frac{27}{50}+\frac{29}{60}+\frac{31}{80}+\frac{33}{100}+\frac{37}{240}$.
29. $2+\frac{2}{3}+1\frac{3}{4}+4\frac{8}{9}+5\frac{13}{24}$.
30. $3\frac{5}{8}+6+\frac{4}{11}+2\frac{3}{10}+5\frac{5}{16}+\frac{9}{20}$.
31. $\frac{8}{15}+\frac{7}{18}+3\frac{17}{20}+1\frac{19}{24}+2\frac{19}{120}$.
32. $\frac{5}{14}+\frac{6}{11}+9\frac{1}{2}$.
33. $20\frac{5}{12}+11\frac{7}{20}+5\frac{1}{8}+305$.
34. $\frac{11}{38}+\frac{14}{57}+\frac{17}{76}$.
35. $\frac{5}{17}+\frac{11}{34}+\frac{14}{51}+\frac{19}{68}$.
36. $317\frac{2}{5}+17\frac{3}{51}+4\frac{9}{10}+\frac{7}{15}+6\frac{2}{3}+\frac{5}{17}$.
37. $4\frac{7}{15}+8\frac{5}{21}+4\frac{7}{11}+5\frac{2}{7}+5\frac{4}{5}+\frac{2}{3}$.
38. $3\frac{2}{3}+5\frac{3}{40}+8\frac{7}{240}+\frac{39}{60}+1\frac{29}{2880}$.
39. $4\frac{5}{13}+7\frac{5}{39}+5\frac{47}{78}+275\frac{37}{156}+2\frac{57}{91}$.
40. $\frac{17}{33}+7\frac{5}{12}+6\frac{3}{44}+400\frac{3}{28}+51\frac{25}{56}$.

SUBTRACTION OF FRACTIONS.

(1) From $\frac{17}{24}$ take $\frac{5}{18}$.

$$24 = 2^3 \times 3,$$
$$18 = 2 \times 3^2.$$

Hence, the L. C. D. $= 2^3 \times 3^2 = 72$.

$$\tfrac{17}{24} - \tfrac{5}{18} = \tfrac{51-20}{72} = \tfrac{31}{72}. \quad \textit{Ans.}$$

271. Therefore, to subtract one fraction from another:

Reduce the fractions to equivalent fractions having the least common denominator.

Subtract the numerator of the subtrahend from the numerator of the minuend.

Write the result over the least common denominator.

(2) Subtract $3\frac{11}{24}$ from $4\frac{11}{15}$.

$$24 = 2^3 \times 3,$$
$$15 = 3 \times 5.$$

Hence, the L. C. D. $= 2^3 \times 3 \times 5 = 120$.

$$4\tfrac{11}{15} - 3\tfrac{11}{24} = 1\tfrac{88-55}{120} = 1\tfrac{33}{120} = 1\tfrac{11}{40}. \quad \textit{Ans.}$$

If the terms are mixed numbers, subtract *separately* the integers and the fractions, and unite the results.

(3) Subtract $2\frac{7}{8}$ from $5\frac{5}{12}$.

$$5\tfrac{5}{12} - 2\tfrac{7}{8} = 3\tfrac{10-21}{24} = 2\tfrac{34-21}{24} = 2\tfrac{13}{24}. \quad \textit{Ans.}$$

The difference between $5\frac{5}{12}$ and $2\frac{7}{8}$ is $3\frac{10-21}{24}$.

Since $\frac{21}{24}$ cannot be subtracted from $\frac{10}{24}$, 1 is taken from 3 and added to $\frac{10}{24}$, making $\frac{34}{24}$.

(4) From 8 take $2\frac{31}{84}$.

$$8 = 7\tfrac{84}{84},$$
$$7\tfrac{84}{84} - 2\tfrac{31}{84} = 5\tfrac{84-31}{84} = 5\tfrac{53}{84}. \quad \textit{Ans.}$$

Exercise XXVI.

Find the value of:

1. $52\frac{1}{3} - 46$.
2. $\frac{8}{9} - \frac{3}{9}$.
3. $\frac{3}{4} - \frac{2}{3}$.
4. $\frac{8}{15} - \frac{5}{12}$.
5. $\frac{11}{18} - \frac{3}{14}$.
6. $4 - \frac{1}{2}$.
7. $7 - \frac{2}{3}$.
8. $3 - \frac{5}{6}$.
9. $8 - \frac{3}{7}$.
10. $5 - \frac{4}{5}$.
11. $5 - \frac{7}{9}$.
12. $6\frac{1}{3} - 5\frac{1}{6}$.
13. $4\frac{2}{5} - 3\frac{3}{7}$.
14. $7\frac{1}{3} - 2\frac{3}{10}$.
15. $7\frac{2}{5} - 4\frac{8}{9}$.
16. $6\frac{2}{3} - 2\frac{3}{4}$.
17. $9\frac{4}{5} - 4\frac{5}{6}$.
18. $4\frac{2}{3} - \frac{1}{2}$.
19. $6\frac{3}{4} - 4\frac{2}{3}$.
20. $7\frac{1}{2} - 2\frac{3}{4}$.
21. $8\frac{1}{5} - 4\frac{4}{7}$.
22. $85\frac{7}{22} - 27\frac{11}{18}$.
23. $8\frac{7}{10} - 2\frac{11}{16}$.
24. $10 - 3\frac{5}{8}$.
25. $120\frac{21}{32} - 110\frac{13}{24}$.
26. $5\frac{17}{28} - \frac{27}{35}$.
27. $13\frac{3}{40} - 2\frac{15}{44}$.
28. $2\frac{151}{240} - 1\frac{163}{192}$.
29. $4 - 1\frac{2817}{4000}$.
30. $1473 - 279\frac{11}{12}$.
31. $1473\frac{5}{18} - 279\frac{11}{12}$.
32. $1473\frac{7}{18} - 279\frac{11}{12}$.
33. $278\frac{15}{16} - 30\frac{5}{12}$.
34. $125\frac{5}{22} - 10\frac{17}{33}$.
35. $118\frac{5}{11} - 17\frac{3}{14}$.
36. $94\frac{5}{11} - 91\frac{13}{14}$.
37. $7\frac{5}{21} - 2\frac{11}{14}$.
38. $\frac{235}{357} - \frac{13}{51}$.
39. $\frac{17}{63} - \frac{29}{108}$.
40. $\frac{9}{88} - \frac{43}{209}$.
41. $\frac{146}{273} - \frac{268}{637}$.
42. $\frac{359}{860} - \frac{199}{200}$.

Plus and Minus Terms.

Simplify $5\frac{4}{5} - 4\frac{3}{4} + 3\frac{2}{3} - 2\frac{7}{10}$.

$$5\frac{4}{5} + 3\frac{2}{3} = 8\frac{12+10}{15} = 8\frac{22}{15} = 9\frac{7}{15},$$
$$4\frac{3}{4} + 2\frac{7}{10} = 6\frac{15+14}{20} = 6\frac{29}{20} = 7\frac{9}{20},$$
and $9\frac{7}{15} - 7\frac{9}{20} = 2\frac{28-27}{60} = 2\frac{1}{60}$.

272. Hence, to simplify an expression consisting of plus and minus terms,

Subtract the sum of the minus terms from the sum of the plus terms.

EXERCISE XXVII.

Simplify:

1. $3\frac{2}{5} - 2\frac{5}{8} + 4\frac{3}{10} + 1\frac{7}{9} - 5\frac{8}{15}$.

2. $1\frac{5}{11} - \frac{11}{12} + 7\frac{3}{8} - 2\frac{1}{3} - 1\frac{11}{16}$.

3. $12 - 3\frac{2}{7} - 1\frac{3}{10} - 4\frac{5}{28} + 2\frac{13}{20} - 4\frac{7}{\ }$.

4. $43\frac{7}{15} - 1\frac{1}{3} - 1\frac{31}{48} - 1\frac{23}{24} - 2\frac{13}{48} - 2\frac{7}{12} - 2\frac{43}{48} - 3\frac{5}{12}$.

5. $\frac{1}{2} + \frac{4}{15} + 7\frac{9}{40} + 8\frac{14}{35} + 7\frac{1}{4} + 8\frac{3}{10} + 4\frac{1}{12} - 36\frac{1}{40}$.

6. $(8\frac{5}{18} + 1\frac{10}{27} + 17\frac{11}{36} + 40) - (30\frac{13}{40} + 11\frac{11}{20})$.

7. $(172\frac{19}{78} + 93\frac{14}{117}) + (172\frac{19}{78} - 93\frac{14}{117})$.

8. $(172\frac{19}{78} + 93\frac{14}{117}) - (172\frac{19}{78} - 93\frac{14}{117})$.

9. $(\frac{3}{13} - \frac{2}{39}) + (\frac{5}{78} + \frac{7}{156})$.

10. $\frac{4}{9} - \frac{3}{11} - 2\frac{3}{4} + 3\frac{2}{3} + 7\frac{7}{12} - 1\frac{3}{5} - \frac{3}{22}$.

11. $\frac{3}{10} - \frac{7}{100} - \frac{9}{1000} - \frac{5}{10000}$.

12. $9\frac{5}{8} - 7 - \frac{3}{4} - \frac{5}{6}$.

13. $5\frac{2}{3} + 8\frac{3}{4} - 1\frac{3}{5} - 4\frac{7}{9}$.

14. $6\frac{3}{4} - 5\frac{2}{3} + 4\frac{2}{5} - 4\frac{5}{12}$.

15. $14\frac{7}{18} + 9\frac{3}{5} - 6\frac{3}{4} - 12\frac{4}{5} - 3\frac{3}{5}$.

16. $20\frac{2}{3} - 2\frac{5}{8} - 9\frac{5}{9} + 10\frac{3}{10} - 14\frac{7}{12}$.

17. $95\frac{2}{3} - 9\frac{7}{20} - 8\frac{3}{7} - 14\frac{8}{15} + 74\frac{2}{9}$.

18. $12\frac{3}{4} + 23\frac{3}{8} - (4\frac{3}{10} + 12\frac{2}{5} + 7\frac{14}{15})$.

19. $16\frac{2}{15} + 18\frac{5}{24} - (5\frac{2}{5} + 9\frac{9}{40} + 14\frac{5}{24})$.

20. $97\frac{8}{9} - (20 + 9\frac{3}{4} + 18\frac{2}{25} + 24\frac{49}{50})$.

21. $2\frac{13}{16} + 3\frac{15}{35} - (1\frac{53}{70} + 1\frac{31}{32} + \frac{49}{56})$.

22. $\frac{143}{100} + \frac{2471}{1000} - \frac{82643}{100000}$.

Complex Fractions.

273. A **complex fraction** is one which has a fraction in the numerator or in the denominator, or in both.

274. The simplest meaning to give to a complex fraction is that of an indicated division. Hence,

To simplify a complex fraction,

Divide the numerator by the denominator.

$$(1)\quad \frac{3\frac{4}{5}}{20}=\frac{19}{5}\div 20=\frac{19}{5}\times\frac{1}{20}=\frac{19}{100}.$$

$$(2)\quad \frac{2\frac{5}{6}}{4\frac{6}{7}}=\frac{17}{6}\div\frac{34}{7}=\frac{\cancel{17}}{6}\times\frac{7}{\underset{2}{\cancel{34}}}=\frac{7}{12}.$$

$$(3)\quad \frac{2\frac{2}{3}-1\frac{5}{9}}{1\frac{5}{6}-1\frac{1}{8}}.$$

$$2\frac{2}{3}-1\frac{5}{9}=1\frac{6-5}{9}=1\frac{1}{9};\quad 1\frac{5}{6}-1\frac{1}{8}=\frac{20-3}{24}=\frac{17}{24};$$

$$1\frac{1}{9}\div\frac{17}{24}=\frac{10}{\underset{3}{\cancel{9}}}\times\frac{\overset{8}{\cancel{24}}}{17}=\frac{80}{51}=1\frac{29}{51}.\quad \textit{Ans.}$$

A complex fraction may be simplified by multiplying both its terms by the smallest number that will make them integral. This multiplier will always be the L. C. M. of the denominators of the fractions contained in the terms of the given fraction.

$$(4)\quad \frac{4\frac{7}{9}-3\frac{1}{2}-2\frac{1}{3}+1\frac{7}{18}}{3\frac{5}{9}-2\frac{2}{3}+2\frac{1}{2}-\frac{7}{18}}=\frac{86-63-42+25}{64-48+45-7}=\frac{6}{54}=\frac{1}{9}.$$

Here each term of the numerator and denominator is made integral by multiplying it by 18, the L. C. M. of the denominators of the fractions contained in the numerator and denominator of the given fraction.

$$(5)\quad \frac{6\frac{3}{4}-1\frac{8}{9}}{\frac{7}{12}\text{ of }1\frac{2}{3}}=\frac{6\frac{3}{4}-1\frac{8}{9}}{\frac{35}{36}}=\frac{243-68}{35}=\frac{175}{35}=5.$$

Here the compound term $\frac{7}{12}$ of $1\frac{2}{3}$ is first reduced to the simple term $\frac{35}{36}$, and then the numerator and denominator of the resulting fraction is multiplied by 36, the L. C. M. of the small fractions.

Compound terms must first be reduced to simple terms.

Exercise XXVIII.

Simplify:

1. $\dfrac{2\frac{3}{11}}{3\frac{3}{4}}$.

2. $\dfrac{3}{7\frac{1}{5}}$.

3. $\dfrac{17\frac{1}{7}}{13\frac{1}{3}}$.

4. $\dfrac{\frac{5}{9}}{8\frac{1}{3}}$.

5. $\dfrac{5\frac{1}{9}}{8\frac{4}{11}}$.

6. $\dfrac{1\frac{4}{5}\text{ of }3\frac{1}{7}}{4\frac{1}{6}\text{ of }\frac{9}{10}}$.

7. $\dfrac{2\frac{1}{2}-1\frac{5}{9}}{1\frac{5}{6}-1\frac{5}{8}}$.

8. $\dfrac{10\frac{2}{5}-1\frac{6}{7}}{7\frac{1}{5}-3\frac{3}{40}}$.

9. $\dfrac{\frac{3}{7}\text{ of }2\frac{1}{17}}{1\frac{2}{3}+2\frac{3}{7}}$.

10. $\dfrac{6\frac{3}{4}-1\frac{5}{14}}{2\frac{1}{6}+1\frac{5}{7}}$.

11. $\dfrac{5\frac{4}{5}+2\frac{3}{7}}{4\frac{2}{3}-3\frac{11}{15}}$.

12. $\dfrac{8\frac{3}{4}}{14}-\dfrac{\frac{2}{3}}{1\frac{1}{7}}$.

13. $\dfrac{3\frac{3}{7}}{11\frac{1}{4}}$ of $\dfrac{3\frac{3}{8}}{2\frac{2}{5}}$.

14. $\dfrac{5\frac{8}{9}-4\frac{11}{12}}{5\frac{3}{8}-2\frac{17}{18}}$.

15. $\dfrac{2\frac{3}{4}+2\frac{7}{8}}{4\frac{3}{4}-3\frac{1}{7}}$.

16. $\dfrac{2\frac{3}{5}\times\frac{9}{11}}{3\frac{5}{7}+4\frac{1}{8}}$.

17. $\dfrac{\frac{17}{20}+\frac{11}{15}+\frac{7}{10}+\frac{4}{5}}{\frac{17}{20}-\frac{11}{15}+\frac{7}{10}-\frac{4}{5}}$.

18. $\dfrac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}-2\frac{1}{7}}$.

19. $\dfrac{2\frac{27}{40}-4\frac{4}{7}+3\frac{1}{8}}{5\frac{6}{7}-4\frac{7}{8}+\frac{2}{5}}$.

20. $\dfrac{1\frac{1}{4}\times1\frac{2}{7}+\frac{1}{8}\text{ of }2\frac{1}{4}-\frac{13}{28}\times 2}{\frac{13}{28}\text{ of }2+\frac{1}{3}\text{ of }2\frac{1}{4}-1\frac{1}{4}\text{ of }1\frac{2}{7}}$.

21. $2\frac{1}{4}\times\dfrac{10\frac{3}{4}-4\frac{11}{12}}{6\frac{3}{16}+7\frac{2}{3}}\times\dfrac{3\frac{5}{11}}{1\frac{2}{3}\times 9\frac{1}{11}}$.

22. $\dfrac{8\frac{7}{8}-7\frac{6}{7}+5\frac{5}{6}-4\frac{4}{5}}{9\frac{9}{10}-8\frac{13}{15}+7\frac{7}{8}-6\frac{6}{7}}$.

To Express one Number as the Fraction of Another.

275. What fraction of 8 is 7? *Ans.* $\frac{7}{8}$.

For since $1 = \frac{1}{8}$ of 8,
$7 = 7$ *times* $\frac{1}{8}$ of 8;
that is, $7 = \frac{7}{8}$ of 8.

It will be noticed that the number which follows "of" is the denominator, and the other number the numerator, of the required fraction. See, also, § 166.

EXERCISE XXIX.

What fraction of

1. 8 is 3?
2. 3 is 8?
3. 9 is 7?
4. 7 is 9?
5. 8 is 12?
6. 12 is 8?
7. $2\frac{1}{5}$ is $\frac{3}{5}$?
8. $\frac{3}{5}$ is $2\frac{1}{5}$?
9. $2\frac{3}{4}$ is $1\frac{1}{4}$?
10. $1\frac{1}{4}$ is $2\frac{3}{4}$?
11. $2\frac{1}{3}$ is $7\frac{1}{8}$?
12. $7\frac{1}{3}$ is $2\frac{1}{8}$?
13. $3\frac{1}{3}$ is $8\frac{1}{7}$?
14. \$2 is $\$1\frac{1}{2}$?
15. $\$2\frac{1}{2}$ is \$5?
16. $\$\frac{3}{4}$ is $\$\frac{1}{4}$?
17. $\$\frac{5}{4}$ is $\$\frac{3}{8}$?
18. $\$2\frac{3}{4}$ is $\$\frac{1}{8}$?
19. $\$\frac{1}{2}$ is $\$\frac{1}{10}$?
20. \$1 is $\$\frac{7}{8}$?
21. \$10 is $\$\frac{2}{3}$?
22. \$100 is \$6?
23. \$100 is $\$4\frac{1}{2}$?
24. \$4 is \$25?
25. $100\frac{5}{9}$ is $8\frac{4}{9}$?
26. 21 is $\frac{15}{19}$ of $3\frac{4}{5}$?
27. $18\frac{117}{256}$ is $\frac{5}{8}$ of $33\frac{3}{4}$?
28. $3\frac{1}{3}$ is $\frac{2}{3} \times 1\frac{1}{3}$?
29. $3\frac{1}{11} \times 5\frac{1}{27}$ is 1720?
30. $3\frac{1}{2} \times \frac{8}{9}$ of $\frac{4}{7}$ is $1\frac{3}{5}$?

What part of

31. $\frac{28}{59} \times \frac{59}{63}$ is $\frac{1}{6} \times 4 \times \frac{2}{3}$?
32. $13\frac{5}{8} \times \frac{2}{3} \times \frac{9}{65}$ is $\frac{2}{3}$ of $1\frac{44}{65}$ of $1\frac{1}{8}$?
33. $\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}$ is $\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}$?
34. $4\frac{1}{7} - 2\frac{1}{4}$ is $6\frac{1}{2} - 2\frac{1}{7}$?
35. $17\frac{2}{3} - 12\frac{6}{7}$ is $5 - \frac{1}{13} - \frac{4}{39} - \frac{1}{25}$?
36. $24 - 17\frac{4}{13}$ is $7 + \frac{2}{15} - \frac{5}{81} - \frac{11}{45}$?
37. $\frac{3}{7}$ of $2\frac{1}{17}$ is $1\frac{2}{3} \div 2\frac{3}{7}$?
38. $\left(\frac{7}{4-\frac{5}{6}} - \frac{5}{6-\frac{3}{8}}\right) \div \left(\frac{4}{7-\frac{4}{7}} + \frac{2}{4-\frac{2}{5}}\right)$ is $\left(14 - \frac{1}{\frac{1}{2}-\frac{6}{31}}\right) \div \left(\frac{1}{\frac{1}{2}-\frac{27}{59}} - 13\right)$?

TO REDUCE A DECIMAL TO A COMMON FRACTION.

(1) Reduce .527 to a common fraction.

.527 means $\frac{5}{10} + \frac{2}{100} + \frac{7}{1000} = \frac{500+20+7}{1000} = \frac{527}{1000}$.

(2) $.525 = \frac{525}{1000} = \frac{21}{40}$.

(3) $18.375 = 18\frac{375}{1000} = 18\frac{15}{40} = 18\frac{3}{8}$.

276. Hence, to reduce a decimal to a common fraction,

Write the figures of the decimal for the numerator; and 1 *with as many zeros as there are figures in the decimal for the denominator.*

Exercise XXX.

Reduce to common fractions in their lowest terms:

1. .125.	9. 7.015625.	17. 125.6048.
2. .625.	10. 20.100256.	18. .128.
3. .675.	11. 10.012575.	19. .73125.
4. 10.864.	12. 104.235.	20. 1.1875.
5. 50.84.	13. 50.0004.	21. .603125.
6. 3.00025.	14. 100.001.	22. 6.03125.
7. 8.1075.	15. 8.00725.	23. 60.3125.
8. 35.01024.	16. 20.018375.	24. 7.0315.

To Reduce a Common Fraction to a Decimal.

(1) Change $\frac{5}{8}$ to a decimal.

$$8\overline{)5.000}\quad .625$$

By this operation the *form* of the quotient is changed from $\frac{5}{8}$ to that of .625, but the *value* remains unaltered.

277. Hence, to reduce a common fraction to a decimal,

Divide the numerator by the denominator.

Exercise XXXI.

Reduce to decimals:

1. $\frac{7}{8}$.	6. $4\frac{11}{800}$.	11. $\frac{17}{4000}$.	16. $\frac{124}{16}$.
2. $\frac{15}{16}$.	7. $5\frac{5}{32000}$.	12. $\frac{119}{128}$.	17. $\frac{2}{3}$ of $1\frac{4}{5}$.
3. $\frac{9}{32}$.	8. $9\frac{123}{25600}$.	13. $\frac{18}{625}$.	18. $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{7}{10}$.
4. $\frac{9}{25}$.	9. $11\frac{19}{4000}$.	14. $\frac{11}{256}$.	19. $3\frac{3}{5}$ of $4\frac{1}{9}$.
5. $\frac{5}{64}$.	10. $\frac{9}{125}$.	15. $\frac{8}{160}$.	20. $\frac{29}{32}$ of $\frac{49}{64}$.

278. Simplify by common fractions, then by reducing the common fractions to decimals, and show that the results agree :

$$3\tfrac{2}{5} + 7\tfrac{7}{20} = 3.4 + 7.35 = 10.75.$$
$$3\tfrac{2}{5} + 7\tfrac{7}{20} = 10\tfrac{8+7}{20} = 10\tfrac{15}{20} = 10\tfrac{3}{4} = 10.75.$$

EXERCISE XXXII.

In like manner simplify :

1. $7\frac{2}{5} + 4\frac{5}{8} + 9\frac{13}{20} + 11\frac{29}{32}$.
2. $84\frac{13}{20} + 19\frac{11}{21} + \frac{41}{50}$.
3. $4\frac{27}{64} + 13\frac{17}{20} + 42\frac{37}{50} + 2\frac{13}{16} + 1\frac{1}{2}$.
4. $5\frac{7}{8} + 13\frac{4}{5} + 19\frac{7}{16} + 7\frac{3}{20}$.
5. $5\frac{5}{10} + \frac{2}{3}$ of $1\frac{4}{5} + \frac{7}{8}$ of $2\frac{2}{7} + \frac{3}{4}$ of $\frac{5}{8}$.
6. $1\frac{5}{12}$ of $2\frac{5}{8}$.
7. $3\frac{5}{16} + 2\frac{19}{20}$.
8. $7\frac{2}{5} - 4\frac{5}{8}$.
9. $82\frac{1}{5} - 37\frac{11}{16}$.
10. $100 - 17\frac{113}{625}$.
11. $5\frac{1}{2} - 1\frac{1}{2}$ of $1\frac{13}{24}$.
12. $\frac{14}{25} - \frac{11}{64}$.
13. $8\frac{1}{5} - 1\frac{1}{2}$ of $\frac{3}{16}$.
14. $\frac{19}{64} \times 1000$.

CIRCULATING DECIMALS.

279. If a fraction when reduced to its lowest terms contain in the denominator any other factor than 2 or 5 (the prime factors of 10), the division of the numerator by the denominator will give rise to a figure, or series of figures, which will be repeated without end. Thus, $\frac{49}{88} = .5568181.....$ in which the series 81 will repeat itself as long as we choose.

280. The series of figures which is thus repeated is called a *recurring period*, and is indicated by placing a dot over the first and last figures of the period. Thus, $\frac{49}{88} = .556\dot{8}\dot{1}$.

281. If the decimal have no other figures except the recurring period, it is called a *pure* circulator ; if the decimal have other figures preceding the period, it is called a *mixed* circulator.

(1) **Reduce $12\frac{15}{28}$.**

$$
\begin{array}{r l}
 & .53571428 \\
28) & 15.0 \\
 & \underline{140} \\
 & 100 \\
 & \underline{84} \\
 & 160 \ldots\ldots (1) \\
 & \underline{140} \\
 & 200 \\
 & \underline{196} \\
 & 40 \\
 & \underline{28} \\
 & 120 \\
 & \underline{112} \\
 & 80 \\
 & \underline{56} \\
 & 240 \\
 & \underline{224} \\
 & 16 \text{ the same remainder as (1).}
\end{array}
$$

It is to be observed that,

The *end* of the recurring period is reached *when a previous remainder reappears.*

When 2 or 5 occurs as a factor in the denominator, *the number of decimal places preceding the recurring period is equal to the highest exponent which either of these factors has* in the denominator. Thus, the denominator 28, in (1), is equal to $2^2 \times 7$, and the quotient contains *two* decimal places preceding the recurring period.

Hence, at the *beginning* of the recurring period it is easily determined what the remainder will be at the *end* of the recurring period, and when this remainder reappears the division need be carried no further.

Hence, $12\frac{15}{28} = 12.53\dot{5}7142\dot{8}$.

The work may be shortened by cancelling the factor 4 common to 28 and the second dividend 100. Thus:

$$
\begin{array}{r|l}
 & .53571428 \\
28 & 15.0 \\
 & \underline{14.0} \\
 & 100 \\
7 & 25000000
\end{array}
$$

(2) Reduce $\frac{6}{7}$ to a decimal.

$$
\begin{array}{r l}
 & .\dot{8}5714\dot{2} \\
7) & 6.000000
\end{array}
$$

Since in this fraction the denominator contains neither 2 nor 5, the first figure of the decimal begins the recurring period, and the period ends when 6 reappears.

(3) Reduce $\frac{125}{592}$ to a decimal.

$592 = 2^4 \times 37$, and four decimal places will precede the recurring period.

$$
\begin{array}{l}
\qquad\qquad .211\dot{1}48\dot{6} \\
592 \,)\, 125.0000000 \\
\qquad\quad 1184 \\
\qquad\qquad 660 \ldots\ldots \text{Cancel 4 from divisor and dividend.} \\
148 \,)\, 165 \\
\qquad\quad 148 \\
\qquad\qquad 170 \ldots\ldots \text{Cancel 2 from divisor and dividend.} \\
74 \,)\, 85 \\
\qquad 74 \\
37 \,)\, 110 \ldots\ldots \text{Cancel 2 from divisor and dividend.} \\
\qquad 55 \\
\qquad 37 \\
\qquad 180 \ldots\ldots \text{remainder at \textit{beginning} of recurring period.} \\
\qquad 148 \\
\qquad\; 320 \\
\qquad\; 296 \\
\qquad\quad 240 \\
\qquad\quad 222 \\
\qquad\qquad 18 \ldots\ldots \text{remainder at \textit{end} of recurring period.}
\end{array}
$$

$\therefore \frac{125}{592} = .211\dot{1}48\dot{6}$.

Exercise XXXIII.

Reduce to decimals:

1. $\frac{5}{9}$.
2. $\frac{5}{11}$.
3. $3\frac{5}{12}$.
4. $\frac{11}{60}$.
5. $3\frac{17}{48}$.
6. $2\frac{5}{37}$.
7. $\frac{3}{3700}$.
8. $11\frac{11}{84}$.
9. $9\frac{11}{108}$.
10. $11\frac{4}{35}$.
11. $\frac{15}{56}$.
12. $\frac{8}{21}$.
13. $\frac{13}{33}$.
14. $\frac{37}{70}$.
15. $2\frac{58}{255}$.
16. $5\frac{6}{26}$.
17. If $\frac{117}{5^7 \times 2^3}$ be expressed as a decimal, the quotient will contain how many decimal places?

18. If $\frac{119}{2^5 \times 13}$ be expressed as a decimal, how many decimal places will *precede* the recurring period?

19. If $\frac{57}{5^3 \times 7}$ be reduced to a decimal, how many decimal places will precede the recurring period?

To Reduce a Circulating Decimal to a Common Fraction.

Reduce $.5\dot{2}4\dot{3}$ to a common fraction.

From 10000 times the decimal, or 5243.243243.....
Take $\underline{\quad 10 \quad}$ " the decimal, or $\underline{\quad 5.243243}$.....
Then 9990 times the decimal $= 5243 - 5$.
Therefore the decimal $= \frac{5243-5}{9990} = \frac{5238}{9990} = \frac{97}{185}$.

Reduce $.\dot{2}\dot{7}$ to a common fraction.

From 100 times the decimal, or 27.2727.....
Take $\underline{\quad 1 \quad}$ " the decimal, or $\underline{\quad .2727}$.....
Then 99 times the decimal $= 27$.
Therefore the decimal $= \frac{27}{99} = \frac{3}{11}$.

The decimal is multiplied by such a power of 10 as will remove the decimal point to the *end of the recurring period*, then by such a power of 10 as will bring the decimal point at the *beginning of the recurring period*. The difference of these two products will contain so many times the decimal as will be indicated by a row of as many nines as there are recurring figures, and as many zeros following the nines as there are non-recurring figures in the given decimal.

282. Hence, to reduce a circulating decimal to a common fraction,

For the numerator, subtract the figures which precede the recurring period (if any) from the figures up to the end of the first period.

For the denominator, write a 9 *for each recurring figure, and annex a* 0 *for each figure that precedes the recurring period.*

Exercise XXXIV.

Reduce to common fractions in their lowest terms:

1. $.2\dot{4}\dot{5}$.	**9.** $1.41\dot{6}$.	**17.** $.2\dot{3}6\dot{8}$
2. $.4\dot{2}\dot{5}$.	**10.** $.5\dot{5}7\dot{5}$.	**18.** $1.1\dot{3}\dot{6}$.
3. $53.00\dot{2}4\dot{3}$.	**11.** $2.0\dot{8}\dot{1}$.	**19.** $1.\dot{5}3\dot{1}$.
4. $7.2\dot{0}1\dot{1}$.	**12.** $5.12\dot{2}9\dot{7}$.	**20.** $3.28\dot{9}6\dot{3}$.
5. $2.53\dot{0}\dot{6}$.	**13.** $.35\dot{9}\dot{0}$.	**21.** $5.878\dot{3}$.
6. $.004\dot{2}\dot{6}$.	**14.** $4.3\dot{1}6\dot{2}$.	**22.** $1.6\dot{9}40\dot{8}$.
7. $31.2\dot{0}\dot{3}$.	**15.** $.7\dot{2}8\dot{3}$.	**23.** $.48\dot{3}2\dot{4}$.
8. $.\dot{3}5\dot{1}$.	**16.** $5.1\dot{4}285\dot{7}$.	**24.** $.00\dot{1}221\dot{3}$.

283. Recurring decimals in questions requiring very great accuracy should be reduced to common fractions. Generally, however, it will be sufficient to treat them as terminating decimals, if care be taken to add 1 to the last figure retained in the decimal when the next figure is 5 or more.

To Find the G. C. M. and L. C. M. of Fractions.

If we divide $\frac{8}{5}$ by $\frac{2}{15}$ we obtain the quotient 12.

If we divide $\frac{8}{5}$ by $\frac{2}{7}$ we obtain the quotient $5\frac{6}{10}$.

It will be seen that the quotient is integral only when the *numerator* of the divisor is a *measure* of the numerator of the dividend and the *denominator* of the divisor is a *multiple* of the denominator of the dividend. Therefore, in order that a fraction may be a measure of a series of

fractions (expressed in their lowest terms), its numerator must be a measure of each numerator, and its denominator must be a multiple of each denominator.

284. Hence, to find the G. C. M. of a series of fractions,

Find the G. C. M. *of the numerators for the numerator, and the* L. C. M. *of the denominators for the denominator, of the required measure.*

Conversely, to find the L. C. M. of a series of fractions,

Find the L. C. M. *of the numerators for the numerator, and the* G. C. M. *of the denominators for the denominator, of the required multiple.*

(1) Find the G. C. M. of $\frac{5}{36}$, $\frac{25}{9}$, $\frac{35}{99}$.

G. C. M. of 5, 25, 35 = 5;
L. C. M. of 36, 9, 99 = 396.

Hence, $\frac{5}{396}$ is the G. C. M. required.

(2) Find the L. C. M. of $\frac{5}{36}$, $\frac{25}{9}$, $\frac{35}{99}$.

L. C. M. of 5, 25, 35 = 175;
G. C. M. of 36, 9, 99 = 9.

Hence, $\frac{175}{9}$ is the L. C. M. required.

EXERCISE XXXV.

Find the G. C. M. and L. C. M. of:

1. $\frac{7}{9}$, $\frac{14}{27}$, $\frac{13}{45}$.
2. $2\frac{2}{9}$, $2\frac{2}{5}$, $\frac{4}{40}$.
3. $33\frac{3}{7}$, $50\frac{5}{8}$.
4. $\frac{7}{24}$, $\frac{35}{36}$, $\frac{49}{60}$.
5. $5\frac{1}{2}$, $7\frac{1}{3}$, $8\frac{1}{4}$, $4\frac{8}{9}$, $9\frac{1}{6}$, $6\frac{5}{12}$.
6. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{12}$.
7. $50\frac{1}{2}$, $67\frac{1}{3}$, $44\frac{8}{9}$, $84\frac{1}{6}$, 707.
8. $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$.
9. $1\frac{1}{14}$, $1\frac{19}{21}$, $4\frac{2}{7}$, $\frac{25}{42}$.
10. $18\frac{2}{5}$, $57\frac{1}{2}$.
11. $134\frac{3}{4}$, $128\frac{1}{3}$, $115\frac{1}{2}$.
12. $2\frac{22}{25}$, $1\frac{37}{75}$, $\frac{63}{100}$.

13. A, B, and C start together and travel round a circular island, in the same direction. It takes A $2\frac{1}{3}$ days, B $2\frac{5}{6}$, C $2\frac{7}{8}$ days to walk round the island. They travel until they all meet at the point of starting. In how many days will they be together at the point of starting?

14. If the step of a man be $2\frac{1}{3}$ft and that of a horse be $2\frac{3}{4}$ft, find the smallest number of feet which is an exact number of man-paces and of horse-paces.

15. Find the largest number that is contained without remainder in $2\frac{5}{9}$, $6\frac{7}{18}$, $11\frac{1}{2}$, and $19\frac{1}{6}$.

Exercise XXXVI.

MISCELLANEOUS EXAMPLES IN FRACTIONS.

1. Simplify $\frac{2709}{6966}$, $\frac{48785}{56835}$, $\frac{2436}{567216}$, $\frac{4087}{5063}$.

2. Which is greater, and by how much, $\frac{7}{9}$ or $\frac{19}{24}$?

3. Find the sum of $3\frac{2}{5}$, $2\frac{4}{11}$, $5\frac{1}{2}$, $7\frac{7}{10}$, $1\frac{3}{22}$.

4. Simplify $5\frac{1}{2} - 3\frac{3}{7} + 2\frac{9}{10} - 1\frac{3}{5}$.

5. Simplify $1\frac{4}{5} + 3\frac{5}{6} - 2\frac{7}{12} + 4\frac{3}{20} - 3\frac{7}{15}$.

6. Simplify $\dfrac{3\frac{1}{2} + 3\frac{5}{6}}{4\frac{1}{3} - 2\frac{7}{12}}$.

7. Simplify the expressions: $7 \div 2\frac{3}{4}$; $\dfrac{7}{1\frac{3}{8}}$; $\dfrac{95\frac{1}{2}}{8\frac{7}{11}}$; $15 \div \frac{2}{3}$; $\dfrac{16}{5\frac{1}{3}}$; $7\frac{4}{11} \div 9$; $43\frac{1}{4} \div 37\frac{1}{3}$; $\dfrac{6\frac{7}{11}}{18\frac{1}{4}}$; $5\frac{4}{5} \div 4\frac{5}{6}$; $\dfrac{\frac{3}{8} \text{ of } 4\frac{1}{2}}{\frac{7}{4} \times \frac{3}{2}}$; $106 \div 8\frac{5}{6}$; $\dfrac{17}{4\frac{7}{17}}$.

8. Simplify the expressions: $7\frac{19}{32} \times 8$; $43\frac{13}{58} \times 6\frac{5}{8}$; $6\frac{5}{6} \div 8\frac{1}{5}$; $5\frac{1}{17} \times 51$; $\frac{11}{12}$ of $\frac{11}{13}$; $\frac{30}{49}$ of $\frac{7}{15}$ of $\frac{7}{8}$ of $\frac{2}{3}$ of $\frac{2}{5}$; $\frac{17}{19}$ of $\frac{228}{561}$; $\frac{1}{2} \times \frac{3}{4} \times \frac{7}{11} \times \frac{8}{9} \times \frac{3}{7}$.

9. By what must $\frac{1}{6}$ be multiplied to obtain $\frac{1}{2}$? $\frac{1}{6}$ to obtain $\frac{2}{3}$? $\frac{1}{2}$ to obtain $\frac{5}{6}$? $\frac{2}{3}$ to obtain $\frac{5}{6}$? $\frac{3}{5}$ to obtain $\frac{7}{8}$?

10. By what must $\frac{1}{6}$ be divided to obtain $\frac{1}{2}$? $\frac{2}{3}$ to obtain $\frac{1}{6}$? $\frac{5}{6}$ to obtain $\frac{5}{6}$? $\frac{7}{8}$ to obtain $\frac{4}{5}$? $\frac{3}{5}$ to obtain $\frac{7}{8}$? 8 to obtain $7\frac{19}{32}$?

11. What number exceeds $5\frac{2}{9}$ by $4\frac{7}{8}$?

12. From what must $6\frac{3}{5}$ be subtracted to leave $\frac{1}{2}$ of $3\frac{1}{9}$?

13. What fraction falls short of $\frac{7}{12}$ by $\frac{3}{20}$?

14. What fraction is that to which $\frac{5}{76}$ must be added to give $\frac{11}{57}$?

15. Convert into decimals: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{3}{4}$; $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$; $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$; $\frac{1}{6}$, $\frac{5}{6}$; $\frac{3}{7}$; $\frac{5}{9}$; $\frac{3}{11}$; $\frac{7}{40}$.

16. Convert into common fractions: .16; .016; .125; .13; .725; .625; .00625; .8125; .03125; .08; $.\dot{5}\dot{4}$; $.01\dot{6}$; $.5\dot{4}3\dot{7}$; $.0\dot{2}\dot{7}$; $.27\dot{7}$; $.68\dot{4}9\dot{4}$; $1.3\dot{4}\dot{5}$.

17. Simplify $\dfrac{2.8 \text{ of } 2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}}$.

18. Multiply $6.9\dot{5}\dot{4}$ by $5.3\dot{0}\dot{3}$, and express the result as a whole number and common fraction.

19. Simplify $1\frac{1}{2}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4}$ and reduce the result to a decimal.

20. From what number can $4\frac{17}{36}$ be taken 9 times and leave no remainder?

21. Of what fraction is $17\frac{1}{3}$ the 7th part?

22. Add $\frac{4}{5}$, 0.35, $\frac{5}{8}$, $\frac{3}{4}$, 0.112, 45.28.

23. Convert into decimals $\frac{13}{15}$; $\frac{3}{11}$; $\frac{4}{85}$; $\frac{17}{60}$; $\frac{14}{19}$; $\frac{5}{18}$; $\frac{7}{58}$.

24. What part of $\frac{15}{73}$ is $\frac{3}{1241}$?

25. Divide .0015 by .012, and express the result as a common fraction in lowest terms.

26. Convert into decimals: $\frac{3}{32}$; $\frac{3}{32000}$; $\frac{17}{74}$; $\frac{1}{7}$.

27. If the product of two factors is $\frac{5}{8}$, and one factor is $1\frac{1}{4}$, find the other factor.

28. If the dividend is $\frac{11}{12}$ and the quotient $6\frac{1}{2}$, find the divisor.

29. The dividend is $12\frac{17}{72}$, quotient 3, remainder $1\frac{5}{12}$; find the divisor.

30. Find the G. C. M. and the L. C. M. of 833, 1127, 1421, 343.

31. Arrange in order of magnitude $\frac{9}{11}$, $\frac{32}{45}$, $\frac{19}{27}$, $\frac{17}{20}$, $\frac{29}{38}$.

32. Find the L. C. M. of $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$.

33. Find the G. C. M. of $\frac{65}{68}$, $\frac{39}{2}$, $\frac{91}{84}$, and $6\frac{1}{2}$.

34. Convert into common fractions in lowest terms: $7.\dot{2}01\dot{1}$; $6.9\dot{5}\dot{4}$; $5.\dot{3}0\dot{3}$; $21.\dot{3}9\dot{6}$.

35. Simplify $\dfrac{3\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{8}}$.

36. Simplify $\dfrac{6\frac{3}{4} + 5\frac{1}{2} \times 3\frac{1}{7} - 7\frac{1}{4}}{3\frac{1}{5} + 2\frac{1}{2} - 4\frac{1}{10}}$.

37. Simplify $\dfrac{2\frac{4}{5} - 1\frac{1}{2} + 9\frac{1}{11}}{4\frac{1}{5} - 2\frac{1}{4} + 13\frac{7}{11}}$.

38. Simplify $\dfrac{(3.71 - 1.908) \times 7.03}{2.\dot{2} - \frac{74}{333}}$.

39. Simplify $\dfrac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{2}{5} \text{ of } \dfrac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}$.

40. Simplify $1\frac{1}{2} \text{ of } 2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4} + \left(5\frac{1}{2} + \dfrac{.24 + .5\dot{3}}{2.2 - .6\dot{4}}\right)$.

41. Simplify $.9$ of $\frac{5}{8}$ of $\frac{4}{7}$ of $15\frac{3}{4}$.

42. What part of $\frac{2}{3}$ is $\frac{1}{2}$?

43. What part of .390625 is .05?

44. $.\dot{0}\dot{9}$ is what fraction of $.20\dot{4}\dot{5}$?

45. Convert into decimals $\frac{49}{67}$; $\frac{14}{37}$; $\frac{62}{73}$.

46. The G. C. M. of three numbers is 15, and their L. C. M. is 450. What are the numbers?

CHAPTER XII.

MEASURES IN COMMON USE.

285. The unit of measure for lengths is a **yard**; and from this are derived the units of surface and volume.

286. The **standard yard** of Great Britain, as defined by act of Parliament, is the distance between the centres of two cylindrical holes in a certain bar of gun-metal, when the metal has a temperature of 62 degrees Fahrenheit.

The standard yard of the United States conforms as nearly as possible to that of Great Britain.

LINEAR MEASURES.

287.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod (rd.).
320 rods, 1760 yards, or 5280 feet	= 1 mile (mi.).

A line	= $\frac{1}{12}$ in.
A barleycorn	= $\frac{1}{3}$ in.
A hand (used in horse measure)	= 4 in.
A palm	= 3 in.
A span	= 9 in.
A cubit	= 18 in.
A military pace	= $2\frac{1}{2}$ ft.
A chain	= 4 rds.
A link	= $\frac{1}{100}$ chain.
A furlong	= $\frac{1}{8}$ m.
A knot (used in navigation)	= 6086 ft.
A league	= 3 knots.
A fathom (used in measuring depths at sea)	= 6 ft.
A cable-length	= 120 fathoms.

288. A quantity expressed with reference to a *single unit* is called a **simple quantity**; but a quantity expressed with reference to *different units* is called a **compound quantity.** Thus, 20 miles is a simple quantity; but 20 mi. 70 rds. 5 ft. is a compound quantity.

NOTE. A number expressed with reference to a particular unit is sometimes called a *denominate*, or *concrete*, number; and, in distinction, a number expressed without reference to a unit is called an *abstract* number. But it is to be observed that all *numbers* are abstract.

(1) Change 20 mi. to rods.

$$\begin{array}{r} 320 \text{ rds.} \\ \underline{20} \\ 6400 \text{ rds.} \end{array}$$

Since a mile contains 320 rds., 20 miles contain twenty *times* 320 rds. = 6400 rds.

(2) Change 20 mi. 70 rds. 5 ft. to feet.

$$\begin{array}{r} 20 \text{ mi. } 70 \text{ rds. } 5 \text{ ft.} \\ \underline{320} \\ 6400 \\ \underline{70} \\ 6470 \text{ rds.} \\ \underline{16\frac{1}{2}} \\ 3235 \\ 38820 \\ \underline{6470} \\ 106755 \\ \underline{5} \\ 106760 \text{ ft.} \end{array}$$

20 times 320 rds. = 6400 rds., to which the 70 rds. are added. Again, 6470 times $16\frac{1}{2}$ ft. = 106,755 ft., to which the 5 ft. are added.

Observe that multiplicand and multiplier are interchanged in the operation.

(3) Change 6400 rds. to miles.

$$\begin{array}{r} 320\,\underline{)\,6400} \\ 20 \text{ mi.} \end{array}$$

Since there are 320 rds. in 1 mi., in 6400 rds. there are (6400 ÷ 320) mi. = 20 mi.

(4) Change 106,760 ft. to miles.

$$\begin{array}{rl} 16\tfrac{1}{2})\,106760 & \text{ft.} \\ 2 & \\ \hline 33)\,213520 & \ldots\ldots \text{half-feet.} \\ 320)\,6470 & \text{rds.} \ldots 10 \text{ half-feet} = 5 \text{ ft.} \\ 20 & \text{mi.} \ldots 70 \text{ rds.} \end{array}$$

20 mi. 70 rds. 5 ft. *Ans.*

There are 16½ ft., or 33 half-feet, in a rod; so the 106,760 ft. are changed to half-feet, and *these* to rods, by dividing by 33. The remainder is 10 half-feet = 5 ft.

6470 rds. are changed to miles by dividing by 320, the number of rods in a mile. The remainder is 70 rds.

289. The process of changing the *unit* in which a quantity is expressed, without changing the *value* of the quantity, is called **reduction.**

290. If the change be from a higher denomination to a lower, it is called *reduction descending;* if from a lower to a higher, it is called *reduction ascending.*

291. To reduce a higher denomination to a lower,

Multiply by the number required of the lower denomination to make one of the higher.

If there be intermediate denominations, reduce step by step, and add to each result the given number of the denomination reached.

292. To reduce a lower denomination to a higher,

Divide by the number required of the lower denomination to make one of the higher.

If there be intermediate denominations, reduce step by step.

Exercise XXXVII.

Reduce:

1. 3 yds. 2 ft. to inches.	8. 712 mi. to inches.
2. 4 mi. 124 rds. to feet.	9. 540,451 ft. to miles.
3. 27 rds. $4\frac{1}{4}$ yds. to inches.	10. 271,256 in. to miles.
4. 290 leagues to feet.	11. 723,964 ft. to miles.
5. 82,976,432 in. to miles.	12. 233,205 in. to miles.
6. 7 mi. $3\frac{1}{2}$ yds. to inches.	13. 10 chains to inches.
7. 27 mi. 222 rds. to inches.	14. 233,185 in. to fathoms.

15. If the height of a horse be 16 hands, how many feet is his height?

16. If a train move 40 ft. in a second, what is its rate in miles per hour? (One hour = 3600 seconds.)

Measures of Surface.

293.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet	= 1 square rod (sq. rd.).
160 square rods, or 10 square chains	= 1 acre (A.).
640 acres	= 1 square mile (sq. mi.).

A square of flooring or roofing	= 100 sq. ft.
A section of land	= 1 mile square.
A township	= 36 sq. mi.

Note. The units of surface measure, except the acre, are *squares*, the sides of which are *linear units*. Thus, a square inch is a square each side of which is one inch in length.

Hence, in surface measurement, the scale ascends and descends by *squares* of the linear units; $144 = 12^2$; $9 = 3^2$; $30\frac{1}{4} = (5\frac{1}{2})^2$; $272\frac{1}{4} = (16\frac{1}{2})^2$.

Reduce 740 sq. yds. to sq. rds.

$$\begin{array}{r l} 30\tfrac{1}{4})740 & \text{sq. yds.} \\ 4 & \\ 121\overline{)2960} & \text{quarter-yards.} \\ 24 \text{ sq. rds.} \ldots 56 & \text{quarter-yards} = 14 \text{ sq. yds.} \end{array}$$

$30\frac{1}{4}$ sq. yds. made a square rod; that is, 121 quarter-yards make a square rod. Therefore, the 740 sq. yds. are reduced to quarter-yards; and these quarter-yards to square rods, by dividing by 121. The remainder, 56, is 56 quarter-yards = 14 yds.

Exercise XXXVIII.

Reduce.

1. 92,638 sq. yds. to inches.
2. 1,223,527 sq. in. to yards.
3. 721 sq. mi. to rods.
4. 34,729 sq. yds. to rods.
5. 3 A. 107 sq. rds. 27 sq. yds. 7 sq. ft. 23 sq. in. to inches.
6. 1 A. to feet.
7. 99,894,712 sq. in. to acres.
8. 15,376 sq. yds. to acres.
9. 562,934 sq. in. to rods.

294. Measures of Volume.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).
128 cubic feet	= 1 cord (cd.).

40 feet of round timber, or 50 feet of hewn timber	= 1 ton, or load.
40 cubic feet	= 1 ton of shipping.

Note I. A pile of wood 4 ft. wide, 4 ft. high, and 8 ft. long, is a *cord*. One foot of the length of such a pile is called a *cord foot*, and contains 16 cubic feet.

Note II. The units of volume, except the cord, are *cubes*, the edges of which are *linear units*. Thus, a cubic inch is a cube each edge of which is one inch in length. Hence, in the measurement of volumes the scale ascends and descends by *cubes* of the linear units; thus, $1728 = 12^3$; $27 = 3^3$.

Exercise XXXIX.

Reduce:

1. 7 cu. yds. 13 cu. ft. to cubic feet.
2. 25 cu. yds. 5 cu. ft. 143 cu. in. to cubic inches.
3. 74,325 cu. in. to cubic feet.
4. 439,000 cu. in. to cubic yards.
5. 921,730 cu. in. to cubic yards.
6. Wood cut in lengths of 4 ft. is piled to a height of 3½ ft. How long must the pile be to contain a cord?
7. A pile of wood 127 ft. long, 4 ft. wide, and 3 ft. 8 in. high is sold for $7 a cord. How much money is received for it?

Measures of Capacity.

Dry Measure.

295.

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

Liquid Measure.

296.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

31½ gallons = 1 barrel (bbl.).
2 barrels = 1 hogshead (hhd.).

Note. The gallon of liquid measure contains 231 cubic inches. The bushel of dry measure contains 2150.42 cubic inches. Therefore the quart of liquid measure contains 57¾ cubic inches, and the quart of dry measure 67⅕ cubic inches. The imperial gallon of *Great Britain* contains 277.274 cubic inches. The imperial bushel contains 2218.192 cubic inches. A quarter contains 8 imperial bushels.

Exercise XL.

Reduce:

1. 3 pks. 5 qts. 1 pt. to pints.
2. 4234 pts. to bushels.
3. 24 gals. 2 qts. 1 pt. 2 gi. to gills.
4. 272 liquid quarts to dry quarts.
5. 400 dry quarts to liquid quarts.
6. Express a bushel in cubic feet, carrying the decimal to three places.
7. Express a cubic foot as the decimal fraction of a bushel.
8. 1715½ bushels to pints.
9. 3047 gals. to barrels.

Weights.

Troy Weight.

297. 24 grains (grs.) = 1 pennyweight (dwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

A carat (of diamond) = $3\frac{1}{5}$ grs.

Avoirdupois Weight.

298. 16 drams (drs.) = 1 ounce (oz.).
16 ounces = 1 pound (lb.).
100 pounds = 1 hundred-weight (cwt.).
20 hundred-weight = 1 ton (t.).

112 pounds = 1 long hundred-weight.
2240 pounds = 1 long ton.

Note. Troy weight is used for weighing gold, silver, and precious stones. The troy pound is the standard unit of weight.

Avoirdupois weight is used for weighing all other articles.

In the United States custom house and in wholesale transactions in coal and iron the long ton is used.

The pound troy contains 5760 grains.

The pound avoirdupois contains 7000 troy grains.

In preparing medicines, apothecaries use the following:

Apothecaries' Weight.

299.

20 grains (grs.)	=	1 scruple (℈).
3 ℈	=	1 dram (ʒ).
8 ʒ	=	1 ounce (℥).
12 ℥	=	1 lb. troy.

Apothecaries' Measure.

300.

60 minims (♏)	=	1 dram (♏ lx.).
8 drams	=	1 ounce (fl. drm. viij.).
16 ounces	=	1 pint (fl. oz. xvj.).

Exercise XLI.

Reduce:

1. 27,587 grs. to pounds troy.
2. 34,652 lbs. to long tons.
3. 136,851 oz. to tons.
4. 864,205 grs. (troy) to pounds.
5. 864,205 grs. (apoth.) to pounds avoirdupois.
6. 5 lbs. 7 oz. 6 dwts. 12 grs. to grains.
7. 745 lbs. avoirdupois to troy weight.
8. 745 lbs. troy to avoirdupois weight.
9. 23,047,125 drs. to tons.
10. 90,252,381 drs. to tons.
11. 1 pt. to minims.
12. 8000 ♏ to ounces.

301. Time.

60 seconds (sec.)	=	1 minute (min.).
60 minutes	=	1 hour (hr.).
24 hours	=	1 day (dy.).
7 days	=	1 week (wk.).
365 days (or 52 wks. 1 dy.)	=	1 common-year (yr.).
366 days	=	1 leap-year.
100 years	=	1 century.

NOTE. The interval measured from the instant the sun is due south until it is due south the next day, is called a *solar day*. Its length varies slightly, and its average length, called the *mean solar day*, is the day of which an hour is the twenty-fourth part.

A *year* is the time in which the earth performs one revolution around the sun, and consists of 365.242218 mean solar days.

Before the time of Julius Cæsar, the year was reckoned as 365 days. On the supposition that 365¼ days was the true length, he introduced a calendar in which every fourth year (*every year which will give an integral quotient when its number is divided by* 4) was to consist of 366 days. The year of 366 days is called a *leap-year*.

The error of the Julian calendar, (365.25 – 365.242218 =) .007781 of a day, would amount to 3.1124 days in four centuries. To correct this error Pope Gregory XIII., in 1582, introduced a calendar in which three leap-years in every four centuries were reckoned as common years. Hence, the centuries are not leap-years unless *the number of the century* divided by 4 gives an integral quotient.

The year is divided into twelve *calendar months*. Four months, April, June, September, and November, have each thirty days; February has twenty-eight days, and in leap-year twenty-nine; the rest have each thirty-one days.

A *lunar month* is the time between two new moons, and is a little more than 29 dys. 12 hrs. 44 min.

EXERCISE XLII.

Reduce:

1. 6 hrs. 17 min. 25 sec. to seconds.
2. 1 yr. 13 dys. 4 min. to minutes.
3. 48,567 min. to days.
4. 742,392 sec. to days.
5. Find the number of days, reckoning from noon of the one to noon of the other, between the following days in the year 1880: July 4 and December 2; February 1 and May 29; January 3 and October 15; also, between December 25, 1880, and May 25, 1881.
6. How many minutes are there from midnight of March 7 to midnight of June 20?
7. Find the number of seconds from eight o'clock Monday morning till six o'clock the next Saturday evening.
8. Which of the years 1600, 1656, 1700, 1734, 1800, 1818, 1880, 1900, 1924, 2000 are leap years?

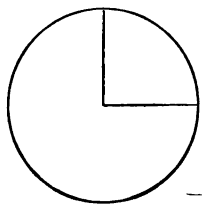
CIRCULAR AND ANGULAR MEASURES.

302. The circumference of every circle is divided into 360 equal parts, called degrees; each degree into 60 minutes, and each minute into 60 seconds.

60 seconds ($''$)	= 1 minute ($'$).
$60'$	= 1 degree ($^\circ$).
360°	= 1 circumference.

NOTE. As the circumference of every circle has 360 degrees, the length of the degree differs in different circles. A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

If two lines drawn from the centre of a circle include *one-fourth* of the circumference, they form at the centre an angle of 90°, and this is called a right angle. The lines are said to be perpendicular to each other.



The number of degrees in *the angle* formed by two lines drawn from the centre of a circle is the same as the number of degrees which the lines intercept on *the circumference.*

EXERCISE XLIII.

Reduce:

1. 2° 30′ 25″ to seconds.
2. 15° 3′ 22″ to seconds.
3. 56,760″ to degrees.
4. 212,221″ to degrees.
5. The hour and minute hands of a watch form an angle of how many degrees at 3 o'clock? at 4 o'clock? at 6 o'clock? at 7½ o'clock? at 11 o'clock? at 12 o'clock?
6. How many geographical miles in the width of the torrid zone (46° 55′)? How many statute miles?

MONEY.

For United States Money see § 182.

English Money.

303. 4 farthings = 1 penny (*d.*).
12 *d.* = 1 shilling (*s.*).
20 *s.* = 1 pound (£).

A guinea	= 21 shillings.
A crown	= 5 *s.*
A half-crown	= 2 *s.* 6 *d.*
A florin	= 2 *s.*
A sovereign	= 20 *s.*
A half-sovereign	= 10 *s.*
A sovereign	= $4.866½.

French Money:	100 centimes	= 1 franc (fr.)	= $0.193.
German Money:	100 pfennigs	= 1 mark	= $0.238.
Russian Money:	100 kopecks	= 1 rouble	= $0.734.
Austrian Money:	100 kreutzers	= 1 florin (fl.)	= $0.453.

EXERCISE XLIV.

Reduce:

1. £583 6 *s.* 8 *d.* to pence.
2. £79 18 *s.* 11½ *d.* to farthings.
3. 28,572 *d.* to pounds.
4. 272,191 *d.* to half-sovereigns.
5. 27,281 crowns to guineas.
6. 1,716,114 guineas to pounds.
7. 291,374 far. to pounds.
8. 709,126 *d.* to guineas.
9. 286,347 far. to crowns.
10. 20 francs to dollars.
11. 20 marks to dollars.
12. 5 roubles to dollars.
13. Find the whole sum of money in a box containing 35 sovereigns, 27 half-sovereigns, 13 crowns, 41 half-crowns, and 85 shillings.

Temperature.

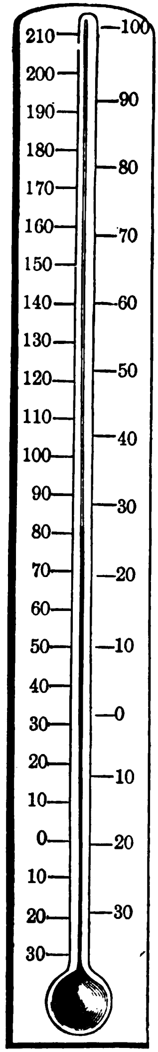
304. There are three scales for registering temperature by means of the thermometer.

Fahrenheit's has the freezing point of water marked 32°, and boiling point 212°.

The Centigrade has the freezing point zero, and the boiling point 100.

Reaumur's has the freezing point zero, and the boiling point 80°.

The reduction from one scale to another may be effected as follows:



(1) Express 80° C. in Réaumur's scale.

100° C. = 80° R.

Therefore, 80° C. = $\frac{80}{100}$ of 80° R.

= 64° R.

(2) Express 50° F. in Centigrade scale.

The number of degrees F. between the freezing and boiling points is 180;

Therefore, 1° F. = $\frac{100}{180}$° C. = $\frac{5}{9}$° C.

But 50° F. is 18° F. above freezing point,

and $\frac{5}{9}$ of 18° = 10°.

That is, 50° F. = 10° C.

(3) Express 60° C. on Fahrenheit's scale.

100° C. = 180° F.

Therefore, 60° C. = $\frac{60}{100}$ of 180° F.

= 108° F.

This will be the height above the freezing point, and will be marked 32° + 108° = 140°.

EXERCISE XLV.

Express:

1. 59° F. in Centigrade scale; in Réaumur's scale.
2. 77° F. in Centigrade scale; in Réaumur's scale.
3. 950° F. in Centigrade scale; in Réaumur's scale.
4. $-40°$ F. in Centigrade scale; in Réaumur's scale.
5. $-4°$ F. in Centigrade scale; in Réaumur's scale.
6. 10° C. in Fahrenheit's scale; in Réaumur's scale.
7. 22° C. in Fahrenheit's scale; in Réaumur's scale.
8. $-30°$ C. in Fahrenheit's scale; in Réaumur's scale.
9. $-11\frac{3}{7}°$ C. in Fahrenheit's scale; in Réaumur's scale.
10. 4° R. in Fahrenheit's scale; in Centigrade scale.
11. 12° R. in Fahrenheit's scale; in Centigrade scale.
12. $-20°$ R. in Fahrenheit's scale; in Centigrade scale.
13. 4° C. in Fahrenheit's scale; in Réaumur's scale.
14. 0° F. in Centigrade scale; in Réaumur's scale.

MISCELLANEOUS.

Numbers.	*Paper.*
12 units = 1 dozen.	24 sheets = 1 quire.
12 dozen = 1 gross.	20 quires = 1 ream.
12 gross = 1 great gross.	2 reams = 1 bundle.
20 units = 1 score.	5 bundles = 1 bale.

Weights.

A bushel of corn or rye	= 56 lbs.	A bushel of barley	= 48 lbs.
A bushel of corn meal, rye meal, or cracked corn	= 50 lbs.	A bushel of timothy-seed	= 45 lbs.
A bushel of wheat	= 60 lbs.	A stone of iron or lead	= 14 lbs.
A bushel of potatoes	= 60 lbs.	A pig of iron or lead	= $21\frac{1}{2}$ stone.
A bushel of beans	= 60 lbs.	A fother of iron or lead	= 8 pigs.
A bushel of oats	= 32 lbs.		

The weight of a bushel of potatoes, corn, etc., varies slightly in different States, but the weights here given are those generally adopted in business transactions.

A barrel of flour	= 196 lbs.
A barrel of pork or beef	= 200 lbs.
A cask of lime	= 240 lbs.
A cental of grain	= 100 lbs.
A quintal of fish	= 100 lbs.

Books.

A book formed of sheets folded in

2 leaves is a folio;
4 leaves is a quarto;
8 leaves is an octavo;
12 leaves is a duodecimo;
16 leaves is a 16mo.

COMPOUND ADDITION.

305. Compound Addition is the operation of finding the sum of two or more compound quantities expressed in the same measure.

	wks.	dys.	hrs.	min.
Ex.	13	3	6	20
	8	6	14	34
	11	4	20	28
	5	2	10	56
	39	3	4	18

Write the numbers so that units of the same denomination shall be in the same column.

The sum of the minutes is 138. Divide 138 min. by 60 (60 min. = 1 hr.).

The result is 2 hrs. 18 min.

Write 18 under the column of minutes, and carry 2 to the column of hours.

The sum of the hours, including the two hrs. obtained from the 138 min., is 52. Divide by 24 (24 hrs. = 1 dy.).

The result is 2 dys. 4 hrs.

Write 4 under the column of hours, and carry 2 to the column of days.

The sum of the days, including the 2 dys. obtained from the 52 hrs., is 17. Divide by 7 (7 dys. = 1 wk.).

The result is 2 wks. 3 dys.

Write 3 under the column of days, and carry 2 to the column of weeks.

The sum of the weeks, including the 2 wks. from the 17 dys., is 39.

	mi.	rds.	yds.	ft.		
(2)	4	110	5	2		
	6	25	4	2		
	10	136	$4\frac{1}{2}$	1		
			$\frac{1}{2}$=	1	6 in.	
	10	136	4	2	6 in.	*Ans.*

In the result, the $\frac{1}{2}$ yd. is changed to 1 ft. 6 in.

Exercise XLVI.

Add:

	hrs.	min.	sec.
1.	14	21	37
	17	13	32
	9	47	43
	12	53	54
	22	17	50

	cu. yds.	cu. ft.	cu. in.
2.	130	5	820
	56	20	304
	37	4	86
	8	10	129
	12	19	175

	£	s.	d.
3.	35	2	$6\frac{3}{4}$
	18	5	4
	27	3	10
	12	0	5

	mi.	rds.	yds.	ft.	in.
4.	6	120	3	2	2
	18	15	1	1	6
	3	215	2	2	3
	7	95	1	1	8

	A.	sq. rds.	sq. yds.	sq. ft.	sq. in.
5.	74	21	5	4	100
	23	37	13	5	83
	12	106	17	8	7
	41	50	23	0	24

6. 5 bu. 3 pks. 6 qts. 1 pt.; 6 bu. 2 pks. 7 qts.; 7 bu. 1 pk. 1 qt. 1 pt.; 1 pk. 7 qts.; 2 bu. 3 pks. 1 pt.
7. 48 t. 13 cwt. 75 lbs. 6 oz. 4 drms.; 25 t. 12 cwt. 27 lbs. 8 oz. 13 drms.; 51 t. 10 cwt. 44 lbs. 15 drms.; 80 t. 5 cwt. 6 oz.; 19 cwt. 27 lbs.; 25 lbs. 8 oz. 10 drms.; 5 t. 5 cwt. 5 oz.
8. 50 gals. 3 qts. 1 pt. 3 gi.; 12 gals. 1 qt. 1 pt. 1 gi.; 5 gals. 2 qts. 1 pt. 2 gi.; 75 gals. 3 qts. 1 pt. 3 gi.; 80 gals. 3 qts. 0 pts. 1 gi.; 17 gals. 1 qt. 1 pt. 3 gi.
9. 13 lbs. 4 oz. 8 dwt. 6 grs.; 25 lbs. 8 oz. 13 dwt. 20 grs.; 8 lbs. 11 oz. 14 grs.; 20 lbs. 16 dwt. 8 grs.; 15 lbs. 9 oz. 12 dwt.; 4 oz. 3 dwt.

10. 4 gals. 3 qts. 1 pt.; 3 gals. 2 qts. 1½ pts.; 12 gals. 3 qts.; 14 gals. 1½ pts.; 5 gals. 2 qts. 1 pt.
11. 60° 50′ 50″; 20° 41′ 52″; 30° 25′ 20″; 20° 32′ 43″.

COMPOUND SUBTRACTION.

306. Compound Subtraction is the operation of finding the difference between two compound quantities expressed in the same weight or measure.

	£	s.	d.
Ex.	36	6	8¾
	24	8	3¼
	11	18	5½

Here 6 *s.* is less than 8 *s.*, so £1 is reduced to shillings and added to the 6 shillings. Then 8 *s.* are taken from the 20 *s.* + 6 *s.*, and 1 is added to the £24 of the subtrahend.

Exercise XLVII.

Subtract:

1. 23 lbs. 8 oz. 19 dwt. 10 grs. from 58 lbs. 6 oz. 17 dwt. 21 grs.
2. 5 bu. 1 pk. 6 qts. 1 pt. from 5 bu. 3 pks. 3 qts.
3. 32 cu. yds. 13 cu. ft. 1600 cu. in. from 39 cu. yds. 17 cu. ft. 1400 cu. in.
4. £92 15 *s.* 1¼ *d.* from £120 13 *s.* 4 *d.*
5. 22 gals. 3 qts. 1 pt. from 30 gals. 2 qts.
6. 17 t. 7 cwt. 17 lbs. 6 oz. 10 drs. from 25 t. 13 cwt. 15 lbs. 12 oz. 5 drs.
7. 13 A. 150 sq. rds. 98 sq. ft. 10 sq. in. from 20 A.
8. 58° 33′ 36″ from 90° 11′ 21″.
9. 2 yrs. 213 dys. 17 hrs. from 3 yrs. 147 dys. 14 hrs.
10. 3 mi. 217 rds. 4 yds. 1 ft. 3 in. from 4 mi. 100 rds. 3 yds.

COMPOUND MULTIPLICATION.

307. Compound Multiplication is the operation of finding the amount of a quantity taken any number of times, when the quantity is expressed in several denominations.

(1) Multiply £13 14 *s.* 7 *d.* by 9.

£	s.	d.	
13	14	7	9×7 *d.* = 63 *d.* = 5 *s.* 3 *d.*
		9	9×14 *s.* = 126 *s.*; and with the 5 *s.* added = 131 *s.* = £6 11 *s.*
123	11	3	$9 \times$ £13 = £117; and with the £6 added = £123.

NOTE. If the multiplier be the product of two factors, multiply the given quantity by one of the factors, and the resulting product by the other. Thus, if the multiplier be 72, multiply the given quantity by 8 and the resulting product by 9.

If the multiplier be a large number, the work may be arranged as in the following example:

(2) Multiply 25 lbs. 10 oz. 11 dwt. 16 grs. by 439.

```
I.        439                  II.   439
           16                         11
         ----                       ----
         2634                       4829
         439                         292 . . . from I.
        -----                    20)5121
24 { 4 | 7024                       ----
   { 6 | 1756                        256 . . . 1 dwt.
        -----
          292 . . . 2/3 dwt. = 16 grs.

III.      439                  IV.   439
           10                         25
         ----                       ----
         4390                       2195
          256 . . . from II.        878
       ------                      -----
     12)4646                       10975
         ----                        387 . . . from III.
          387 . . . 2 oz.          -----
                                   11362 lbs.
```

11,362 lbs. 2 oz. 1 dwt. 16 grs. *Ans.*

EXERCISE XLVIII.

Multiply:

1. £31 2 *s.* 6½ *d.* by 8.
2. 19 gals. 3 qts. 1 pt. by 70.
3. 3 lbs. 4 oz. 8 dwt. 10 grs. by 10.
4. 5 t. 10 cwt. 67 lbs. by 10.
5. 43 bu. 2 pks. by 63.

6. 15 wks. 3 dys. 5 hrs. 12 min. by 7.
7. 5 cu. yds. 10 cu. ft. 371 cu. in. by 6.
8. 27 A. 76 sq. rds. 22 sq. yds. 5 sq. ft. by 90.
9. 32 rds. 3 yds. 1 ft. by 57.
10. 34 dys. 10 hrs. 13 min. 12 sec. by 108.
11. 5 mi. 126 rds. 19 yds. 6 in. by 7125.
12. 11 ℥ 5 ʒ 2 ℈ 11 grs. by 2197.

COMPOUND DIVISION.

308. Compound Division is

I. The operation of finding a quantity expressed in several denominations, which, when multiplied by a given number, shall equal a given quantity of the same kind.

II. The operation of finding the number of times that a quantity expressed in several denominations is contained in another quantity of the same kind.

NOTE. It is to be observed that the result of the division in I. is a quantity of the same kind as the dividend; in II. it is an abstract number.

(1) Divide £28 11*s.* 8*d.* by 7.

$$\begin{array}{r|rrr} & \text{£} & s. & d. \\ 7) & 28 & 11 & 8 \\ \hline & 4 & 1 & 8 \end{array}$$

The remainder from dividing the shillings by 7 is 4 *s.* = 48 *d.*, which, with the 8 *d.* added, is 56 *d.*; and this divided by 7 gives 8 *d.*

(2) Divide 40 t. 12 cwt. 20 lbs. 14 oz. by 36.

$$\begin{array}{ll|rrrr} & & \text{t.} & \text{cwt.} & \text{lbs.} & \text{oz.} \\ 36 \begin{cases} \\ \\ \end{cases} & 4 & 40 & 12 & 20 & 14 \\ \cline{3-6} & 9 & 10 & 3 & 5 & 3\frac{1}{2} \\ \cline{3-6} & & 1 & 2 & 56 & 2\frac{1}{6} \end{array}$$

1 t. 2 cwt. 56 lbs. 2⅙ oz. *Ans.*

(3) Divide 2334 cu. yds. 2 cu. ft. 1693 cu. in. by 649.

```
       cu. yds. cu. ft. cu. in.
   649)2334    2    1693(3
       1947
        387
         27
       2711
       774
  649)10451(16
       649
       3961
       3894
         67
       1728
        536
       134
      469
      67
       1693
 649)117469(181
      649
      5256
      5192
        649
        649
```

3 cu. yds. 16 cu. ft. 181 cu. in. *Ans.*

In (3) the remainder from dividing 2334 cu. yds. by 649 is 387 cu. yds., which are reduced to cubic feet by multiplying by 27 (27 cu. ft. = 1 cu. yd.). The result, with the 2 cu. ft. added, is 10451 cu. ft. The remainder from dividing 10451 cu. ft. by 649 is 67 cu. ft., which are reduced to cubic inches by multiplying by 1728 (1728 cu. in. = 1 cu. ft.). The result, with the 1693 cu. in. added, is 117469 cu. in.

(4) Divide £26 11*s.* by 3*s.* 8*d.*

Reduce both quantities to pence.

£26 11*s.* = 6372*d.*
3*s.* 8*d.* = 44*d.*

$44)\underline{6372}$
$144\frac{9}{11}$

$144\frac{9}{11}$. *Ans.*

Exercise XLIX.

Divide:

1. 54 mi. 124 rds. 1 yd. 2 ft. 6 in. by 33.
2. 5 cu. yds. 1 cu. ft. 84 cu. in. by 1716 cu. in.
3. 8426 wks. 6 dys. 21 hrs. 10 min. 21 sec. by 1029.
4. £394 2*s.* 10½ *d.* by £5 2*s.* 4½ *d.*
5. 22 wks. 2 dys. by 11 hrs. 31 min. 12 sec.
6. 74,128 sq. mi. 517 A. 80 sq. rds. by 10,000.
7. 38° 37′ 42″ by 5° 31′ 6″.

To Express the Value of the Fraction of a Simple Quantity in Lower Denominations.

309. (1) Express $\frac{2}{3}$ rd. in yards, feet, and inches.

$\frac{2}{3}$ rd. $= \frac{2}{3}$ of $5\frac{1}{2}$ yds. $= 3\frac{2}{3}$ yds.
$\frac{2}{3}$ yds. $= \frac{2}{3}$ of 3 ft. $= 2$ ft.

3 yds. 2 ft. *Ans.*

(2) Find the value of .3975 of a mile.

$$\begin{array}{r l}
.3975 & \text{mi.} \\
320 & \\
\hline
79500 & \\
11925 & \\
\hline
127.2 & \text{rds.} \\
16\frac{1}{2} & \\
\hline
3.3 & \text{ft.} \\
12 & \\
\hline
3.6 & \text{in.}
\end{array}$$

In (2) the multiplicand and multiplier are interchanged.
.3975 of 320 rds. = 127.2 rds.
.2 of a rod = 3.3 ft., and
.3 of a foot = 3.6 in.

127 rds. 3 ft. 3.6 in. *Ans.*

Exercise L.

Find the value of:

1. $\frac{4}{5}$ of a mile.
2. $\frac{3}{16}$ of an acre.
3. $\frac{5}{8}$ of a hundred-weight.
4. $\frac{2}{3}$ of a pound sterling.
5. $\frac{9}{11}$ of a mile.
6. $\frac{7}{11}$ of an acre.
7. $\frac{4}{9}$ of a degree.
8. $\frac{1}{3}$ of a year.
9. 0.15625 of a bushel.
10. 0.625 of a gallon.
11. 0.875 of a leap-year.
12. 0.325 of a pound troy.

To Find the Value of the Fraction of a Compound Quantity.

310. (1) Find the value of $\frac{3}{8}$ of £3 2*s.* 4*d.*

$$\begin{array}{r r r r}
 & \pounds & s. & d. \\
 & 3 & 2 & 4 \\
 & & & 3 \\
\hline
8) & 9 & 7 & 0 \\
\hline
 & 1 & 3 & 4\frac{1}{2}
\end{array}$$

£1 3*s.* 4½*d.* *Ans.*

(3) Find the value of $2\frac{5}{6}$ of £13 4*s*. 9*d*.

£	s.	d.
13	4	9
		5
6)66	3	9
11	0	$7\frac{1}{2}$

£	s.	d.
13	4	9
		2
26	9	6
11	0	$7\frac{1}{2}$
37	10	$1\frac{1}{2}$

£37 10*s*. $1\frac{1}{2}$*d*. *Ans.*

NOTE. When the multiplier is a mixed number, multiply by the integer and the fractional parts separately, and add the results.

EXERCISE LI.

Find the value of:

1. $6\frac{2}{5}$ of 3 A. $101\frac{1}{3}$ sq. rds.
2. $1\frac{3}{7}$ of 7 hrs. 21 min. 27 sec.
3. 10.0175 of 1 dy. 13 hrs.
4. $17\frac{7}{12}$ of 10 yds. 2 ft. $3\frac{1}{5}$ in.
5. 0.01284 of 14 mi.
6. 0.42776 of 12 t. 10 cwt.
7. $\frac{2}{5}$ of 1 lb. + $3\frac{2}{9}$ oz. + $5\frac{2}{3}$ dwt.
8. 0.35 of 4 lbs. 5 oz. 6 dwt. 16 grs.
9. 3.726 mi. — 33.57 rds.
10. $\frac{3}{73}$ of a year + $\frac{9}{56}$ of a week + $\frac{7}{12}$ of an hour.
11. 5.268 of 2 dys. + 2.829 of 16 hrs. + 0.9528 of 25 min.
12. $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of 40 rds. + $\frac{3}{8}$ of a yd.
13. $\frac{3}{4}$ of 2 cwt. 84 lbs. + $\frac{3}{7}$ of 5 cwt. 98 lbs. + $\frac{2}{5}$ of $7\frac{1}{2}$ lbs.

TO EXPRESS ONE QUANTITY AS THE FRACTION OF ANOTHER.

311. (1) Express 10 hrs. 33 min. 36 sec. as the fraction of a day.

$$36 \text{ sec.} = \tfrac{36}{60} \text{ min.} = \tfrac{3}{5} \text{ min.}$$

$$33\tfrac{3}{5} \text{ min.} = \frac{33\frac{3}{5}}{60} \text{ hr.} = \tfrac{168}{300} \text{ hr.} = \tfrac{14}{25} \text{ hr.}$$

$$10\tfrac{14}{25} \text{ hrs.} = \frac{10\frac{14}{25}}{24} \text{ dy.} = \tfrac{264}{600} \text{ dy.} = \tfrac{11}{25} \text{ dy.}$$

$\frac{11}{25}$ dy. *Ans.*

(2) Express 5 yds. 1 ft. 4 in. as the fraction of 1 yd. 2 ft. 4 in.

5 yds. 1 ft. 4 in.:

$4 \text{ in.} = \frac{4}{12} \text{ ft.} = \frac{1}{3} \text{ ft.}$

$1\frac{1}{3} \text{ ft.} = \frac{1\frac{1}{3}}{3} \text{ yd.} = \frac{4}{9} \text{ yd.}$

$5\frac{4}{9} \text{ yds.}$

1 yd. 2 ft. 4 in.:

$4 \text{ in.} = \frac{4}{12} \text{ ft.} = \frac{1}{3} \text{ ft.}$

$2\frac{1}{3} \text{ ft.} = \frac{2\frac{1}{3}}{3} \text{ yd.} = \frac{7}{9} \text{ yd.}$

$1\frac{7}{9} \text{ yds.}$

$$\frac{5\frac{4}{9}}{1\frac{7}{9}} = \frac{49}{16}. \quad \textit{Ans.}$$

Exercise LII.

Express:

1. A pound avoirdupois as the fraction of a pound troy.
2. An ounce avoirdupois as the fraction of an ounce troy.
3. 363 sq. yds. as the fraction of an acre.
4. $\frac{3}{5}$ of £2 1 *s.* 3 *d.* + $\frac{5}{11}$ of £1 4 *s.* 9 *d.* as the fraction of £2 14 *s.*
5. 2 mi. 138 rds. 1 yd. as the fraction of 3 mi. 265 rds. 3 yds. 1 ft. 6 in.
6. $\frac{2}{7}$ of 560 lbs. as the fraction of 5 long tons.
7. $\frac{2}{3}$ of 200 rds. as the fraction of 4 miles.
8. $\frac{10}{27}$ of 2 dys. 2 hrs. 24 min. as the fraction of 2 wks. 1 d.
9. $\frac{4}{5}$ of the difference between 3 yds. 2 ft. 11 in. and 10 yds. 7 in. as the fraction of 8 yds.
10. $\frac{10}{21}$ of the difference between $\frac{5}{8}$ of 7 hrs. and $\frac{7}{25}$ of 15 min. as the fraction of 12 hrs. 18 min.
11. $\frac{2}{5}$ pt. as the fraction of a gallon.
12. What part of 4 lbs. 1 oz. 8 dwt. 15 grs. is 1 lb. 1 oz. 9 dwt. 15 grs.?
13. What part of 2 mi. is $\frac{2}{3}$ of 6 rds. 3 yds. 2 in.?
14. What part of a bushel is 1 pk. 2 qts. 1 pt.?
15. What part of 20 acres are 19 A. 3.5 sq. ch.?
16. What part of 5 tons are 3 t. 240 lbs.?
17. 38 sq. rds. 194 sq. ft. 108 sq. in. = what part of an acre?

To Express one Quantity as the Decimal of Another.

312. (1) Express 1 oz. 8 dwt. 19.2 grs. as the decimal of a pound.

$$\begin{array}{r|l} 24 & 19.2 \text{ grs.} \\ 20 & 8.8 \text{ dwt.} \\ 12 & 1.44 \text{ oz.} \\ \hline \end{array}$$
Ans. .12 lb.

19.2 grs. = .8 dwt., which is written at the right of the 8 dwt.

8.8 dwt. = .44 oz., which is written at the right of the 1 oz.

1.44 oz. = .12 lb.

(2) Express 132 sq. yds. 2 sq. ft. 36 sq. in. as the decimal of 42 sq. yds. 4 sq. ft. 72 sq. in.

$$\begin{array}{r|l} 144 & 36.00 \text{ sq. in.} \\ 9 & 2.25 \text{ sq. ft.} \\ \hline & 132.25 \text{ sq. yds.} \end{array} \qquad \begin{array}{r|l} 144 & 72.00 \text{ sq. in.} \\ 9 & 4.5 \text{ sq. ft.} \\ \hline & 42.5 \text{ sq. yds.} \end{array}$$

$$\frac{132.25}{42.5} = 3.11176. \quad \textit{Ans.}$$

36 sq. in. = .25 sq. ft., which is written at the right of the 2 sq. ft.

2.25 sq. ft. = .25 sq. yd., which is written at the right of the 132 sq. yds.

In like manner 42 sq. yds. 4 sq. ft. 72 sq. in. are reduced to square yards and decimal of a yard; then 132.25 sq. yds. are divided by the resulting 42.5 sq. yds.

Exercise LIII.

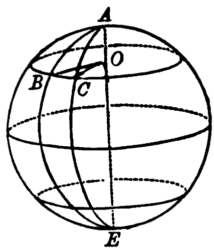
Express:

1. 16*s.* $3\frac{3}{4}$*d.* as the decimal of a pound.
2. 233 rds. 9 ft. 10.8 in. as the decimal of a mile.
3. 71 sq. rds. 54 sq. ft. 64.8 sq. in. as the decimal of an acre.
4. 15 hrs. 14 min. 6 sec. as the decimal of 2 days.
5. 38 sq. rds. 21 sq. yds. 5 sq. ft. 108 sq. in. as the decimal of an acre.
6. 3 mi. 242 rds. 2 yds. 2 ft. 3 in. as the decimal of 7 mi. 160 rds.
7. 5 hrs. 13 min. 30 sec. as the decimal of a week.
8. 27° 14′ 45″ as the decimal of 90°.

9. 54 dys. 2 hrs. 40 min. as the decimal of $365\frac{1}{4}$ dys.
10. 2 lbs. avoirdupois as the decimal of 10 lbs. troy.
11. 44,920.9025 hrs. as the decimal of a year.
12. 1 drm. avoirdupois as the decimal of 1 dwt. troy.
13. 10 milligrams as the decimal of a grain, if a kilogram equals 2 lbs. 8 oz. 3 dwt. 1 gr.
14. 14.52 sq. yds. as the decimal of a square chain.
15. 8 cwt. 77 lbs. 9.6 oz. as the decimal of a ton.

Longitude and Time.

313. A **meridian** is any line drawn straight around the earth and passing through both poles.



314. The **longitude** of a place is the angle of inclination of the two planes which are supposed to pass through the centre of the earth, and contain, the one the meridian of that place, and the other the standard meridian. Thus, the longitude of *C*, reckoned from meridian *ABE*, is the angle *BOC*. A plane passing through *ABE* divides the earth into Eastern and Western Hemispheres. Places on the Eastern Hemisphere are in East Longitude; on the Western Hemisphere, in West Longitude.

315. As the earth turns upon its axis once in twenty-four hours, a point on the earth's surface will describe a circumference (360°) in twenty-four hours. Therefore longitude may be reckoned in *time* as well as in degrees.

In one hour a point on the earth's surface describes $\frac{1}{24}$ of $360° = 15°$; in one minute $\frac{1}{60}$ of $15° = \frac{1}{4}°$ (or $15'$); and in one second $\frac{1}{60}$ of $15' = \frac{1}{4}'$.

Again, since it requires one hour (60 min.) for a point to pass over 15°, to pass over 1° it requires $\frac{1}{15}$ of 60 min. $= 4$ min.; and to pass over $1'$ it requires $\frac{1}{60}$ of 4 min. $= 4$ sec.

(1) Express 17° 35′ 15″ of longitude in time.

17° 35′ 15″
= 17° 35.25′
= 4 × (17 min. 35.25 sec.)
= 1 hr. 10 min. 21 sec. *Ans.*

Express the longitude in degrees and minutes.

Since it takes 4 × 1 min. to pass over 1°, and 4 × 1 sec. to pass over 1′, it takes 4 × (17 min. 35.25 sec.) to pass over 17° 35.25′.

That is, 1 hr. 10 min. 21 sec.

(2) Express 2 hrs. 13 min. 3.5 sec. in degrees, etc.

2 hrs. 13 min. 3.5 sec.
= 133 min. 3.5 sec.
= $\frac{1}{4}$ (133° 3.5′)
= 33° 15′ 52.5″. *Ans.*

Express the time in minutes and seconds.

Since a point passes over $\frac{1}{4}$ of 1° in 1 min., and $\frac{1}{4}$ of 1′ in 1 sec., in 133 min. 3.5 sec. it passes over $\frac{1}{4}$ of (133° 3.5′).

That is, 33° 15′ 52.5″.

Hence, if longitude be expressed in **degrees, multiply** the *number* of degrees by 4; the product is the equivalent *number of minutes of time.*

If longitude be expressed in **minutes of time, divide** the *number* of minutes by 4; the quotient is the equivalent *number of degrees.*

Exercise LIV.

Find the difference in longitude between two places, if the difference in time be:

1. 1 hr. 15 min.
2. 2 hrs. 11 min.
3. 5 hrs. 10 min. 10 sec.
4. 3 hrs. 25 min. 35 sec.
5. 6 hrs. 12 min. 30 sec.
6. 4 hrs. 8 min. 12 sec.
7. 18 hrs. 10 min.
8. 15 hrs. 15 min. 15 sec.

Find the difference in time between two places, if the difference in longitude be:

9. 9° 20′.
10. 70° 30′.
11. 56° 36′ 12″.
12. 108° 32′ 36″.
13. 120° 14′ 30″.
14. 100° 45′ 54″.
15. 2° 2′ 2″.
16. 75° 10′.

316. Since the sun *appears* to move from east to west, sunrise will occur earlier at all points east, and later at all points west, of a given place. Hence, clock-time will be later in all places east, and earlier in all places west, of a given meridian.

Therefore, if the time of a place be given,

To find the time of a place **east, add** to the given time the difference of time between the two places.

To find the time of a place **west, subtract** from the given time the difference of time between the two places.

To Find the Difference in Clock-Time when the Difference in Longitude is Known.

(1) When it is noon at Boston (long. 71° 3′ 30″ W.), what is the time at Paris (long. 2° 20′ 22″ E.)?

$$\begin{array}{l} 71^\circ\ \ 3'\ 30''\ \text{W.} \\ \underline{\ \ 2^\circ\ 20'\ 22''\ \text{E.}} \\ 73^\circ\ 23'\ 52'' \ldots\ldots \text{ difference in longitude.} \\ = 4 \times (73 \text{ min. } 23.867 \text{ sec.}) \\ = 4 \text{ hrs. } 53 \text{ min. } 35.47 \text{ sec.} \end{array}$$

53 min. 35½ sec. past 4 o'clock. *Ans.*

Since Boston is west and Paris is east of the meridian of Greenwich, the difference between their longitudes is found by taking the sum of their longitudes.

Their difference in longitude, 73° 23′ 52″, is equivalent to 4 hrs. 53 min. 35.47 sec., and as Paris is *east* of Boston, the time at Paris is found by *adding* the 4 hrs. 53 min. 35.47 sec. to the time at Boston.

Exercise LV.

The longitude of some public building in:

(1) Berlin is 13° 23′ 43″ E.

(2) Rome, 12° 27′ 14″ E.

(3) Constantinople, 28° 59′ E.

(4) Pekin, 116° 23′ 45″ E.

(5) San Francisco, 122° 26′ 15″ W.

(6) St. Louis, 90° 15′ 15″ W.

(7) Jerusalem, 35° 32′ E.
(8) Bombay, 72° 54′ E.
(9) Calcutta, 88° 19′ 2″ E.
(10) Chicago, 87° 35′ W.
(11) New York, 74° 0′ 3″ W.
(12) Montreal, 73° 25′ W.

1. When it is noon at Greenwich, what is the clock-time at each of the above places?
2. When it is half-past four P.M. at Chicago, what is the clock-time at each of the above places?
3. When it is eight o'clock A.M. at Constantinople, what is the clock-time at each of the above places?

To Find the Difference in Longitude when the Difference in Clock-Time is Known.

Ex. When it is noon at Greenwich, what is the longitude of a place where the clock-time is half-past four A.M.?

hrs.	min.
12	
4	30
7	30 = 450 min.

450 min. = ¼ of 450°,
= 112° 30′ W. *Ans.*

Exercise LVI.

When it is noon at Greenwich the time at

(1) Boston, Mass., is 7 hrs. 15 min. 46 sec. A.M.
(2) Augusta, Me., 7 hrs. 20 min. 40 sec. A.M.
(3) Columbia, S.C., 6 hrs. 35 min. 32 sec. A.M.
(4) Little Rock, Ark., 5 hrs. 51 min. 12 sec. A.M.
(5) Salt Lake, 4 hrs. 30 min. A.M.
(6) Albany, N.Y., 7 hrs. 5 min. 1 sec. A.M.
(7) Columbus, O., 6 hrs. 27 min. 48 sec. A.M.
(8) Harrisburg, Penn., 6 hrs. 52 min. 40 sec. A.M.
(9) New Orleans, La., 6 hrs. A.M.
(10) Springfield, Ill., 6 hrs. 1 min. 48 sec. A.M.
(11) Washington, D.C., 6 hrs. 51 min. 44 sec. A.M.

1. What is the longitude of each of the above places?

Exercise LVII.

1. Reduce 7 gals. 3 qts. 1 pt. to gallons and decimal of a gallon.
2. Reduce £4.375 to pounds, shillings, and pence.
3. Reduce 7.6875 gals. to gallons, quarts, and pints.
4. Reduce to pounds, shillings, and pence $5.875; $7.38; $17.85; $21.75; if $4.85 be equal to a pound.
5. How many square yards in 3.7156 acres?
6. If 2 qts. of linseed oil be mixed with ½ pt. spirits of turpentine, what fraction of the mixture is turpentine? How much turpentine in one pint of the mixture?
7. Reduce 5.1732 mi. to yards, feet, and inches.
8. If a man walk 88 mi. in 26 hrs. how many feet does he walk each second?
9. Of a mixture of sand and lime .27 of the weight is lime. How many ounces of lime in a pound of the mixture? How many troy grains of lime in an avoirdupois pound of the mixture?
10. A gill of water is put into a quart measure, and the measure filled with milk. What part of the mixture is water?
11. Reduce 555 ft. to the decimal of a mile.
12. Reduce 1 mi. 13 rds. 2 yds. 2 ft. 6 in. to inches.
13. How many cubic inches in 2½ cubic feet?
14. How many pounds avoirdupois does a cubic yard of water weigh if a cubic foot weigh 1000 ounces?
15. Express the weight of a cubic yard of water as the decimal of a ton.
16. What is the weight of 7 bu. 3½ pks. of potatoes?
17. A farmer sowed 5 bu. 1 pk. 1 qt. of seed, and harvested from it 103 bu. 3 pks. 5 qts. How much did he raise from a bushel of seed?

18. How many bushels in 5 t. of oats?
19. How many bottles, each holding 1 pt. 3 gi., can be filled from a barrel of cider?
20. If a steamer make 13 mi. 6 rds. an hour, how far will she go between 6 A.M. and 6 P.M.? How many hours will she require to make 113 miles?
21. If a locomotive run at the rate of 111 rds. a minute, how many hours will it require to run from Boston to Buffalo, 498 miles?
22. What is the cost of 12 A. 146 sq. rds. land at $16.25 an acre?
23. What is the cost of 8 t. 3 cwt. 27 lbs. of coal at $5.75 a ton?
24. What is the cost of 7 t. 1560 lbs. of hay at $15.50 a ton?
25. What is the cost of a car-load of wheat weighing 20,000 lbs. at $1.05 a bushel?
26. Reduce 5 rds. 4 yds. 2½ ft. to the decimal of a mile.
27. Reduce 9 sq. ch. 11.25 sq. rds. to the decimal of an acre.
28. Reduce .09375 bu. to quarts.
29. Reduce 7560 chains to miles.
30. How many gross are 2000 pens?
31. Find the cost of 27.248 A., at $93.75 an acre.
32. Which is the greater, 2.8 of 3 ft. 11 in. or 3.11 of 2 ft. 8 in., and by how much?
33. Reduce 171 lbs. 6 oz. troy to the decimal of a ton avoirdupois.
34. Express 14.52 sq. yds. as the decimal of a square chain.
35. If a sovereign be equal to 25.22 francs, or to $4.85, what decimal of a dollar is a franc?
36. Express 2.805 florins − 1.89 half-crowns as the decimal of £.472.
37. If .327 of some work be done in 3 hrs. 38 min., how long will the whole work require?

38. A can run a mile in 7.68 min.; B can run at the rate of 7.68 mi. an hour. Which is the faster runner?

39. How many miles an hour does a person walk who takes 2 steps a second and 1900 steps in a mile?

40. If an ounce troy of gold be worth \$20, what is the value of a pound avoirdupois?

41. Two stars cross the meridian at 6 hrs. 4 min. 42.3 sec. and 7 hrs. 2 min. 57.21 sec., respectively. What is the interval between the observations?

42. How long will it take to fill $\frac{13}{25}$ of a cistern, when the whole requires 6 hrs. 10 min.

43. The circumference of a circle is 6 yds. 1 ft. 5.1 in., and is divided into 360 degrees. What is the length of 55 degrees?

44. Multiply 2 t. 16 cwt. $63\frac{2}{5}$ lbs. by $1\frac{4}{9}$.

45. Into how many shares has £120 been divided when each share is £3 8*s.* $6\frac{6}{7}$*d.*?

46. If $\frac{13}{15}$ of one line be equal to $\frac{8}{9}$ of another line, which is the greater? and what fraction of it is the less?

47. Multiply 5 mi. 206 rds. 2 ft. 2 in. by 786.

48. The returns of a gold mine are 241 t. of ore yielding 2 oz. 1 dwt. 15 grs. of fine gold a ton, and 193 t. yielding 1 oz. 12 dwt. 9 grs. a ton. Find the value of the whole yield, at \$19.45 an ounce.

49. Divide 93 long tons 56 lbs. by 23 lbs. 5 oz.

50. Telegraph poles on railroads are generally erected at intervals of 88 yds. Show that if a passenger count the number of poles which the train passes in three minutes, that number will express the number of miles an hour the train is going.

51. If Greenwich time be 5 hrs. 8 min. 16 sec. later than Washington time, and Chicago be 87° 35′ W., what is the difference between Washington and Chicago time?

CHAPTER XIII.

PROBLEMS.

Exercise LVIII.

(1) A train travels 24 miles in .8 of an hour. Find its rate per hour.

If the question had been, a train travels 70 mi. in 2 hrs., its rate per hour would obviously be found by dividing the whole distance, 70 miles, by 2. The application of the same method to this question gives,

$$24 \text{ mi.} \div .8 = 30 \text{ mi. } \textit{Ans.}$$

(2) A train runs from New York to Philadelphia, 90 miles, in 1 hr. 33 min. What is its rate per hour?

$$1 \text{ hr. } 33 \text{ min.} = 1\tfrac{11}{20} \text{ hrs.,}$$
$$\text{and } 90 \text{ mi.} \div 1\tfrac{11}{20} = 58\tfrac{2}{31} \text{ mi. } \textit{Ans.}$$

1. A train from New York to Philadelphia, 90 miles, makes the whole distance in 2 hrs. 5 min. What is its rate?
2. Winlock, in 1869, found that electricity went through 7200 miles of wire in $\frac{3}{8}$ of a second. What was its rate per second?
3. If the time required for a signal to pass through the cable from Brest to Duxbury, 3799 miles, be .816 of a second, what is the rate per second?
4. If the report of a gun $1\frac{1}{4}$ miles distant is heard in $5\frac{5}{8}$ seconds after the flash is seen, what is the velocity of sound, in feet, per second?
5. If a man walk $3\frac{1}{5}$ miles in 46 minutes, what is his rate per hour?

6. If a horse go $47\frac{1}{2}$ miles in 10 hrs. 40 min., what is his average rate per hour?
7. If a stone on a glacier move $95\frac{1}{2}$ feet in 188 days, what is its rate, in inches, per day?

It is often required to express speed with reference to the unit of distance instead of the unit of time.

Ex. If a horse made $5\frac{1}{2}$ miles in 33 minutes, how long did it take him to make a mile?

$$33 \text{ min.} \div 5\frac{1}{2} = 6 \text{ min.} \quad \textit{Ans.}$$

Hence, when the time is required for the unit of distance,

Divide the time by the distance.

8. If a horse trot $\frac{5}{6}$ of a mile in $2\frac{1}{3}$ minutes, in what time can he trot a mile?
9. If a train run 18 miles in 39 minutes, how long does it take to run one mile?
10. If sound travel 1125 feet a second, how long will it take to travel one mile?
11. If a train require 3 hours to travel $104\frac{1}{4}$ miles, find its average time for travelling a mile.
12. If a mower cut $7\frac{1}{2}$ acres of grass in $3\frac{1}{2}$ days, what part of a day will it take him to cut one acre? If a day consist of 10 working-hours, what part of an acre does he cut in an hour?
13. If a mower cut $3\frac{1}{2}$ square rods in $\frac{1}{8}$ of an hour, how many acres can he cut in a day of 10 hours?
14. If a fountain yield $117\frac{1}{2}$ gallons in $\frac{3}{4}$ of an hour, at what rate per hour is it flowing?
15. If a merchant's profits be $3147 in $7\frac{1}{2}$ months, what are his profits for a year?
16. If a wheel turn 17° 30′ in 35 minutes, in how many hours does it make a complete revolution?

17. If a man's expenditures be \$4358 in $13\frac{1}{3}$ months, what is his yearly rate of expenditure?

18. If a cistern lose by leakage 7 gals. 1 pt. in 49 hrs. 40 min., what is its hourly rate of loss?

19. If the circumference of the earth at the equator be 24,900 miles, at what rate per hour is a person there carried round, one whole rotation being made in 23 hrs. 56 min.?

20. If a man travel $3\frac{3}{5}$ miles in $7\frac{1}{2}$ minutes, how many miles will he travel in 50 minutes? and how long will he take to travel 50 miles?

If a man can do a piece of work in 9 days, in 1 day he can do $\frac{1}{9}$ of the work.

If another man can do the same work in 8 days, in 1 day he can do $\frac{1}{8}$ of it; and

Both men together can do $\frac{1}{9} + \frac{1}{8} = \frac{17}{72}$ of it in 1 day.

Therefore, if the whole work be considered to be divided into 72 equal parts, they can do 17 of these parts in 1 day, and the time required to do the whole work will be $72 \div 17 = 4\frac{4}{17}$ days.

Ex. A cistern can be filled by means of a water-pipe in 30 minutes, and can be emptied by a waste-pipe in 20 minutes. If the cistern be full, and both pipes be opened, in what time would it be emptied?

The waste-pipe empties $\frac{1}{20}$ every minute.
The water-pipe fills $\frac{1}{30}$ every minute.
When both are open $\frac{1}{20} - \frac{1}{30}$ is emptied every minute.

$$\frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}.$$

Therefore the whole will be emptied in 60 minutes.

21. If A can mow a certain meadow in 4 days, and B in 3 days, how long will it take both?

22. If A can lay a certain wall in $4\frac{1}{2}$ days, and B in $5\frac{1}{2}$ days, how long will it take both?

23. If a pipe will fill a vessel in $4\frac{1}{2}$ hours, and another in $3\frac{1}{2}$ hours, how long will it take both to fill the vessel?

24. If A can go from Boston to Albany in $9\frac{1}{4}$ hours, and B from Albany to Boston in $11\frac{1}{3}$ hours, and they start at the same time, in how many hours will they meet?

25. A requires 4 days, B 3 days, and C $4\frac{1}{2}$ days, to do a certain piece of work. How long will it take, all three working together?

26. A can mow $\frac{5}{9}$ of a field in 3 days; B can mow $\frac{2}{3}$ of it in 4 days. How long will it take both to mow the field?

27. One pipe can fill a cistern half full in $\frac{3}{4}$ of an hour, and another can fill it three-quarters full in $\frac{1}{2}$ an hour. How long will it take both pipes to fill the cistern?

28. A cistern which holds 100 gallons can be filled from a pipe in 25 minutes, and emptied by a waste-pipe in 45 minutes. If both are opened together, how long will it take to fill the cistern, and how much water will be wasted?

29. A pipe can fill a cistern one-third full in $\frac{1}{4}$ of an hour; a waste-pipe can empty $\frac{1}{4}$ of the cistern in 20 minutes. If both pipes are opened, in what time will the cistern be filled?

30. If one pipe runs into a cistern at the rate of 2 gallons in 3 minutes, and another at the rate of 5 gallons in 4 minutes, while the water is running out of a third pipe at the rate of 4 gallons in 5 minutes, how long will it take to gain 71 gallons in the cistern?

31. A and B can do a piece of work in $2\frac{1}{2}$ days; A and C in $3\frac{1}{3}$ days; B and C in $4\frac{1}{4}$ days. Required the time in which all three, working together, can do the work, and in which each can do it alone.

Notice that by working *two days each* they can do $\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{3}} + \frac{1}{4\frac{1}{4}}$ of the work.

Ex. A person spends $\frac{3}{8}$ of his money for dry goods, $\frac{7}{9}$ of the remainder for groceries, and has \$15 left. How much had he at first?

After spending $\frac{3}{8}$ of his money he had $\frac{5}{8}$ left.
After spending $\frac{7}{9}$ of $\frac{5}{8}$ of his money he had left $\frac{2}{9}$ of $\frac{5}{8} = \frac{5}{36}$.
Then, $\$15 = \frac{5}{36}$ of his money.
Therefore, the whole money $= \$15 \div \frac{5}{36} = \$108$. *Ans.*

32. Sampson & Reed sold $\frac{5}{8}$ of a lot of wheat to one party, $\frac{3}{4}$ of the remainder to another, and had 93 bushels left. How much had they at first?
33. In a certain school $\frac{9}{16}$ of the scholars are girls, $\frac{4}{7}$ of the boys are over 16 years old, and 6 boys are under 16. How many girls, and how many scholars in all?
34. In a certain school $\frac{13}{24}$ are boys; $\frac{9}{22}$ of the girls are under 16, and 13 girls are over 16. How many boys and how many girls in the school?
35. If from a certain number $\frac{3}{4}$ of it be subtracted, then $\frac{1}{5}$ of the remainder, then $\frac{1}{7}$ of that remainder, and 6 still remain, what is the number?
36. 20 is $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{2}{3}$ of what number?
37. 6 is $\frac{6}{7}$ of $\frac{4}{5}$ of $\frac{1}{4}$ of what number?
38. Express $\frac{1}{29}$ of 1 lb. troy $+ \frac{1}{29}$ of 1 lb. avoirdupois as troy and as avoirdupois weights.
39. The cargo of a ship, worth \$45,000, belongs to three partners. A owns $\frac{7}{9}$ of $\frac{3}{5}$ of it, B's share is equal to $3\frac{3}{14}$ of $\frac{2}{9}$ of A's share, and C owns the remainder. What ought each to receive from the sale?
40. Find the largest number which is contained an integral number of times in each of the following: $2\frac{5}{9}$, $6\frac{7}{18}$, $11\frac{1}{2}$, $19\frac{1}{6}$.
41. A person bequeathed $\frac{5}{12}$ of his property to A, $\frac{1}{4}$ of it to B, $\frac{1}{6}$ to C, $\frac{1}{8}$ to D, and the remainder, \$550, to E. What was the value of the whole property?
42. Arrange in descending order of magnitude, $\frac{13}{25}$, $\frac{15}{26}$, $\frac{16}{27}$.

43. A bankrupt's debts are \$2520, and the value of his property is \$1890. How much can he pay on a dollar?
44. A bankrupt's debts are \$4264, and he pays $62\frac{1}{2}$ cents on a dollar, what are his assets?
45. If 15 yards of silk cost \$18.75, how much will $20\frac{1}{8}$ yards cost?
46. If $3\frac{4}{5}$ pounds of tea cost \$3.80, how much can I buy for \$21.87?
47. If $\frac{3}{14}$ of a ton of coal cost \$1.12, what is the price of $5\frac{1}{2}$ cwt.?
48. If $\frac{2}{11}$ of a piece of work be done in 25 days, how much will be done in $11\frac{2}{3}$ days?
49. A man walks 18 mi. 106 rds. $3\frac{2}{3}$ yards in $5\frac{1}{2}$ hours. How long does he take to walk a mile and a half?
50. When an ounce of gold is worth \$19.45, what is the value of .04 of a pound?

Ex. If 27 men can do a piece of work in 14 days, working 10 hours a day, how many hours a day must 12 men work to do the same work in 45 days?

Since 27 men can do the work in 14×10 hours = 140 hours,
1 man can do it in 27×140 hours,
and 12 men can do it in $\frac{27 \times 140}{12}$ hours = 315 hours.
Hence, the number of hours they work each day
$= \frac{315}{45}$, or 7. *Ans.*

51. If 9 horses can plow 46 acres in a certain time, how many acres can 12 horses plow in the same time?
52. If 12 men can reap a field in 4 days, in what time will 32 men reap it?
53. If 72 men dig a trench in 63 days, in how many days will 42 men dig another trench three times as great?
54. If a man travel 540 miles in 24 days, walking 6 hours a day, how many miles can he travel in 3 days, walking 8 hours a day?

55. If 15 men can perform a piece of work in 22 days, how many men will finish another piece of work four times as large in $\frac{1}{5}$ of the time?

56. A garrison of 2100 has provisions for 9 months, but receives reinforcements of 600 men. How long will the provisions last?

57. If a cubic foot of ice weigh $57\frac{3}{8}$ pounds, how many cubic feet of ice will weigh a ton?

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(1) If 12 horses can plow 96 acres in 6 days, how many horses will plow 64 acres in 8 days?

In 6 days 96 acres can be plowed by 12 horses.
In 1 day 96 acres can be plowed by 6×12 horses.
In 1 day 1 acre can be plowed by $\frac{6 \times 12}{96}$ horses.
In 8 days 1 acre can be plowed by $\frac{6 \times 12}{8 \times 96}$ horses.
In 8 days 64 acres can be plowed by $\frac{64 \times 6 \times 12}{8 \times 96}$ horses.

$$\frac{\overset{8}{\cancel{64}} \times 6 \times \cancel{12}}{\cancel{8} \times \underset{8}{\cancel{96}}} \text{ horses} = 6 \text{ horses. } \textit{Ans.}$$

(2) If 40 acres of grass be mowed by 8 men in 7 days, how many acres will be mowed by 24 men in 28 days?

24 men will mow *three times* as much as 8 men in the same time.
And the same number of men will mow *four times* as much in 28 days as in 7 days.
Hence, 24 men in 28 days will mow 3×4 times as much as 8 men in 7 days.

$3 \times 4 \times 40$ acres $= 480$ acres. *Ans.*

58. How many bushels of wheat will serve 72 people 8 days when 4 bushels serve 6 people 24 days?

59. If 2 horses eat 8 bushels of oats in 16 days, how many horses will eat 3000 bushels in 24 days?

60. If a man travel 150 miles in 5 days, when the days are 12 hours long, in how many days of 10 hours each will he travel 500 miles?

61. If a regiment of 939 soldiers consume 351 bushels of wheat in 21 days, how many soldiers will consume 1404 bushels in 7 days?

62. If 5 men can reap a field of $12\frac{1}{2}$ acres in $3\frac{1}{2}$ days, working 16 hours a day, in what time can 7 men reap a field of 15 acres, working 12 hours a day?

63. If 7 men mow 22 acres in 8 days, working 11 hours a day, in how many days, working 10 hours a day, will 12 men mow 360 acres?

64. If 44 cannon, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day?

65. How many times will a wheel $2\frac{4}{7}$ feet in circumference turn round in travelling over $12\frac{6}{7}$ yards?

66. How much ground will be travelled over by a wheel $1\frac{3}{7}$ yards in circumference, when it has made $4\frac{1}{2}$ turns?

67. Find the circumference of a wheel which makes 9 turns in travelling over $7\frac{1}{5}$ yards.

68. If the circumference of a wheel be $\frac{22}{7}$ of 1 yd. $1\frac{1}{8}$ ft., how many times will it turn in travelling $3\frac{3}{7}$ miles?

69. If the wheel of a locomotive be $3\frac{1}{7}$ times 5.52 feet in circumference, how many times does it turn in a minute, when the locomotive is running at the rate of 13.34 miles an hour?

70. A can run $\frac{4}{55}$ of a mile in $\frac{3}{5}$ of a minute, B can run $\frac{5}{56}$ of a mile in $\frac{2}{3}$ of a minute, and C $\frac{2}{19}$ of a mile in $\frac{5}{7}$ of a minute. Which is the fastest runner? and if he can run a certain distance in 3 min. 10 sec., how much longer will each of the others take to run the same distance?

71. Find the amount of the following bill:

Boston, Nov. 23, 1880.

Mr. Richard Rowe,

To John Doe, Dr.

To 125 lbs. sugar @ 10 cts.	*$*	
" 1 bag coffee, 115 lbs. @ 32 cts. .		
" 25 gals. molasses @ 62 cts. . .		
" 8 lbs. Japan tea @ 92 cts. . .		
" 28 lbs. crackers @ 8 cts. . . .		
" 2 bbls. flour @ $7.50		
	$	

Received Payment, *John Doe.*

Make bills for the following transactions:

72. James Hardy bought of C. H. Mills 275 bbls. of flour, at $6.75; 324 bbls. of flour, at $6.25; 300 bu. of potatoes, at 48 cents; 1578 lbs. of butter, at 32 cents; 2000 bbls. of apples, at $1.25; a car-load (20,000 lbs.) of oats, at 42 cents a bushel; a car-load (28,575 lbs.) corn, at 55 cents a bushel.

73. James Harlow bought of John Pike 12 bales, 480 lbs. each, Texas cotton, at 9¼ cents; 7 bales, 502 lbs. each, upland, at 10¼ cents; 3 bales, 492 lbs. each, low middling, at 9¾ cents; 18 bales, 490 lbs. each, good ordinary, at 9 cents.

Measures of Surface.

It will be remembered that if the length and breadth of a rectangle be expressed in the same linear unit, the product of these two numbers will express its area in square units of the same name as the linear units of the sides.

And, conversely, the number of square units divided by the number of linear units in one side will give the number of linear units in the other side. § 203.

(1) Find the area of a floor 16 ft. 3 in. long, 12 ft. 6 in. wide.

16 ft. 3 in. = $16\frac{1}{4}$ ft. 12 ft. 6 in. = $12\frac{1}{2}$ ft.

$16\frac{1}{4} \times 12\frac{1}{2} = \frac{65}{4} \times \frac{25}{2} = \frac{1625}{8}$ sq. ft.

$= 203\frac{1}{8}$ sq. ft. *Ans.*

(2) A rectangle contains 670 sq. ft. 108 sq. in., and is 19 ft. 6 in. wide. Find its length.

670 sq. ft. 108 sq. in. = $670\frac{3}{4}$ sq. ft. 19 ft. 6 in. = $19\frac{1}{2}$ ft.

$$670\frac{3}{4} \div 19\frac{1}{2} = \frac{2683}{\cancel{4}_{2}} \times \frac{\cancel{2}}{39} = \frac{2683}{78}$$

$= 34\frac{31}{78}$ ft. *Ans.*

Exercise LIX.

1. What length of board 15 in. wide will contain 11 sq. ft. 36 sq. in.?
2. What length of road 44 ft. wide will contain an acre?
3. Find the area of a rectangular field 13.12 chains long, 10.35 chains broad.
4. A path 216 ft. long measured 72 sq. yds. Find its breadth.
5. A rectangular field of 21.66 acres is 250.8 yds. broad. Find its length.
6. What is the area of a table if length and breadth be 4 ft. $3\frac{3}{7}$ in. and 2 ft. $9\frac{3}{5}$ in., respectively?

7. From each corner of a square, the side of which is 2 ft. 5 in., a square measuring 5 in. on a side is cut out. Find the area of the remainder of the figure.
8. The length and breadth of a map are $4\frac{1}{2}$ ft. and $3\frac{1}{3}$ ft., respectively. If the map represent 77,760 sq. mi. of country, how many miles are there to a square inch?
9. In rolling a grass plot 24 yds. long and containing 400 sq. yds., how many times must a roller 3 ft. 4 in. wide be drawn over it lengthwise so that the whole may be rolled?
10. How many sods, each 2 ft. $3\frac{1}{2}$ in. long and $8\frac{1}{4}$ in. broad, would be required to turf an acre of ground?
11. Find the area of a picture-frame $2\frac{1}{4}$ in. broad and having an outside measurement of 4 ft. $6\frac{1}{2}$ in. in length and 2 ft. 8 in. in width.
12. Find the expense of glazing four windows, each containing 12 panes, the panes being each a foot long and 10 in. wide, and the price of the glass 38 cents per square foot.
13. A garden 76 yds. long and 56 yds. broad, enclosed by a wall, has a border 4 ft. wide within the wall, and within this a path 5 ft. wide, the middle being grass. Find the areas of the border, path, and grass, respectively.

It will be remembered that the area of a circle is found by multiplying the square of the radius by 3.1416. § 204.

14. Find the area of a circle which has a radius of 3 ft.
15. What is the area of a circular field with a radius of 400 yards?
16. The radius of the rotunda of the Pantheon at Rome is 71 ft. 6 in. Find the area of the floor.
17. The diameter of a cistern is 13 ft. What is the area of the bottom?

18. The two dials of the clock of St. Paul's, London, are each $18\frac{1}{7}$ feet in diameter. What is the area of each in square feet?

It will be remembered that the surface of a sphere is found by multiplying the square of the diameter by 3.1416. § 205.

19. How many square inches on the surface of a ball 3 in. in diameter?

20. How many square inches of surface in a spherical black-board 12 in. in diameter?

21. What is the interior surface of a hemispherical vase 20 inches in diameter?

Carpeting Rooms.

It will be remembered that in determining the number of yards of carpeting required for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed.

The number of yards in a strip multiplied by the number of strips will give the required number of yards. § 207.

22. How many yards of carpeting $\frac{3}{4}$ of a yard wide will be required for a floor 26 ft. long, $15\frac{3}{4}$ ft. wide, if the strips run lengthwise? How many if the strips run across the room? How much will be turned under in each case?

23. How many yards $\frac{7}{8}$ of a yard wide will be required for a room $8\frac{1}{2}$ yds. long and 17 ft. wide, if the strips run lengthwise, and there is a waste of $\frac{1}{16}$ of a yard in each strip, in matching patterns?

24. How many square yards of oil-cloth will be required for a hall floor $5\frac{1}{4}$ yds. long and 10 ft. wide?

25. What will be the cost of carpet $\frac{5}{4}$ of a yard wide for a room $28\frac{1}{2}$ ft. by $18\frac{3}{4}$ ft., if the strips run lengthwise, and the cost per yard is 92 cents?

26. Find the cost of carpet 30 inches wide, at \$1.25 per yd. for a room 18 ft. by 14 ft., if the strips run lengthwise? if the strips run across the room?

27. Find the cost of carpeting 27 inches wide, at \$1.12½ per yard, for a room 29 ft. 9 in. by 23 ft. 6 in., if the strips run across the room?

28. Find the cost of carpeting ¾ of a yard wide, at \$2.75 per yard, for a room 34 ft. 8 in. by 13 ft. 3 in., if the strips run lengthwise, and if there be a waste of ¼ of a yard on each strip in matching the pattern?

29. Which way must the strips of carpet ¾ of a yard wide run in order to carpet most economically a room 20 ft. 6 in. long and 19 ft. 6 in. wide, if there be no waste for matching the pattern?

PAPERING AND PLASTERING.

It will be remembered that the area of the four walls of a room is equal to the perimeter × height. § 208.

30. Find the number of yards of plastering in the walls of a room 21¾ ft. long, 16½ ft. wide, and 11 ft. high, if 12 sq. yds. be allowed for doors, windows, and base-boards.

31. How many square yards of plastering in the walls and ceiling of a room 30 ft. 8 in. long, 26 ft. 5 in. wide, 10 ft. 6 in. high, if 24 sq. yds. be allowed for doors, windows, and base-board?

32. What will be the cost of plastering the walls and ceiling of a room 27 ft. 4 in. long, 20 ft. wide, and 12 ft. 6 in. high, at 27 cents per square yard, if 20 sq. yds. be deducted for doors, windows, and base-board?

33. Find the cost of whitening the ceiling and walls of a room 14 ft. 4 in. wide, 15 ft. 6 in. long, 10 ft. 6 in. high, at 5 cents per square yard, allowing 9 square yards for doors and windows.

34. Find the cost of plastering a room 21 ft. long, 15 ft. wide, 12 ft. high, at 40 cents per square yard, allowing for a door 7 ft. high, 3 ft. wide; 3 windows, each 5 ft. high, 3 ft. wide; and a dado 2 ft. 9 in. high around the room?

35. Find the cost of papering a room 20 ft. 6 in. long, 17 ft. 4 in. wide, 9 ft. high, with paper 18 in. wide, 8 yards in a roll, at 75 cents a roll; allowing for 2 doors, each 7 ft. high, 3 ft. wide, and for 3 windows, each 5 ft. 6 in. high and 3 ft. 3 in. wide.

36. Find the cost of papering a room 32 ft. long, 22 ft. wide, 13 ft. high, with paper 18 in. wide, 8 yards in a roll, at $1.25 a roll, if 50 sq. yds. be allowed for doors, windows, and base-board.

37. Find the cost of papering a room 26 ft. long, 21 ft. wide, 12 ft. high, with paper 20 in. wide, 8 yards in a roll, at $1.50 a roll, and a border at 25 cents per running foot; allowing for a fire-place 5 ft. 3 in. by 4 ft., a door 7 ft. by $4\frac{1}{2}$ ft., and 3 windows, each 6 ft. by $3\frac{1}{2}$ ft.

Board Measure.

317. Boards 1 inch or less in thickness are sold by the square foot.

Boards more than 1 inch in thickness, and all squared lumber, are sold by the number of square feet of boards 1 inch in thickness to which they are equivalent.

Thus, a board 16 feet long, 1 foot wide, and 1 inch thick, contains 16 feet *board measure.* If only $\frac{7}{8}$, $\frac{3}{4}$, or $\frac{1}{2}$ of an inch thick, it still contains 16 feet; but if $1\frac{1}{4}$ inches thick, it contains 20 feet board measure.

In practice, the width of a board, unless sawed to order, is reckoned only to the next smaller half-inch. Thus, a width of $11\frac{3}{8}$ inches is reckoned 11 inches; of $13\frac{5}{8}$ or $13\frac{3}{4}$ inches, is reckoned $13\frac{1}{2}$ inches.

How many feet board measure in:

38. A board 18 ft. long, 9 in. wide, $\frac{7}{8}$ in. thick?
A board 16 ft. long, 11 in. wide, 1 in. thick?

39. Twenty boards averaging 14 ft. long, 10 in. wide, $\frac{1}{2}$ in. thick?

40. Three joists 13 ft. long, 8 in. wide, 3 in. thick?

41. A stick of timber 8 in. by 9 in. and 27 ft. long.

42. Two beams, each 6 in. by 9 in. and 23 ft. long?

43. Three joists, each 3 in. by 4 in. and 11 ft. long?

44. Five joists, each 6 in. by 4 in. and 14 ft. long?

45. A stick of timber 10 in. square and 36 ft. long?

46. Ten planks, each 13 ft. long, 15 in. wide, 2 in. thick?

Find the cost of:

47. Nine joists, each 15 ft. long, $3\frac{1}{2}$ in. by 5 in., at $12 per M.

NOTE. The abbreviation "per M." means by the thousand.

48. Thirty planks, each 12 ft. long, 11 in. wide, 3 in. thick, at $15 per M.

49. Four sticks of timber, each 8 in. by 9 in. and 23 ft. long, at $18 per M.

50. A board 24 ft. long, 23 in. wide at one end and 17 in. at the other, and $1\frac{1}{2}$ in. thick, at $30 per M.

51. A stick of timber 29 ft. long, 10 in. by 12 in., at $13.50 per M.

52. The flooring for two floors, each 23 ft. by 17 ft., each floor double, and of boards $\frac{7}{8}$ in. thick; the lower floor at $18, and the upper at $24, per M.

53. The flooring timbers for a room 23 ft. by 17 ft., at $18 per M., if they are 2 in. by 10 in., 17 ft. long, and are placed on edge, two close to the walls and the others with spaces of $\frac{83}{40}$ of a foot between them.

318. Round logs are sold by the amount of square lumber that can be cut from them. When they do not exceed

16 ft. in length, the length and the diameter of the small end (not the average diameter) are taken, and a table stamped upon calipers gives the calculated number of feet board measure. This table may be calculated by the following rule:

Express the diameter in inches; subtract twice the diameter from the square of the diameter, and $\frac{21}{40}$ of the remainder will express the number of feet board measure in a log ten feet long.

Ex. Find the number of feet board measure in a log 12 ft. long and 20 in. in diameter.

$20^2 - 2 \times 20 = 400 - 40 = 360,$

$\frac{21}{40}$ of $360 = 189,$

$\frac{12}{10}$ of $189 = 226.8$ ft. board measure.

The square of the diameter − twice the diameter = 360, and $\frac{21}{40}$ of 360 = 189, which would be the number of feet board measure if the log were 10 ft. long. As the log is 12 ft. long, it is necessary to take $\frac{12}{10}$ of 189 to obtain the number of feet in the whole log.

The rule may be remembered by the formula $\frac{21}{40}(d^2 - 2d)$, in which d is the diameter of the log in inches.

By this rule find the number of feet board measure in:

54. A log 14 ft. long, 17 in. in diameter.
55. A log 11 ft. long, 13 in. in diameter.
56. A log 16 ft. long, 20 in. in diameter.
57. A log 12 ft. long, 15 in. in diameter.

Find the value, at $9 per M. of:

58. A log 17 ft. long, averaging 11 in. in diameter.
59. A log 18 ft. long, averaging 13 in. in diameter.
60. A log 13 ft. long, 16 in. in diameter.
61. A log 14 ft. long, 12 in. in diameter.

319. Large, heavy timber is often sold by the ton, signifying 50 cubic feet, or 600 feet board measure.

320. Clapboards are usually 4 ft. long and 6 in. wide, and are nailed on to expose $3\frac{1}{2}$ or 4 inches of their width to the weather.

62. How many clapboards will be required to cover the front of a house 60 ft. long and 20 ft. high, if they are laid 4 in. to the weather, and if 120 sq. ft. be deducted for doors and windows?

321. Shingles are 16 in. long, and are laid with $\frac{1}{3}$ of their surface exposed to the weather.

63. If one thousand shingles cover 120 sq. ft. of roof, what is the average width of a shingle?

64. Allowing one thousand shingles for 120 sq. ft., how many thousand will be required to cover the pitched roof of a house 60 ft. long, if the width of each side of the roof be $24\frac{1}{2}$ ft.?

Measures of Volume.

It will be remembered that if the length, breadth, and height of a rectangular solid be expressed in the same linear unit, the product of these numbers will express its volume in cubic units of the same name as the linear unit of the edges.

Exercise LX.

1. Find the volume of a rectangular solid whose length, breadth, and thickness are 7 ft., 2 ft. 6 in., and 11 in. respectively.
2. How many cubic feet of air in a hall 54 ft. long, 33 ft. wide, 21 ft. 4 in. high?
3. Find the volume of a cube whose edge is $2\frac{1}{2}$ yds.
4. A cellar is dug 21 ft. long, 17 ft. 3 in. wide, 9 ft. deep. How many cubic yards of earth are taken out?
5. How many cubic feet of water does a cistern hold whose length, breadth, and height are 5 ft. 4 in., 3 ft. 6 in., 2 ft. 10 in., respectively?

6. If the dimensions of a brick be 8 in. by $3\frac{1}{2}$ in. by $2\frac{1}{4}$ in., find its volume.
7. In a bar of iron 21 ft. long, 3 in. wide, 2 in. thick, how many cubic inches are there?
8. What is the value of a bar of gold 8 in. long and $\frac{3}{4}$ of an inch square, at $190 a cubic inch?

If two of the dimensions of a rectangular solid be given, and also its volume, the third dimension is found by dividing the volume by the product of the two given dimensions.

9. A reservoir whose length and breadth are 15 yds. and 12 yds., respectively, holds 330 cu. yds. of water. What is its depth?
10. What length must be cut off a beam 9 in. by 15 in. to contain $2\frac{1}{2}$ cu. ft.?
11. How high should a room be made, if its length be 31 ft. 3 in. and breadth 24 ft., in order that it may contain 10,000 cu. ft. of air?
12. A piece of wood 5 ft. long, 1 ft. broad, and 9 in. thick, is cut up into matches $2\frac{1}{2}$ in. long and .1 of an inch square. How many will there be if no allowance be made for waste in cutting?
13. How long a wall 6 ft. high, $12\frac{3}{4}$ in. thick, could be built with the bricks forming a pile 17 ft. 6 in. long, 5 ft. wide, 4 ft. 3 in. high?

---

14. Find the surface of a cube whose edge is 3 ft. $5\frac{7}{8}$ in.
15. Find the surface of a rectangular block of stone 4 ft. long, $2\frac{1}{2}$ ft. broad, $1\frac{1}{4}$ ft. thick.
16. A lake whose area is 45 A. is covered with ice 3 in. thick. Find the weight of the ice in tons, if a cubic foot weigh 920 oz. avoirdupois.
17. How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 16 in. thick, each brick being 8 in. long, 4 in. wide, $2\frac{1}{4}$ in. thick?

18. Find the cost of making a road 110 yds. in length and 18. ft wide, the soil being first removed to the depth of 1 ft. at a cost of 25 cts. a cubic yard; rubble being then laid 8 in. deep, at 25 cts. a cubic yard, and gravel placed on top 9 in. thick, at 62½ cents a cubic yard.

19. A room whose length is 27 ft., breadth 24 ft., height 10 ft., is to have its ceiling raised so as to increase the space by 84 cu. yds. What will then be its height?

20. A block of wood 5 ft. 4.8 in. long, 1 ft. 9 in. wide and thick, weighs 7.56 cwt. Determine the weight, in pounds, of a cubic foot.

It will be remembered that a cord is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

21. How many cords in a pile of wood 40 ft. long, 4 ft. wide, 5 ft. 4 in. high?

22. A pile of wood containing 67½ cords is 270 ft. long, and 4 ft. wide. How high is it?

23. What will be the cost of a pile of wood 25 ft. long, 4 ft. wide, 4 ft. 8 in. high, at $3.75 a cord?

24. What must be the length of a load of wood 3½ ft. high and 5 ft. wide, to contain a cord?

25. How high must manure be in a cart 6 ft. by 4 ft., in order to be ½ a cord?

Capacity of Bins and Cisterns.

It will be remembered that the standard bushel of the United States contains 2150.42 cubic inches; and the standard liquid gallon 231 cubic inches.

Ex. Find the number of bushels in a bin that is 6 ft. long, 5 ft. wide, 4 ft. deep.

$6 \times 5 \times 4 = 120$;
120 cu. ft. $= 120 \times 1728$ cu. in. $= 207,360$ cu. in.
207,360 cu. in. $\div$ 2150.42 cu. in. $= 96.4$.
Therefore, the bin will hold 96.4 bu.

NOTE. Since a bushel contains 2150.42 cu. in., and a cubic foot 1728 cu. in., a bushel contains $1\frac{1}{4}$ cu. ft., nearly. Hence, practically, *any number of cubic feet diminished by $\frac{1}{5}$ of itself will represent an equivalent number of bushels.*

And *any number of bushels increased by $\frac{1}{4}$ of itself will represent an equivalent number of cubic feet.*

Thus, if a bin contain 120 cu. ft., $120 - \frac{1}{5}$ of $120 = 96$, the number of bushels the bin will hold.

And, if a bin will hold 96 bu., $96 + \frac{1}{4}$ of $96 = 120$, the number of cubic feet in the bin.

26. Find the number of bushels in a bin that is 8 ft. long, 4 ft. wide, 3 ft. deep.

27. Find the number of bushels in a bin 9 ft. long, 6 ft. 6 in. wide, 3 ft. 4 in. deep.

28. Find the depth of a bin to hold 360 bu., if its length be 12 ft. and its width 6 ft.

29. Find the length of a bin that is 6 ft. wide and 5 ft. deep, if it hold 400 bu.

30. Find the number of bushels that will fill a bin 8.5 ft. long, 4.5 ft. wide, 3.5 ft. deep.

31. A bin 20 ft. long, 12 ft. wide, and 6 ft. deep, is full of wheat. What is its value, at $1.25 a bushel?

32. If a ton of coal occupy 40 cu. ft., how many tons will a bin hold that is 21 ft. long, 10 ft. wide, 5 ft. deep?

33. If a ton of Lehigh coal occupy 35 cu. ft., how many tons will a bin hold that is 8 ft. long, 5 ft. 9 in. wide, 4 ft. 6 in. deep?

Ex. How many gallons will a cistern hold that is 5 ft. square and 6 ft. deep?

$5 \times 5 \times 6 = 150$;

150 cu. ft. $= 150 \times 1728$ cu. in. $= 259{,}200$ cu. in.

259,200 cu. in. $\div$ 231 cu. in. $= 1122\frac{6}{77}$.

Therefore, the cistern will hold $1122\frac{6}{77}$ gals.

34. Find the number of gallons that a cistern will hold that is 13 ft. long, 6 ft. wide, 7 ft. 4 in. deep.

35. Find the number of gallons that a tank will hold that is 4 ft. long, 2 ft. 8 in. wide, 1 ft. 8 in. deep.

36. Find the number of gallons in a cubic foot.

NOTE. In practice, $7\frac{1}{2}$ gallons are allowed for a cubic foot.

37. Find the capacity of a cistern, in cubic feet, that will hold 200 barrels of water.

It is to be observed that the capacity of a round cistern is found by multiplying the area of the base by the depth.

38. Find the number of gallons that a round cistern will hold that is 6 ft. in diameter and 7 feet deep.

39. Find the number of gallons that a vessel will hold that is 12 in. in diameter and 10 in. deep?

40. How many quarts will a round vessel hold $5\frac{1}{4}$ in. in diameter and 6 in. deep?

It will be remembered that the capacity of a sphere is found by multiplying the cube of the diameter by .5236 ($\frac{1}{6}$ of 3.1416). § 211.

41. Find the number of cubic inches in a sphere 11 in. in diameter.

42. How many quarts will a sphere hold that is 12 in. in diameter?

43. What part of a bushel will a hemispherical bowl hold that is 13 in. in diameter?

44. If a cubical box 2 ft. on an edge contain a solid sphere 2 ft. in diameter, how many gallons of water can be poured into the box?

45. If 64 qts. of water be poured into a vessel that will hold 2 bu. of wheat, what part of the vessel will be filled?

SPECIFIC GRAVITY.

It will be remembered that the specific gravity of any substance is the *number of times* the weight of the substance contains the weight of an equal bulk of water. § 212.

One cubic foot of water weighs 1000 ounces avoirdupois.

46. Find the number of cubic inches in 1 oz. (av.) of water.
47. Find the weight in ounces (av.) of 1 cu. in. of water.
48. Find the weight in ounces (av.) of 1 pt. of water.
49. Find the number of pints in 1 lb. of water.
50. Find the weight, in grains, of 1 cu. in. of water.
51. Find the specific gravity of a bar of iron 5 in. long, and 2 in. square, if it weigh 5 lbs.
52. Find the specific gravity of a bar of iron 18 in. long, $2\frac{1}{8}$ in. wide, $1\frac{3}{4}$ in. thick, if it weigh 18 lbs. 9 oz.
53. Find the number of cubic inches to the pound of iron, if its specific gravity be 7.48.
54. Find the number of cubic inches in 2 lbs. $6\frac{1}{2}$ oz. of gold, if its specific gravity be 19.36.
55. How many pounds does a boy lift in raising a cubic foot of stone under water, if its specific gravity be $2\frac{1}{2}$?
56. A square-built scow 12 ft. long, $6\frac{1}{2}$ ft. wide, sinks 5 in. in water. What does it weigh, and how many pounds will be required to sink it 7 in. deeper?
57. A square-built scow 11 ft. long, $5\frac{1}{4}$ ft. wide, weighs 320 lbs. and is loaded with 750 lbs. of stone. How deep will it sink in the water?
58. How many tons of ice, specific gravity .93, can be packed in a building 50 ft. long, 40 ft. wide, 20 ft. high?
59. If an iceberg weigh .9 of an equal bulk of sea-water, how many cubic yards in an iceberg 40 rds. long, 6 yds. wide, and rising 160 ft. out of the sea?
60. If a cubic foot of brick wall weigh 90 lbs. and contain 22 bricks, with the mortar, what is the weight and the specific gravity of a brick and its share of mortar.
61. What is the weight of a brick wall 40 ft. long, 20 ft. high, and 1 ft. thick, if the specific gravity of a brick with its mortar be 1.46; and how many thousand bricks will be required for the wall, allowing 22 for a cubic foot?

CHAPTER XIV.

METRIC AND COMMON MEASURES.

322. TABLE OF EQUIVALENTS.

Length.

Meter = 39.37043 in.		Inch = 2.53998 ^{cm}.
= 1.09362 yds.		Yard = .91439 ^{m}.
Kilometer = 0.62138 mi.		Mile = 1.60933 ^{km}.

Surface.

Sq. meter = { 1550.031 sq. in.		Sq. inch = 6.45148 ^{qcm}.
1.19601 sq. yds.		Sq. yard = 0.83611 ^{qm}.
Hektar = 2.47110 A.		Acre = 0.40468 ^{ha}.

Volume.

Cu. centimeter = 0.06103 cu. in.		Cu. inch = 16.38662 ^{ccm}.
Cu. meter = 1.30799 cu. yds.		Cu. yard = 0.76453 ^{cbm}.
Ster = 0.27590 cord.		Cord = 3.62446 st.

Capacity.

Liter = 1.05671 liquid qts.		Liquid quart = 0.94633 ^{l}.
= 0.90810 dry qts.		Dry quart = 1.10119 ^{l}.

Weight.

Milligram = .015432 grs.		Grain = 0.06480 ^{g}.
Gram = 15.43235 grs.		Ounce av. = 28.34954 ^{g}.
Kilogram = 2.20462 lbs. av.		Ounce troy = 31.10350 ^{g}.
Metric ton = 2204.62 lbs. av.		Pound av. = 0.45359 ^{kg}.

Approximate Equivalents.

Meter	= 1.1 yds.	Yard	= $.9^{\text{m}}$.
Kilometer	= $\frac{5}{8}$ mi.	Mile	= 1.6^{km}.
Sq. meter	= $1\frac{1}{5}$ sq. yds.	Sq. yard	= $\frac{5}{6}^{\text{qm}}$.
Hektar	= $2\frac{1}{2}$ A.	Acre	= $\frac{2}{5}^{\text{ha}}$.
Cu. centimeter	= $\frac{1}{16}$ cu. in.	Cu. inch	= 16^{ccm}.
Cu. meter	= 1.3 cu. yds.	Cu. yard	= $\frac{10}{13}^{\text{cbm}}$.
Ster	= $\frac{3}{11}$ cord.	Cord	= $3\frac{2}{3}^{\text{st}}$.
Liter	= $1\frac{1}{20}$ liq. qts.	Liquid quart	= $\frac{20}{21}^{\text{l}}$.
	= $\frac{9}{10}$ dry qts.	Dry quart	= $1\frac{1}{9}^{\text{l}}$.
Hektoliter	= $2\frac{5}{8}$ bu.	Bushel	= $\frac{6}{17}^{\text{hl}}$.
Gram	= $15\frac{1}{2}$ grs.	Pound av.	= $\frac{5}{11}^{\text{kg}}$.
Kilogram	= $2\frac{1}{5}$ lbs. av.	Pound troy	= $\frac{5}{13}^{\text{kg}}$.

These tables are given for reference, and not to be committed to memory. It may be well, however, to keep in mind that a meter is 39.37 in.; a square meter is $(39.37)^2 = 1550$ sq. in.; a cubic meter is $(39.37)^3 = 61,025$ cu. in.; a liter is 1.0567 liquid qts.; and .908 dry qts.; and that a gram is 15.432 grs.; as by these equivalents all measures expressed in one system may be converted into measures of the other.

In solving the following examples the student will take his equivalents from the table, taking three places of decimals, or four when the first decimal figure is zero, and add one to the last decimal figure when the next is 5 or more.

(1) Reduce 25.55^{kg} to avoirdupois weight.

$$2.205 \text{ lbs.} = 1^{\text{kg}}.$$

$$\begin{array}{r}
2.205 \\
\underline{25.55} \\
11025 \\
11025 \\
11025 \\
\underline{4410} \\
56.33775 \text{ lbs.} \\
\underline{16} \\
202650 \\
\underline{33775} \\
5.40400 \text{ oz.}
\end{array}$$

56 lb. 5.4 oz. *Ans.*

(2) Reduce 5 sq. yds. 6 sq. ft. 108 sq. in. to square meters.

5 sq. yds. 6 sq. ft. 108 sq. in. = 5.75 sq. yds.

$$.836^{\text{qm}} = 1 \text{ sq. yd.}$$

$$\begin{array}{r}
.836 \\
\underline{5.75} \\
4180 \\
5852 \\
\underline{4180} \\
4.80700
\end{array}$$

4.807^{qm}. *Ans.*

EXERCISE LXI.

Reduce:

1. 24 gals. to liters.
2. 10 lbs. troy to kilograms.
3. 50.5 cu. yds. to cu. meters.
4. $69\frac{17}{100}$ mi. to kilometers.
5. 12 A. 12 rds. to hektars.
6. 10 cords to sters.
7. 4 cwt. 24 lbs. to kilograms.
8. 25 bu. 2 pks. to hektoliters.

Reduce to the common system:

9. 15^{km}.
10. 3^{ha}.
11. 12.125^{cbm}.
12. 101.25^{l}.
13. 20.25^{hl}.
14. 5^{kg} to troy weight.
15. 24^{st}.
16. 62.5^{qm}.
17. 1001^{kg} to avoirdupois weight.

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18. Find in acres, etc., the area of a field if its length be 100^{m} and breadth 75^{m}.
19. Determine the number of cubic meters in a box 2 yds. long, 3 ft. wide, $2\frac{1}{2}$ ft. deep.
20. Determine the number of cubic yards in a box 2^{m} long, 75^{cm} wide, 50^{cm} deep.
21. If a man walk 75^{m} a minute, what is his rate in miles per hour?
22. If cast-iron weigh 7.113^{g} per cubic centimeter, how many pounds does a cubic foot weigh?
23. How many steps 2 ft. 6 in. long will a man take in walking a kilometer?
24. Find the value of a carboy (17 qts.) of sulphuric acid, of 1.841 specific gravity, at $2\frac{1}{2}$ cents a pound.
25. Find the value of a carboy ($17\frac{1}{2}^{l}$) of nitric acid, of 1.451 specific gravity, at 15 cents a pound.
26. Find the weight in pounds and in kilograms of $31\frac{1}{2}$ gals. of the best alcohol, specific gravity .792.
27. If the specific gravity of sea-water be 1.026, and that of olive-oil be .915, what will be the weight of a hektoliter of each in pounds and in kilograms?

28. Find the weight in pounds and in kilograms of the air, specific gravity .00129206, in a room 7^m by 5^m, and 3.5^m high.

29. Find the weight in pounds and in kilograms of the air, specific gravity .00129206, in a room 23 ft. long, 16 ft. wide, and 10 ft. high.

30. If a balloon weigh 2^{kg}, and contain $10{,}000^l$ of hydrogen gas, specific gravity .00008929, what is its lifting force in kilograms and in pounds when the air has a specific gravity of .00129206?

NOTE. The lifting force = the weight of $10{,}000^l$ of air diminished by the weight of the hydrogen and balloon.

31. If a pile of wood be 1.2^m wide, 7^m long, and 2^m high, how much is it worth, at $4.50 a cord?

32. How many miles will be travelled in 1 hr. 28 min. 21 sec., at the rate of 50^{km} an hour?

33. Find the time of travelling 31 mi. 180 yds. at 1 min. 25 sec. per kilometer.

34. What is the weight of 12 cu. yds. 16 cu. ft. 720 cu. in. of earth of which a cubic meter weighs 1 t. 17 cwt.?

35. Find the weight in grams of a liter of mercury, of which a cubic inch weighs .4925 of a pound avoirdupois.

36. How many yards of cloth, at $3.12½ a meter, should be given in exchange for 15^m at $2.75 a yard?

37. If a wine merchant buy 3^{hl} of wine for 1600 francs, at what rate, United States money, does he pay a gallon, reckoning 25 francs equal to $4.85?

38. A mill-wheel is turned by a stream of water running at the rate of a yard per second in a channel 5 ft. wide and 9 in. deep. Determine the weight of water in metric tons, supplied in 12 hrs., if a cubic foot of water weigh 1000 oz.

CHAPTER XV.

PROPORTION.

323. The *relative magnitude* of two numbers is called their **ratio**, and is expressed by the fraction which the first is of the second.

324. The terms of this fraction are called the **terms** of the ratio; the numerator is called the **antecedent**; the denominator is called the **consequent**.

Thus, the ratio of 2 to 3 is expressed by $\frac{2}{3}$, of which the numerator 2 is the antecedent, and the denominator 3 is the consequent.

325. The ratio $\frac{2}{3}$ is often written, $2:3$.

326. If both terms of a ratio be multiplied or divided by the same number, the ratio is not altered.

Thus, if both terms of the ratio $2:3$ be multiplied by 4, the resulting ratio is $8:12$, and the ratio $8:12$ is equal to the ratio $2:3$; for, $\frac{8}{12}=\frac{2}{3}$. Again, if the ratio $2\frac{1}{2}:3\frac{1}{3}$ be multiplied by 6, the resulting ratio is $15:20$, and the ratio $2\frac{1}{2}:3\frac{1}{3}$ is equal to $15:20$; for, $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}=\frac{15}{20}$. Since $\frac{15}{20}$ reduced to its lowest terms $=\frac{3}{4}$, the simplest expression for the ratio of $2\frac{1}{2}:3\frac{1}{3}$ is $3:4$.

327. If the numerator and denominator of a fraction be interchanged, the fraction is said to be *inverted;* likewise, if the antecedent and consequent of a ratio be interchanged, the resulting ratio is called the *inverse* of the given ratio.

Thus, if the fraction $\frac{3}{4}$ be inverted the resulting fraction is $\frac{4}{3}$; and the inverse of the ratio $3:4$ is $4:3$.

328. If two *quantities* be expressed in the *same unit*, their ratio will be the same as the ratio of the two *numbers* by which they are expressed.

Thus, the quantity \$5 is the same fraction of \$11 as 5 is of 11.

329. Since ratio is simply *relative magnitude*, two quantities *different in kind* cannot form the terms of a ratio.

330. Two quantities the same in kind must be expressed in a *common unit* before they can form the terms of a ratio.

Thus, no ratio exists between 5 *tons* and 30 *days*; and the ratio of 5 tons to 3000 pounds can be expressed only when *both* quantities are written as tons or pounds.

331. Ratios are mere numbers, and may be compared.

Ex. Which is the greater ratio, 5 : 7 or 12 : 18?

$5:7 = \frac{5}{7}$, and $12:18 = \frac{12}{18} = \frac{2}{3}$.
Now $\frac{5}{7} = \frac{15}{21}$, and $\frac{2}{3} = \frac{14}{21}$.
As $\frac{15}{21}$ is greater than $\frac{14}{21}$,
the ratio 5 : 7 is greater than the ratio 12 : 18.

Exercise LXII.

Which is the greater ratio:

1. 5 : 8 or 6 : 9?
2. 7 : 10 or 9 : 12?
3. 8 : 9 or 10 : 12?
4. 6 : 12 or 8 : 14?
5. 10 cwt. : 15 cwt. or \$7 : \$9?
6. 5 dys. : 7 dys. or 8 ft. : 11 ft.?
7. 9 yds. : 6 yds. or 5 : 3?
8. $\frac{2}{3}$ lb. : $\frac{1}{2}$ lb. or $\frac{5}{8}$ yd. : $\frac{3}{8}$ yd.?

332. When two ratios are equal the four terms are said to be in **proportion**, and are called **proportionals.**

Thus, 5, 3, 15, 9 are proportionals; for $5:3 = 15:9$, since each of these ratios is represented by the fraction $\frac{5}{3}$.

333. A proportion is also written by putting a double colon between the ratios. Thus, $5 : 3 = 15 : 9$ (read 5 to 3 = 15 to 9), may be written $5:3::15:9$ (read 5 is to 3 as 15 is to 9).

334. The *first* and *last* terms of a proportion are called **the extremes**, and the two *middle* terms are called the **means.**

335. *Test of a proportion.* When four numbers are proportionals, the product of the extremes is equal to the product of the means.

This is seen to be true by expressing the ratios in the form of fractions, and multiplying both by the product of the denominators.

Thus, the proportion $5:3::15:9$ may be written $\frac{5}{3}=\frac{15}{9}$; and if both be multiplied by 3×9, the result will be $5\times 9=3\times 15$.

336. Either extreme, therefore, will be equal to the product of the means divided by the other extreme; and either mean will be equal to the product of the extremes divided by the other mean. Hence, if three terms of a proportion be given, the fourth may be found.

(1) What number is to 4 as 3 is to 6?

This may be written $\frac{\text{What number}}{4}=\frac{3}{6}$?

Multiply both sides of the equation by 4.

The result is, $\text{What number}=\frac{4\times 3}{6}$?

Answer, 2.

(2) 20 is to 24 as what number is to 30?

This may be written $\frac{20}{24}=\frac{\text{What number}}{30}$?

Multiply by 30, $\frac{20\times 30}{24}=\text{What number}$?

Answer, 25.

(3) 18 is to 32 as 45 is to what number?

This may be written $\frac{18}{32}=\frac{45}{\text{What number}}$?

As these fractions are equal, their reciprocals are equal;

that is, $\frac{32}{18}=\frac{\text{What number}}{45}$?

Multiply by 45, $\frac{32\times 45}{18}=\text{What number}$?

Answer, 80.

337. When three terms of a proportion are given, the method of finding the fourth term is called the **Rule of Three.**

It is usual to arrange the quantities (that is, to *state* the question), so that the quantity required for the answer may be the fourth term. Hence, the quantity which *corresponds* to that of the required answer must be the third term.

(1) If 5 tons of hay cost \$87.50, what will 21 tons cost?

Since the *cost* of 21 tons is required, \$87.50 is the third term.

Since 21 tons will cost *more* than 5 tons, 21 tons is the second term and 5 tons the first term.

That is, 5 tons : 21 tons : : \$87.50 : What quantity?

A difficulty presents itself here, inasmuch as no meaning can be given to the product of the means (\$87.50 multiplied by 21 tons). Since, however, the ratio of 5 tons : 21 tons = the ratio of 5 : 21, the ratio 5 : 21 may be substituted for 5 tons : 21 tons.

Then, 5 : 21 : : \$87.50 : What quantity?

That is, What quantity $= \dfrac{21 \times \$87.50}{5}$? *Answer*, \$367.50.

(2) When a post 11.5 ft. high casts a shadow on level ground 17.4 ft. long, a neighboring steeple casts a shadow 63.7 yds. long. How high is the steeple?

Height is required; the height 11.5 ft. is therefore the third term.

Since the *shadow* of the steeple is the longer, the *height* of the steeple must be the greater; therefore, the second term must be the greater of the two remaining quantities expressed in the same unit. 63.7 yds. = 191.1 ft.

	Shadow.	Shadow.	Height.	Height.
	17.4 ft.	: 191.1 ft.	: : 11.5 ft.	: What?
or,	17.4	: 191.1	: : 11.5 ft.	: What?

That is, height of steeple $= \dfrac{191.1 \times 11.5 \text{ ft.}}{17.4} = 126.3$ ft. *Ans.*

(3) If $\frac{2}{5}$ of a peck of oats weigh $\frac{23}{7}$ of a pound, what will $\frac{8}{7}$ of a bushel weigh?

$\frac{2}{5}$ of a peck $= \frac{2}{5}$ of $\frac{1}{4}$ bu. $= \frac{1}{10}$ bu.; $\frac{1}{10} : \frac{8}{7} : : \frac{23}{7}$ lbs. : What?

That is, the weight of $\frac{8}{7}$ bu. $= \frac{8}{7} \times \frac{23}{7}$ lbs. $\div \frac{1}{10}$

$= \frac{8}{7} \times \frac{10}{1} \times \frac{23}{7}$ lbs. $= 37\frac{27}{49}$ lbs.

The beginner will find it of advantage to state the problem thus:
$\frac{1}{10}$ of a bushel *weighs* $\frac{23}{7}$ lbs.
Will $\frac{8}{7}$ of a bushel weigh *more* than $\frac{1}{10}$ bu. or *less?*
If the answer to the question be *more*, the larger number is to be the second term; if *less*, the smaller number is to be the second term, and the remaining number to be the first term.

Hence, in solving problems by the Rule of Three,

338. Make that quantity which is of the same kind as the required answer the third term.

The *numbers* by which the other two quantities are expressed, when expressed *in a common unit*, will be the first and second terms.

If, from the nature of the question, the answer will be *greater* than the third term, make the *greater* of these two numbers the *second* term; if *less*, make the *less* the *second* term, and the remaining number the first term.

Divide the product of the second and third terms by the first term, and the quotient will be the answer required.

Exercise LXIII.

1. If 24 men can finish some work in 14 days, how long will it take 21 men to do it?
2. A well is dug in 13 days of 9 hours each. How many days of 10 hours each would it have taken?
3. A man who steps 2 ft. 5 in. takes 2480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?
4. If $\frac{5}{18}$ ton cost \$6, what will $7\frac{5}{9}$ cwt. cost, at the same rate?
5. If 42 yds. of carpet 2 ft. 3 in. wide are required for a room, how many yards 2 ft. 4 in. wide will be required?

6. A court was paved with 950 stones, each $1\frac{5}{8}$ sq. ft., and is re-paved with 836 stones of a uniform size. Find the size of each.

7. If a train, at the rate of $\frac{5}{18}$ of a mile per minute, take $3\frac{1}{4}$ hours to reach a station, how long will it take at the rate of $\frac{7}{15}$ of a mile a minute?

8. If a post 4 ft. 8 in. high cast a shadow 7 ft. 3 in. long, how long a shadow will a post 11 ft. high cast?

9. When a shadow 8 ft. 5 in. long is cast by a post 5 ft. 7 in. high, how high is a steeple that casts a shadow of 211 ft. at the same time?

10. If 4 men can mow a certain field in 10 hours, how many men will it take to mow it in 5 hours?

11. A tap discharging 4 gals. a minute empties a cistern in 3 hours. How long will it take a tap discharging 7 gals. a minute to empty it?

12. A pipe discharging 3 gals. 1 pt. a minute fills a tub in 4 min. 20 sec. How long will it take a pipe discharging 83 qts. a minute to fill it?

13. If both pipes of Ex. 12 discharge at the same time into the tub, how long will it take to fill it?

14. How long will it take to fill a cistern of 165 gals. by a pipe that fills one of 120 gals. in 7 min. 16 sec.?

15. A ship has sailed 1800 mi. in a fortnight. How long, at the same rate, will it take for a voyage of 5000 mi.?

16. The wheels of a carriage are 6 ft. 9 in. and 9 ft. 6 in. in circumference. How many times will the larger turn while the smaller turns 3762 times?

17. If $\frac{8}{25}$ of a ship be worth $2167, what is the value of $\frac{7}{17}$ of it?

18. What will be the weight of 18 cu. ft. 432 cu. in. of stone of which 10 cu. ft. 864 cu. in. weigh 14 cwt. 7 lbs.?

19. If 230 lbs. of flour make 360 lbs. of bread, how many four-pound loaves can be made from 1 cwt. of flour?

20. If a column of mercury 27.93 in. high weigh .76 of a pound, what will be the weight of a column of the same diameter 29.4 in. high?

21. How many francs will pay a bill of £100, when £42 10*s.* 8*d.* is equivalent to 1090.98 francs?

22. What will be the weight of a cube whose edge is 2 ft. 2 in., when a cube of the same material whose edge is 1 ft. 4 in. weighs 537.6 lbs.?

23. If a square field measuring 50 yds. $10\frac{2}{7}$ in. on each side be worth $\$2710\frac{9}{17}$, what is the value of a square field 62 yds. 1 ft. each way?

24. A gains 4 yds. on B in running 30 yds. How much will he gain while B is running $97\frac{1}{2}$ yds.?

25. If 10 cu. in. of gold weigh as much as 193 cu. in. of water, what is the size of a nugget weighing as much as a cubic foot of water?

26. If a garrison of 1500 men have provisions for 13 months, how long will the provisions last if it be increased by 700 men?

27. If a tree 38 ft. high be represented by a drawing $1\frac{1}{2}$ in. high, what, on the same scale, will represent the height of a house 45 ft. high?

28. If a country 630 mi. long be represented on a raised map by a length of $5\frac{1}{2}$ ft., by what height ought a mountain of 15,750 ft. be represented on the map?

29. A train travels $\frac{1}{4}$ of a mile in 18 sec. How many miles an hour does it travel?

30. If $4\frac{1}{2}$ tons of coal fill a bin 9 ft. long, 5 ft. broad, 5 ft. high, how many cubic feet will be required for the coal of a steamer carrying 3 weeks' consumption at 20 tons a day?

31. If 2 lbs. of rosin be melted with 5 oz. of mutton tallow, to make a grafting wax, how many ounces of tallow will 20 oz. of the wax contain?

Compound Proportion.

339. A ratio is said to be *compounded* of two or more given ratios, when it is expressed by a fraction which is the product of the fractions representing the given ratios.

Thus, the ratios 2 : 3 and 7 : 11 are represented by the fractions $\frac{2}{3}$ and $\frac{7}{11}$; and the ratio 14 : 33, which is represented by $\frac{14}{33}$ (the product of $\frac{2}{3}$ and $\frac{7}{11}$), is said to be compounded of the ratios 2 : 3 and 7 : 11.

340. A proportion which has one of its ratios a compound ratio is called a **compound proportion.**

In stating problems in compound proportion the quantity which corresponds to the answer required is made the third term. Each *pair* of the remaining quantities is then considered *separately* with reference to the answer required. The process will be understood by the following example:

Ex. If 4 men mow 15 acres in 5 days of 14 hours, in how many days of 13 hours can 7 men mow $19\frac{1}{2}$ acres?

As the answer is to be in days, make 5 days the third term.

I. *Will* it require *more* days for 7 men to mow 15 acres than it *did* for 4 men? Evidently, less.

Therefore, make 7 the first term and 4 the second.

II. *Will* it require *more* days for the same number of men to mow $19\frac{1}{2}$ acres than it *did* to mow 15 acres? Evidently, more.

Therefore, make 15 the first term and $19\frac{1}{2}$ the second.

III. *Will* it require *more* days of 13 hours for the same number of men to mow the same number of acres than it *did* of 14 hours? Evidently, more.

Therefore, make 13 the first term and 14 the second.

Hence, the statement is:

$$\left.\begin{array}{r|l} 7 & 4 \\ 15 & 19.5 \\ 13 & 14 \end{array}\right. :: 5 \text{ days} : \text{what?}$$

As the fourth term is obtained by dividing the product of the numbers on the right of the vertical line by the product of the

numbers on the left, the work may be shortened by cancelling. Thus:

$$\begin{array}{r|l}
\cancel{7} & 4 \qquad\qquad 1 \text{ day} \\
\cancel{15} & \cancel{19.5} \quad \cancel{1.3} :: \cancel{5} \text{ days} : \text{what?} \\
\cancel{5} \quad \cancel{10} \quad \cancel{13} & \cancel{14} \quad \cancel{2}
\end{array}$$

Cancel the factor 7, in 7 and 14, and set the 2 at the right of 14.

Cancel the factor 15, in 15 and 19.5.

Cancel the factor 1.3, in 1.3 and 13.

Cancel the factor 2, in 2 and 10.

Cancel the factor 5, in 5 and in 5 days, which leaves 1 day to be multiplied by 4.

Therefore, the answer is 4 days.

Exercise LXIV.

1. How many days 8 hours long will 60 men take to finish some work which 24 men can do in 15 days, working 10 hours a day?
2. What will be the expense of covering a room with drugget 4 ft. wide, at 91$\frac{3}{8}$ cts. a yard, when carpet 2 ft. 3 in. wide for the room costs \$70.50, at \$1.37$\frac{1}{2}$ a yard?
3. If 4418 tons of iron ore produce \$36,190 worth of metal, when iron is at \$37.50 a ton, what will be the value of the iron from 2275 tons of ore, at \$47 a ton?
4. If a bar of iron 3$\frac{1}{3}$ ft. long, 3 in. wide, 2$\frac{3}{4}$ in. thick weigh 93 lbs., what will be the weight of a bar 3$\frac{2}{3}$ ft. long, 4 in. wide, and 2$\frac{1}{2}$ in. thick?
5. If 40 bu. of wheat can be grown on the same area as 48 bu. of barley, and 28 acres produce 840 bu. of wheat, how much barley will be obtained from 38 acres?
6. If 18 men can dig a trench 150 ft. long, 6 ft. broad, and 4 ft. 6 in. deep in 12 days, how long will 16 men take for a trench 210 ft. long, 5 ft. broad, and 4 ft. deep?
7. In the reprint of a book consisting of 810 pages, 50 lines are contained in a page, instead of 40, and 72 letters in a line instead of 60. Of how many pages will the new edition consist?

8. If 3280 42-lb. shot cost $3000, how many 32-lb. shot can be bought for $4200?

9. What must be the rate of wages, that 12 men may earn in 10 days the same amount that 9 men earn in 14 days, at $1.50 a day?

10. A reservoir 15 yds. long and 4 ft. deep holds 32,500 gals. Determine the quantity of water it will hold when it has been increased in length by 18 ft. and in depth 1 ft.?

11. How far can A, who takes 3.1 ft. each step, run, while B, who takes 2.3 ft. each step, runs 220 yds., if A takes 7 steps while B takes 11?

12. If 6 hours be required for travelling a given distance at a given rate, how long will be required when the distance is diminished by one-fourth and the rate is increased by one-half?

13. How many hours a day must 5 men work to mow the same quantity of grass in 8 days that 7 men can mow in 6 days, working 10 hours a day?

14. If a bar 10 ft. $6\frac{1}{2}$ in. long. $3\frac{5}{8}$ in. broad, $3\frac{1}{2}$ in. thick weigh 4 cwt. 8.23 lbs., what length must be taken to weigh a long ton when the breadth and thickness are $4\frac{2}{3}$ in. and $4\frac{1}{8}$ in. respectively?

15. If 27 men, in 28 days of 10 hours each, dig a trench 126 yds. long, $2\frac{1}{2}$ yds. broad, $1\frac{1}{2}$ yds. deep, how long a trench $2\frac{1}{4}$ yds. broad, $1\frac{3}{4}$ yds. deep, will 56 men dig in 25 days of $8\frac{1}{4}$ hours?

16. What must be the length of a bar of silver $\frac{3}{4}$ in. square, that it may weigh the same as a bar of gold $\frac{1}{2}$ in. square and $6\frac{3}{4}$ in. long, if the weight of a cubic inch of silver have to that of a cubic inch of gold the ratio 47 : 88?

17. If it take 34^{kg} of wool to make 25^{m} of cloth $.6^{m}$ wide, how long a piece of cloth $.8^{m}$ wide can be made from 108.8^{kg} of wool?

18. An oak beam 5.40^{m} long, $.63^{m}$ thick, and $.57^{m}$ wide weighs 1469.25^{kg}, find the weight of a beam whose dimensions are 4.87^{m}, $.58^{m}$, $.53^{m}$.

NOTE. The "coefficient of expansion" of a body is the increase of the unit of volume of the body when the temperature is increased one degree. For air, this coefficient is .00367. That is, a cubic foot of air at zero will occupy at 1° C. a volume equal to 1.00367 cu. ft. if allowed to expand, the pressure remaining the same.

19. A certain quantity of air has a volume of 195.5 cu. ft. at 27.8°. What will be its volume at 100°?

NOTE. The volumes occupied by the same quantity of air, the temperature remaining unchanged, are in inverse ratio to the pressures.

20. A quantity of air at a temperature of 15.6° C. has a volume of 4 cu. ft. under a pressure of 12 lbs. to the square inch. What will be its volume at a temperature of 48.7° C., and under a pressure of 14 lbs. to the square inch?

PROPORTIONAL PARTS.

341. If it be required to divide a quantity into parts proportional to 3, 4, 5; the *numbers* 3, 4, 5 may be taken as *representatives* of the parts, and then the whole quantity will be represented by $3 + 4 + 5$; that is, by 12.

(1) Divide $391 into parts proportional to the numbers 5, 7, and 11.

The whole quantity will be represented by $5 + 7 + 11 = 23$.
Therefore, the respective parts will be, $\frac{5}{23}$, $\frac{7}{23}$, $\frac{11}{23}$ of $391.
That is, $85, $119, $187. *Ans.*
Or, the parts of the quantity may be found by the proportions:

$$23 : 5 :: \$391 : ?$$
$$23 : 7 :: \$391 : ?$$
$$23 : 11 :: \$391 : ?$$

In which the required terms will be $85, $119, and $187 respectively.

(3) Divide $248 into parts proportional to the fractions $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{25}$.

Multiply the fractions by 150, the L.C.M. of their denominators. The results are 15, 10, 6. Hence, the parts will be *represented* by the numbers 15, 10, 6, and the whole by 31.

Therefore, the respective parts will be $\frac{15}{31}$, $\frac{10}{31}$, $\frac{6}{31}$ of $248.

That is, $120, $80, $48. *Ans.*

EXERCISE LXV.

1. Divide $12,000 proportionally to the numbers 3, 4, 5.
2. Divide 815 tons proportionally to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.
3. Divide 6853 lbs. of wool proportionally to $1\frac{3}{4}$, $2\frac{4}{5}$, $5\frac{5}{6}$; and also proportionally to the reciprocals of these numbers.
4. Two persons join in purchasing some property, one paying $1250 and the other $1000. If the property rise in value to $3600, what will be the value of each one's share?
5. Gun-metal is composed of 3 parts (by weight) of tin to 100 parts of copper. What weight of each of these metals will there be in cannon weighing 721 lbs.?
6. Bell-metal contains 78 parts copper and 22 parts tin. What weight of each of these metals will there be in a bell weighing 937 lbs.?
7. It takes 75^{kg} of saltpeter, 12.5^{kg} of charcoal, and 12.5^{kg} of sulphur to make 100^{kg} of powder. How much of each of these substances will be required to make 10,000,000 cartridges, each containing 5^{g} of powder?
8. Yellow copper contains 2 parts of red copper and 1 part zinc. How many ounces of red copper are there in an article weighing 1 lb. made of yellow copper?
9. Type-metal is made of an alloy containing 39 parts of lead to 11 parts antimony. How many pounds of each will be required to make 957 lbs. of type?

10. Plumber's solder contains 2 parts lead and 1 part tin. How much of each of these in 100 lbs. of solder?
11. The air is composed of oxygen and nitrogen. In 100 volumes of air there are 21 volumes of oxygen and 79 of nitrogen. Reckoning the weight of a liter of oxygen to be 1.4295^g, that of a liter of nitrogen 1.2577^g, find the number of grams of each gas in 100^g of air.
12. What is the value of the gold in a chain weighing 3 oz. 4 dwt., supposing it to be 18 carats fine (that is, 18 parts of pure gold out of 24), at $19 an ounce?

Partnership.

342. Partnership is separated into *simple* and *compound.* In simple partnership the capital of each partner is invested for *the same time.* In compound partnership the time for which the capital of each partner is invested is taken into account, as well as the amount of the capital; and the division of profits and losses is made proportionally to the amount of the capital and the time it is invested.

Exercise LXVI.

1. Arnold and Baker enter into partnership. Arnold puts in $6000 for 8 months, and Baker $4000 for 6 months. Their profits are $2000. What is each man's share?

Note. Since the use of $6000 for 8 months is equivalent to the use of $48,000 for 1 month; and the use of $4000 for 6 months is equivalent to the use of $24,000 for 1 month, their profits must be divided in the ratio 48,000 : 24,000.

2. Dobson furnishes the firm of Dobson & Fogg with $5000 for 13 months; Fogg furnishes $7000 for 9 months. Their profits are $1700. What is the share of each?
3. In a business speculation, A furnishes $800, and after 3 months $250 more; B furnishes $950, and at the

end of 2 months withdraws $200; C furnishes $650, and at the end of 6 months $400 more. At the end of a year they realize a profit of $2516. How shall it be divided among them?

4. Two partners, A and B, begin business with capitals of $3500 and $8700, and A is to have .12 of the profits for managing the business. How shall a profit of $1906.25 be divided between them?

5. A puts $2100 into a business, and B $1750. At the end of a year each puts in $700 more, and C joins them with $2500. At the end of 18 months from this time how shall a profit of $2166.50 be divided?

6. Three graziers hire a pasture, for which they pay $132.50. One puts in 10 oxen for 3 months, another 12 oxen for 4 months, and the third 14 oxen for 2 months. How much of the rent ought each to pay?

7. A begins business, with a capital of $2400, on the 19th of March; and on the 17th of July admits B as a partner, with a capital of $1800. Dec. 31 the profits are $943. What is the share of each?

8. A and B join capitals in the ratio 7 : 11. At the end of 7 months A withdraws $\frac{1}{2}$ of his, and B $\frac{1}{3}$ of his; and, after 11 months more, they divide a profit of $5148.50. What is the share of each?

9. Divide £65 9*s.* among three persons, so that the first may have as many half-crowns as the second has shillings; and the second as many guineas as the third has pounds.

10. Two partners begin business each with a capital of $2000. A adds $500 at the end of 2 months, and $500 more at the end of 7 months; B adds $800 at the end of 3 months. What is the share of each, at the year's end, of a profit of $3605.25?

CHAPTER XVI.

PERCENTAGE.

343. In considering the increase or decrease in quantities, it is usual to employ, as a **common standard of reference**, the **number 100.**

Thus, if the population of a town at one census were 1200, and at the next 1500, the increase would be 300 in 1200; that is, 25 in every 100; or, as it is generally expressed, 25 *per cent.*

344. The symbol % is used for the words **per cent.**

345. The **representative number** resulting after an increase has taken place will be 100 + increase per cent; and after a decrease has taken place will be 100 − decrease per cent.

346. The *representative numbers* in any particular case may be changed to **quantities** by applying them all to the same unit of quantity.

Thus, if gunpowder be said to contain 75% of saltpetre, the meaning is, that if the number 100 be taken as the representative of the whole weight, the number 75 will represent the weight of saltpetre in it; and if the numbers be applied to any unit of weight, as a pound, the meaning will be, that 100 lbs. of gunpowder will contain 75 lbs. of saltpetre.

(1) Ten years ago the population of a city was 26,275, and has increased 20%. What is its present population?

If 100 be taken to represent the population ten years ago, 100 + 20 will represent the present population.

Therefore, the present population will be $\frac{120}{100}$ of 26,275 = 31,530. *Ans.*

(2) Ten years ago the population of a city was 26,275; its present population is 31,530. Determine the increase per cent.

31,530 – 26,275 = 5255, actual increase.

Since the increase on 26,275 is 5255, the increase on 100 is $\frac{100}{26275}$ of 5255 = 20.

The increase, therefore, is 20%. *Ans.*

(3) A town, after decreasing 11%, has 4539 inhabitants. Find its number at first.

If 100 be taken to represent the population at first, 100 – 11 = 89, will represent the present population.

Therefore, the population at first was $\frac{100}{89}$ of 4539 = 5100. *Ans.*

(4) In a certain school there are 200 girls, and the girls are 40% of the whole number of pupils. How many pupils in the school?

If 100 be taken to represent the whole number of pupils, 40 will represent the number of girls.

Therefore, the whole number of pupils is $\frac{100}{40}$ of 200 = 500. *Ans.*

(5) 50 lbs. is what per cent of 450 lbs.?

If 100 be taken to represent the whole weight, the number required to represent 50 lbs. will be $\frac{50}{450}$ of 100 = $11\frac{1}{9}$.

That is, $11\frac{1}{9}$%. *Ans.*

347. In the process of computing by the hundred, it is generally more convenient to use 1 as the representative number, and to express the per cent as **hundredths.**

Thus, in example (1), if the number 1 be taken to represent 26,275 inhabitants, 1.20 will represent the number of inhabitants after an increase of 20%; and the present population will be 1.20 of 26,275 = 31,530. *Ans.*

In example (2), if the number 1 be taken to represent 26,275, the increase will be represented by $5255 \div 26{,}275 = .20$. *Ans.*

In example (3), if 1 be taken to represent the population at first, .89 will represent its present population. That is, 4539 is .89 of the former population. Therefore, the former population was $4539 \div .89 = 5100$.

In example (4), if the number 1 be taken to represent the whole number of pupils, 200 will represent .40 of the whole number. Therefore, the whole number will be $200 \div .40 = 500$. *Ans.*

In example (5), if the number 1 be taken to represent 450 lbs., 50 lbs. will be represented by $\frac{50}{450}$ of $1 = .11\frac{1}{9}$. *Ans.*

Exercise LXVII.

1. The population of a town in 1870 was 12,275, and it increased 8% in the next ten years. Find its population in 1880.
2. How much metal will be obtained from 365 tons of ore, if the metal be 7% of the ore?
3. If gunpowder contains 75% of saltpetre, 10% of sulphur, 15% of charcoal, how much of each is there in a ton of powder?
4. A manufactory uses 24 tons of coal a day, and 20% of it is lost in smoke. How much coal would be needed if this waste could be prevented?
5. Air consists of 20.0265% (by measure) of oxygen gas and 79.9735% of nitrogen. How much oxygen in 1750 cu. ft. of air?
6. A town, after decreasing 25%, has 4539 inhabitants. Find its number at first.
7. 2% of a regiment of 750 men are killed in an engagement, 6% are wounded, and 4% are missing. What is the number still available for service?
8. If $3\frac{3}{4}$ tons of sulphur are required to make $31\frac{1}{4}$ tons of gunpowder, what is the per cent of sulphur in gunpowder?

9. In a school of 80 children, $17\frac{1}{2}\%$ are girls. Find the number of boys.
10. If goods are bought for \$415, and sold for \$500, what is the gain per cent?
11. If goods are bought for \$415, and sold for \$400, what is the loss per cent?
12. \$500 is 4% of what number?
13. A farmer buys 24 head of cattle at \$80 a head, and, after losing 6, sells the remainder at \$105 a head. How much does he gain or lose per cent?
14. If a ton (2240 lbs.) of ore in a gold mine yields 5 oz. (troy) of gold, what is the yield per cent?
15. If the ore in a mine yields $\frac{3}{80}$ of 1% of pure gold, how many tons (2240 lbs.) of ore must be taken to obtain 7 lbs. (troy) of gold?
16. $12\frac{1}{2}$ tons of iron are obtained from 235 tons of ore. What per cent of the ore is iron?
17. Goods are sold, at a loss of 3%, for \$2667.50. What was the cost?
18. Teas at 68 cents, 86 cents, and 96 cents a pound, are mixed in equal quantities, and sold at 90 cents a pound. Find the gain per cent.
19. By selling goods for \$1173.92, a merchant gains \$153.12. Find the gain per cent on the cost.
20. If to 25 gals. of alcohol 2 gals. of water are added, how much per cent of the mixture is water? how much per cent is alcohol?
21. What was the cost when $17\frac{1}{2}\%$ was gained by selling goods for \$253.80?
22. A wine merchant mixes 24 gallons, at \$7 a gallon, with 18 gallons, at \$5 a gallon, and sells the whole at \$7 a gallon. What does he gain per cent?
23. By selling a horse for \$200, a dealer loses $12\frac{1}{2}\%$. What would he have gained or lost per cent by selling at \$250?

24. A spirit merchant buys 75 gals., at \$3.25 a gallon, and, after drawing off 10 gals., sells the remainder so as to gain 5% on the whole. What is the selling price per gallon?

25. A person owns two estates worth respectively \$9845 and \$12,155. If the first rise in value 32%, and the second fall 13%, determine the rise or fall per cent in the value of his whole property.

26. A tradesman marks an article \$5, but takes off 5% for cash. If his profit is 14%, what was the cost of the article?

27. What would a dishonest dealer gain per cent by using a false weight of 15 oz. instead of a pound?

28. A dishonest dealer gains 12% by using false weights. What is the real weight of his pound?

29. A tradesman, in selling goods, deducts from the marked price 5% for cash. What is the marked price of some goods for which he receives \$7.12½?

30. The lead ore from a certain mine yields 60% of metal, and of the metal ¾ of 1% is silver. How much silver and lead will be obtained from 1200 tons of ore?

31. If ore loses 41⅔% of its weight in roasting, and 43¾% of the remainder in smelting, how much ore will be required to yield 1000 tons of metal?

Ex. How many per cent above cost must a man mark his goods in order that he may take off 20% from the marked price, and still make 20% on the cost?

If 100 be taken to represent the cost, then, as the gain is to be 20%, the selling price will be represented by 120.

As the selling price is to be 20% below the marked price, the selling price (120) will be .80 of the marked price.

Therefore, the marked price will be $\frac{100}{80}$ of 120 = 150.

That is, the goods must be marked 50% above cost.

32. How many per cent above cost must a man mark his goods in order to take off 10%, and still make a profit of 17%?
33. How many per cent above cost must a man mark his goods in order to take off $12\frac{1}{2}$%, and still make a profit of $12\frac{1}{2}$%?
34. How many per cent above cost must a man mark his goods in order to take off 15%, and still make a profit of 15%?
35. How many per cent above cost must a man mark his goods in order to take off $33\frac{1}{3}$%, and still make a profit of $33\frac{1}{3}$%?
36. If 5% of the population of a town has been the increase in the preceding ten years, what per cent of the population ten years ago has been added?
37. If, in a population of 27,000,000, 13% are foreign-born, how many foreign-born are there? What is the ratio of the foreign-born to the native?
38. A man bought a horse for \$70, and sold him for \$80. What per cent did he gain? What per cent of the money received for the horse was gained?
39. If, by selling goods for $12\frac{1}{2}$ per cent profit, a merchant clears \$800, what was the cost of the goods, and for how much were they sold?
40. A man selling eggs at 40 cents a dozen clears $33\frac{1}{3}$% on the cost; what was the cost? Another, selling at the same price, clears $33\frac{1}{3}$% of his receipts; what did his eggs cost?
41. By selling a carriage for \$117, a carriage-maker lost 10% of the cost. What ought he to have sold it for to make 10%?
42. A man gained in January 3% in weight, and in February lost 3%. What per cent of his weight on the first day of January is his weight on the first day of March?

43. 7 lbs. of a certain article lose 3 oz. in weight by drying. What per cent of the original weight is water?

44. 7 lbs. of a dry article have lost 3 oz. by drying. What per cent of the original weight was water?

45. A dry article was exposed to damp air, and absorbed 3 oz. of water; it then weighed 7 lbs. What per cent of its present weight is water?

46. If rosin is melted with 20% of its weight of tallow, what per cent of tallow does the mixture contain?

47. If 20% of a mixture of tallow and rosin is tallow, what per cent of the weight of the rosin is the weight of the tallow?

48. How many pounds of tallow must be mixed with $8\frac{1}{2}$ lbs. of rosin in order that the mixture may contain 15% of tallow?

49. Nitrogen gas, under standard pressure and temperature, is $\frac{1}{8}$ of 1% of the weight of an equal volume of water. What is its specific gravity?

50. Oxygen gas is $\frac{1}{7}$ of 1% of the weight of an equal volume of water; what is its specific gravity? How many gallons of oxygen will it take to weigh as much as a pint of water? How many of nitrogen?

51. If common air consist of 4 volumes of oxygen to 13 of nitrogen, what is its specific gravity?

52. How many gallons of air weigh as much as a pint of water?

53. If, by heating iron 185° F., it expands $\frac{1}{8}$ of 1%, what will be the expansion of iron in passing from $-20°$ F. to $+120°$ F.?

54. A tubular iron bridge is 450 ft. long, and one end is fast to a pier. How much play must be allowed at the other end, if the iron expands at the above rate, and if the climate varies from $-30°$ F. in winter to $+130°$ F. in a July sun?

55. How much longer is 100 miles of iron rail at 118° F. than at 20° below zero?

348. In many kinds of commercial transactions the payment made is reckoned at a rate per cent.

Commission and Brokerage.

349. The *commission* paid to an agent for his services is generally reckoned at a rate per cent.

The sum left after the payment of the commission and other expenses is called *net proceeds*.

Commission paid to a *broker* is called *brokerage*.

(1) A commission merchant has consigned to him 50,000 lbs. of wool, which he sells at 50 cents a pound, and charges $2\frac{1}{2}\%$ commission. He pays \$125 for freight and \$50 for cartage. Find the commission and net proceeds.

\$.50 × 50,000	=		\$25,000	
\$25,000 × .025	=	\$625		= commission.
Freight	=	125		
Cartage	=	50		
Total expenses	=		800	
			\$24,200	= net proceeds.

(2) An agent receives \$5000 with which to purchase goods, after deducting a commission of 1%. What is the amount of the commission, and how much remains for purchasing goods?

If 100 be taken to represent the amount to be paid for the goods, 1 will represent the commission, and 101 will represent the \$5000. Therefore, the amount expended for goods will be $\frac{100}{101}$ of \$5000 = \$4950.50.

And 1% of \$4950.50 = \$49.50 = commission.

In selling, the commission is reckoned on *the amount received;* in buying, the commission is reckoned on *the amount paid.*

Exercise LXVIII.

1. Find the brokerage, at $\frac{1}{8}$ of 1%, to be paid on $10,450.
2. Find the commission on $2595, at $2\frac{1}{2}$%.
3. An agent sells 200 bbls. of flour, at $6.25; 600 gals. molasses, at 65 cents; and charges a commission of $1\frac{3}{4}$%. What are the net proceeds?
4. A commission merchant received $1640 with which to buy corn, after deducting a commission of $2\frac{1}{2}$%. What is the amount of his commission, and how many bushels of corn at $62\frac{1}{2}$ cents a bushel can he buy?
5. A commission merchant sells a consignment of cotton for $5216. He pays $51 for freight and storage, and charges a commission of $2\frac{1}{4}$%. What are the net proceeds?
6. A consignment of butter was sold for $1570, of which $1546.45 were the net proceeds. What was the rate per cent of commission?
7. What are the net proceeds from the sale of 2250 bbls. of flour, at $6.25 a barrel, if the charges for freight and storage be 50 cents a barrel, commission for selling 2%, for guaranteeing payment $1\frac{1}{2}$%?
8. A commission merchant sells 350 crates of peaches, at $2.60. If the commission be $4\frac{1}{2}$%, find the net proceeds.
9. A man sells 420 acres of land, at $40 an acre, and charges $1\frac{1}{4}$% commission. What is his commission?
10. An agent, charging $4\frac{1}{2}$% commission, receives for his services $313. Find the amount of his sales.
11. A merchant buys, through an agent, 730 yds. of carpeting, at $1.25 a yard, and pays the agent $\frac{3}{4}$ of 1% commission; the freight amounted to $7.37. At what price per yard must the carpeting be sold to realize a profit of 20%?

12. An agent sells a consignment of goods for $2100. He pays $33.50 for freight, and, reserving his commission, remits $2024.77. Find the rate of his commission.

13. A commission merchant has consigned to him 5000 lbs. of cotton, which he sells at 14 cents a pound, and charges 2% commission. With the net proceeds he buys cotton cloth, at 10 cents a yard, charging 1½% commission for buying. How many yards of cloth does he buy?

14. A commission merchant has consigned to him 500 bbls. of flour, which he sells at $5.50 a barrel, and charges 2½% commission; the expenses for freight, etc., amounted to $250. With the net proceeds he buys sugar, at 6¼ cents a pound, charging 2½% commission for buying. How much sugar does he buy, and what is the amount of his commissions?

15. A collector's commission for collecting taxes, at 1½%, is $206.55. What was the sum collected?

16. An agent received $2961 with which to purchase goods after deducting his commission at 5%. How much was his commission?

17. An agent buys 3100 bbls. of flour, at $4.50 a barrel, and charges 1½% commission. What is the amount of the bill including the commission?

18. A broker receives $6150 to invest in cotton, at 10¼ cents a pound. His commission is 2½%. How many pounds of cotton can he buy?

19. An agent sells 1100 bbls. of flour, at $4.50 a barrel, and charges 2½% commission. He invests the proceeds in steel, at 1½ cents a pound, charging 1½% commission. What is his entire commission, and how many tons of steel (2240 lbs. to a ton) does he buy?

Insurance.

350. In insurance a payment, called a *premium of insurance*, is made for a guarantee of a specified sum of money in the event of loss from fire or accident; and is reckoned at a rate per cent on the amount insured.

In life insurance an annual payment is made in order to secure a specified sum of money in the event of death.

(1) Find the premium for the insurance of a cargo worth \$36,000, at $6\frac{1}{4}\%$.

$$\begin{array}{r} \$36000 \\ .06\frac{1}{4} \\ \hline \$2250.00 \end{array}$$

(2) For what sum should a cargo worth \$36,000 be insured at $6\frac{1}{4}\%$, so that, in the event of loss, the owner may receive both the value of the cargo and the premium?

If 100 be taken to represent the sum to be insured, then the $6\frac{1}{4}$ will represent the premium; and $100 - 6\frac{1}{4}$, that is, $93\frac{3}{4}$, will represent the value of the cargo.

Hence, the sum to be insured will be

$$\$36,000 \div .93\frac{3}{4} = \$38,400.$$

Exercise LXIX.

1. Find the premium of fire insurance for \$2650, at $\frac{1}{2}$ of 1%.
2. Find the premium to be paid for insuring a person's life for \$2500, at an age for which the rate is $2\frac{1}{4}\%$.
3. At $2\frac{3}{4}\%$, what premium of insurance will be paid on a vessel worth \$36,400?
4. A vessel is worth \$12,052. Determine the sum to be insured, and the premium to be paid at $1\frac{3}{4}\%$, so that in the event of loss the owner may receive both the value of the vessel and the premium.

5. The premium for insurance at $1\frac{1}{4}\%$ is \$150. What is the amount insured?

6. If a premium of insurance at $2\frac{3}{4}\%$ amount to \$28.60, what is the sum insured?

7. A vessel is so insured that if lost the owner may receive both the value of the vessel and the premium. The value of the vessel is \$96,084, and the rate of insurance $1\frac{7}{8}\%$. Find the premium.

8. A building worth \$8000 is insured at $\frac{5}{8}$ of its value, at $\frac{1}{8}$ of 1% per annum. What is the annual premium?

9. Four companies join in insuring a ship and cargo for \$60,000. One company takes $\frac{1}{8}$, at $\frac{3}{8}$ of 1%; a second takes \$10,000, at $\frac{3}{4}$ of 1%; a third, \$15,000, at $\frac{5}{8}$ of 1%; a fourth, the remainder, at $\frac{1}{2}$ of 1%. How much is paid for insurance?

10. If the ship in the last problem receive damage to the amount of \$4500, what ought each company to pay?

11. A man insures his life for \$10,000, paying \$350 a year in advance. He dies the day before the fifth premium was due. The company pay his widow \$10,000. How much have they lost by him, if the interest gained on the premiums paid amount to \$175?

12. A merchant shipped a cargo to London; and to cover both the cargo and the premium, he took out a policy of \$100,800, at $3\frac{1}{2}\%$. What was the value of the cargo?

13. Three companies insure, at $\frac{3}{4}$ of its value, a building worth \$16,000. The first company takes $\frac{1}{3}$ the risk, at $\frac{3}{4}$ of 1%; the second, $\frac{2}{5}$ of it, at $\frac{7}{8}$ of 1%; and the third, the remainder, at $\frac{3}{4}$ of 1%. Find the total premium.

14. S. Williams pays \$18.40 premium for insuring his house for $\frac{2}{3}$ of its value at $1\frac{1}{2}\%$. What is the value of his house?

TAXES AND DUTIES.

351. *Taxes* on property are reckoned at a rate per cent on the assessed value of the property; and *duties* on imported goods are sometimes reckoned at a rate per cent on the cost in the country from which they are imported.

Ex. The assessed valuation of a town is $2,326,112; the number of polls is 800 (that is, the number of persons who pay a poll-tax); the amount required for expenses is $15,830.56. If each poll pays a tax of $1.50, and the sum to be received from the State is estimated at $3000, what is the rate per cent of tax on the property? What is the tax of S. Jones, who pays for one poll, and has property valued at $8216?

The amount of poll-taxes = 800 × $1.50 = $1200.
$15,830.56 – $4200 = $11,630.56 to be assessed on property.
$11,630.56 ÷ $2,326,112 = .005 = ½ of 1%.
That is, the tax is 5 mills on a dollar, or $5 on $1000.
Therefore, S. Jones's property-tax is .005 of $8216 = $41.08.
Total tax = $41.08 + $1.50 = $42.58.

EXERCISE LXX.

1. If James Brown be assessed $2500 on his house and $5200 on personal property, and pays for 2 polls at $1.50 each, how much will his tax be, the rate being $12.18 on $1000?
2. If the rate of tax be $12.25 on $1000, and the tax be $11,788.50, what is the valuation?
3. If the assessed valuation of a town be $1,777,000, and the property-tax be $6870, what is the rate on $1000.
4. What sum must be assessed, in order that $15,000 shall remain after paying a commission of 2% for collecting the taxes?

5. A tax of $1857.60 is levied upon a school district for building a school-house. The assessed valuation of the district is $1,935,000. What is the tax on property valued at $6250?

6. In a certain town there are 1350 polls. The assessed value of the real estate is $713,250; of the personal property is $738,954; the poll-tax is $2. The tax on property is $1\frac{1}{8}\%$. But only 96% of the tax can be collected, and the collector is paid $2\frac{1}{2}\%$ of the amount collected. How much does the town receive from the taxes?

7. What is the duty, at 20% *ad valorem* (that is, 20% of the cost), on 320 boxes of raisins, each containing 40 lbs., and costing 8 cents a pound?

8. What is the duty, at 6 cents a gallon, on 420 hhds. of molasses, 63 gals. in a hogshead?

9. At 40%, what is the duty on 300 tons of iron (2240 lbs. to a ton) invoiced at $1\frac{1}{2}$ cents a pound?

10. Paid $1360.80 duty on 300 hhds. of molasses, each containing 63 gals., at 25 cents a gallon. What was the rate per cent of duty?

11. A sugar refiner imports 50 hhds. of sugar weighing 480 lbs. each, and 120 hhds. of molasses containing 63 gals. each. What is the amount of the duties, if the sugar pay 3 cents a pound, and the molasses 8 cents a gallon, an allowance being made on the sugar of 10%, and 2% on the molasses?

12. An importer paid $825 duty on an invoice of silks, the duty being 24%. But damages of 15% were allowed at the custom-house. What was the entire cost of the goods?

13. Paid $325 duty on goods which had been damaged; allowance for damage is 24%, and the duty was 24%. What was the invoice price of the goods?

CHAPTER XVII.

INTEREST AND DISCOUNT.

352. Interest is the payment made for the use of money.

The interest to be paid for a given sum differs from the payments considered in the last chapter, in that it *depends* on the *time* for which the sum is loaned as well as on the *rate per cent* charged.

353. The sum loaned is called the **principal.** The principal and interest, added together, are called the **amount.**

SIMPLE INTEREST.

354. If 100 be taken as the representative of the principal, the rate will represent *the interest for one year;* the product of *the rate per cent by the number of years* will represent the *whole interest,* and this added to 100 will represent the amount.

Thus, if the time be 3 years and the rate per cent 6, the interest will be represented by 18, and the amount by 118.

(1) Find the interest on \$1024, for 2 yrs. 8 mos., at $5\frac{1}{2}\%$.

Time, 2 yrs. 8 mos. = $2\frac{2}{3}$ yrs.

$$
\begin{array}{r l}
\$1024 & \\
\underline{.05\frac{1}{2}} & \\
512 & \\
\underline{5120} & \\
\$56.32 & = \text{interest for 1 yr.} \\
\underline{2\frac{2}{3}} & = 2 \text{ yrs. } 8 \text{ mos.} \\
3755 & \\
\underline{11264} & \\
\$150.19 & \textit{Ans.}
\end{array}
$$

(2) Find the interest of \$1020, for 5 yrs. 11 mos. 18 dys., at 6%.

The interest at 6% for 1 year is .06 of the principal.
The interest for 1 month is $\frac{1}{12}$ of .06 = .005 of the principal.
The interest for 1 day is $\frac{1}{30}$ of .005 = $\frac{1}{6}$ of .001 of the principal.
Hence, the interest for

5 yrs.	$= 5 \times .06$	$= .30$
11 mos.	$= 11 \times .005$	$= .055$
18 dys.	$= 18 \times \frac{1}{6}$ of .001	$= \underline{.003}$
5 yrs. 11 mos. 18 dys.		$= .358$ of the principal.

.358 of \$1020 = \$365.16. *Ans.*

The method employed in the last example may be employed for any rate per cent, by first finding the interest at 6%, and then taking such a part of the interest as the given rate is of 6%. Thus, the interest at $4\frac{1}{2}\% = \frac{4\frac{1}{2}}{6} = \frac{3}{4}$ of the interest at 6% = interest at 6% $- \frac{1}{4}$ of itself. The interest at 8% is $\frac{8}{6} = \frac{4}{3}$ of the interest at 6% = interest at 6% $+ \frac{1}{3}$ of itself.

In most business transactions the time for which interest is required is 1, 2, 3, or 4 months (30 dys. being reckoned 1 mo.), and the rate of interest is $\frac{1}{2}$% a month. Hence, the interest on a given sum for 2 mos. (or 60 dys.) is found by moving the decimal point two places to the left; for 1 mo., 3 mos., 4 mos., by moving the decimal point two places to the left, and multiplying by $\frac{1}{2}$, $1\frac{1}{2}$, and 2, respectively. Thus, the interest on \$1250 for 2 mos. is \$12.50; for 1 mo., \$6.25; for 3 mos., \$18.75; for 4 mos., \$25.

EXERCISE LXXI.

Find the interest of:

1. \$680.40 for 2 yrs. 4 mos. 6 dys., at 6%.
2. \$25.625 for 30 dys., at 6%.
3. \$85.85 for 1 yr. 7 mos. 21 dys., at 6%.

4. \$1100 for 3 yrs. 4 mos., at 5%.
5. \$1275 for 3 yrs. 2 mos. 15 dys., at 8%.
6. \$475.16 for 27 dys., at $4\frac{1}{4}$%.
7. \$1290.50 for 60 dys., at 6%.
8. \$125 for 1 yr. 2 mos. 2 dys., at 9%.
9. \$250.80 for 10 mos. 10 dys., at $3\frac{1}{2}$%.
10. \$258.85 from Mar. 6, to June 24, at 5%.
11. \$380 for 2 yrs. 11 mos. 27 dys., at $4\frac{1}{2}$%.
12. \$475.05 for 1 yr. 9 mos. 14 dys., at $7\frac{3}{10}$%.
13. \$725.40 for 11 mos. 24 dys., at $5\frac{1}{4}$%.
14. \$680.50 for 2 yrs. 6 dys., at 5%.
15. \$630.50 for 90 dys., at 6%.
16. \$547.60 from Feb. 20 to Dec. 5, at $6\frac{1}{2}$%.
17. \$875 from May 5, 1880, to June 21, 1881, at $5\frac{1}{2}$%.
18. \$758.50 from Jan. 5 to July 1, at $4\frac{1}{2}$%.
19. \$342.42 from Feb. 5, 1879, to Mar. 15, 1881, at 7%.
20. \$540 from Mar. 5 to Sept. 21, at $3\frac{1}{2}$%.

Find the amount of:

21. \$431.50 for 2 yrs. 8 mos., at $4\frac{1}{2}$%.
22. \$476.50 from July 5, 1880, to Feb. 9, 1881, at 4%.
23. \$319.20 from April 7 to Aug. 31, at $3\frac{1}{4}$%.
24. \$6460 from June 15, 1878, to May 7, 1880, at $4\frac{1}{4}$%.
25. \$150 from Aug. 5, 1879, to Mar. 17, 1881, at 7%.
26. \$527.20 from Jan. 1 to Nov. 20, at $4\frac{1}{2}$%.
27. \$1250 from Nov. 15, 1880, to Mar. 1, 1881, at 5%.
28. \$624.36 from Mar. 5 to Dec. 20, at $7\frac{3}{10}$%.
29. \$12,260 from May 6 to Oct. 24, at $3\frac{3}{4}$%.
30. \$11,216 from Oct. 20 to Dec. 31, at 1% a month.

NOTE. In business a year is reckoned at 360 days in computing interest *for a time less than a year expressed in months and days;* hence, the interest is $\frac{5}{365}$ or $\frac{1}{73}$ too great. But general governments take the number of days between the two given dates, and reckon for the interest such a part of a year's interest as this number of days is of 365 days.

355. It is often required to find the rate, time, or principal, when two of these and the interest (or amount) are given.

(a) *When the principal, interest (or amount), and time are given, to find the rate per cent.*

(1) At what rate per cent will \$480 produce \$72 in three years?

Interest on \$480 for 3 years is \$72;
on \$480 for 1 year is $\frac{1}{3}$ of \$72;
on \$1 for 1 year is $\frac{1}{480}$ of $\frac{1}{3}$ of \$72 = \$.05.
Therefore, rate required is 5%.

(2) At what rate per cent will \$8432 amount to \$9437.516 in 2 yrs. 7 mos. 24 dys.?

The interest is \$9437.516 − \$8432 = \$1005.516.
The time is 2 yrs. 7 mos. 24 dys. = 2.65 yrs.
Interest on \$8432 for 2.65 yrs. is \$1005.516;

$$\text{on } \$8432 \text{ for 1 yr. is } \frac{\$1005.516}{2.65};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$1005.516}{2.65 \times 8432} = \$.04\tfrac{1}{2}.$$

Hence, rate required is $4\frac{1}{2}$%.

The rate may also be found by forming a proportion with the representative numbers. Thus,

$$\overset{\text{Capital.}}{8432} : \overset{\text{Capital.}}{100} : : \overset{\text{Interest.}}{1005.516} : \overset{\text{Interest.}}{\text{what?}}$$

The fourth term will be 11.925.

Hence, the interest, which = rate × time, is represented by 11.925.

$$\text{Therefore, the rate} = \frac{11.925}{2.65} = 4.5.$$

That is, the rate is $4\frac{1}{2}$%.

Find the rate per cent:

31. When the interest on $326 for 15 yrs. is $220.05.
32. When the interest on $372.50 for 18 yrs. is $301.725.
33. When $245 amount to $252.96¼ for 9 mos.
34. When the interest on $235.25 is $70.575 for 5 yrs.
35. When $363.125 amount to $371.598 for 7 mos.
36. When the interest on $249.43¾ is $49.88¾ for 5 yrs. 4 mos.
37. When $350 amount to $406.70 for 3 yrs. 7 mos. 6 dys.
38. When the interest on $6875 is $72.05 for 90 dys.
39. When the interest on $642 is $10.70 for 5 mos.
40. When the interest on $8432 for 2 yrs. 7 mos. 23 dys. is $1339.28.
41. When a sum of money is doubled in 14 yrs.
42. When an investment for 5 yrs. 2 mos. produces a sum equal to $\frac{2}{5}$ of the capital.
43. When an investment for 3 yrs. 1 mo. 15 dys. produces a sum equal to $\frac{1}{8}$ of the capital.

(b) *When the principal, interest (or amount), and rate per cent are given, to find the time.*

Ex. In what time will the interest on $8432 amount to $1005.516, at $4\frac{1}{2}\%$?

Interest on $8432, at $4\frac{1}{2}\%$, for 1 yr. is .045 of $8432 = $379.44.

Therefore, the number of years will be $\frac{\$1005.516}{\$379.44} = 2.65$.

And 2.65 yrs. = 2 yrs. 7 mos. 24 dys. *Ans.*

Find the time in which:

44. The interest on $450 will amount to $72, at 4%.
45. The interest on $487.50 will amount to $39, at 4%.
46. The interest on $238.75 will amount to $64.46, at $4\frac{1}{2}\%$.
47. The sum of $793.875 will amount to $805.84, at $5\frac{1}{2}\%$.
48. A sum of money will double itself at 4%.

49. The sum of $10 will amount to $17, at 6%.
50. The sum of $502.67 will amount to $578.07, at $4\frac{1}{2}$%.
51. The interest on $537.50 will amount to $80.625, at 4%.
52. The interest on $6875 will amount to $75.05, at $4\frac{1}{4}$%.
53. The interest on $8520 will amount to $1746.60, at 6%.

(c) *When the interest, time, and rate are given, to find the principal.*

Ex. What principal will in 2 yrs. 7 mos. 24 dys. produce $1005.516 interest, at $4\frac{1}{2}$%?

2 yrs. 7 mos. 24 dys. = 2.65 yrs.

Interest for 1 yr. $= \frac{\$1005.516}{2.65} = \$379.44.$

Interest on $1 for 1 yr., at $4\frac{1}{2}\% = \$.045.$

Hence, principal required $= \frac{379.44}{.045}$ of $\$1 = \$8432.$

Find the principal that:

54. Will produce $90 interest in 3 yrs., at 4%.
55. Will produce $63 interest in 3 yrs., at $6\frac{1}{4}$%.
56. Will produce $100 interest in 8 yrs. 6 mos., at 5%.
57. Will produce $1746.60 interest in 3 yrs. 5 mos., at 6%.
58. Will produce $12 interest in 7 mos., at 5%.
59. Will produce $50 interest in 228 dys., at $4\frac{1}{2}$%.
60. Will produce $1339.28 interest in 2 yrs. 7 mos. 24 dys., at 6%.
61. Will produce $1312.65 interest in 2 yrs. 3 mos., at 6%.
62. Will produce $750 interest in 3 yrs. 8 mos., at 5%.

(d) *When the amount, time, and rate per cent are given, to find the principal.*

Ex. Find the principal that will amount to $748.125 in 3 yrs. 6 mos., at 4%.

3 yrs. 6 mos. = $3\frac{1}{2}$ yrs.

If the principal be represented by 100, the interest will be represented by $3\frac{1}{2} \times 4 = 14$, and the amount will be represented by 114. Hence, the principal $= \frac{100}{114}$ of $\$748.125 = \$656.25.$

Find the principal that will amount:

63. To \$840 in 3 yrs., at 4%.
64. To \$901.1384 in 2 yrs. 6 mos., at $4\frac{1}{8}$%.
65. To \$6000 in 21 dys., at 5%.
66. To \$297.60 in 8 mos., at 6%.
67. To \$6378.75 in 1 yr. 1 mo., at 5%.
68. To \$21,047.95 in 1 yr. 7 mos. 21 dys., at $4\frac{1}{2}$%.
69. To \$185.09 in 2 yrs. 3 mos. 18 dys., at 5%.
70. To \$659.40 in 2 yrs. 11 mos. 15 dys., at 6%.
71. To \$9437.516 in 2 yrs. 7 mos. 24 dys., at $4\frac{1}{2}$%.
72. To \$10,266.60 in 3 yrs. 5 mos., at 6%.

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73. What is the interest of \$195 for 2 yrs. 2 mos. 2 dys., at $6\frac{1}{2}$%?
74. At what rate per cent will \$1025.20 produce \$25.72 in 4 mos. 9 dys.?
75. The principal is \$653; the interest \$5.52; the rate 8%. Find the time.
76. Find the amount of \$520 for 2 mos. 3 dys., at $4\frac{1}{2}$%.
77. What sum bearing interest at $4\frac{1}{2}$% will yield an annual income of \$1000?
78. How long will it take \$4000 to produce \$625 interest, at $5\frac{1}{2}$%?
79. At what rate per cent will \$3000 produce \$250 interest in 1 yr. 2 mos. 24 dys.?
80. Find the interest of \$1721.84 from April 1 to Nov. 12, at $4\frac{1}{2}$%.
81. How long must \$3904.92 be on interest to amount to \$4568.76, at 5%?
82. Find the interest of \$137.60 from July 3 to Dec. 12, at $7\frac{3}{10}$%.
83. Find the interest of \$680.20, at $7\frac{1}{2}$%, for 73 dys., reckoning 365 dys. for a year.

Discount.

356. When the holder of a promissory note sells the note to a bank, or other purchaser, the sum paid by the bank is called the **proceeds** or **avails** of the note, and the difference between the sum named in the note and the proceeds is called the **discount.**

357. Discount is reckoned at so much per cent, and the per cent is called the **rate of discount.**

358. Questions in discount are calculated like questions in simple interest, the terms used being *discount* instead of *interest*, and *rate of discount* instead of *rate of interest.*

Note. The sum named in the note should be written in words, and is called the *face* of the note. The person signing the note is called the *maker;* a person who writes his name on the back of the note is called an *endorser*, and is responsible for the payment of the note.

A note, to be legal, must contain the words "*value received*"; to be negotiable, must be made payable to the *bearer*, or *to the order* of some person who must endorse the note.

When a note bears interest, the discount is computed on the face of the note with the interest added.

A note is *nominally* due at the expiration of the time named in the note, but it does not *mature*, that is, become *legally* due, until three days after this time. These three days are called **days of grace.** And the discount is computed on the time between the day the note is discounted and the day of its maturity.

When the time is expressed in *days*, the day of maturity is found by counting forward from the date of the note the *number of days* named in the note, and the three days of grace. When the time is in *months*, the day of maturity is found by counting the *number of calendar months*, and the three days of grace. When a note falls due on Sunday, or a legal holiday, it is payable on the day previous.

A **protest** is a notice in writing by a notary public to the endorsers that a note has not been paid. If a note be not protested on the last day of grace the endorsers are released from their obligation.

Find the *day of maturity*, the *time to run* (from the day the note is discounted), the *discount*, and the *proceeds* of the following notes:

(1) $520.16. BOSTON, Jan. 12, 1881.

Sixty days after date I promise to pay to the order of G. L. Gage five hundred twenty and $\frac{16}{100}$ dollars, for value received.

Discounted at 6%, Feb. 1. JOHN KELLEY.

Counting 60 days, from Jan. 12, there are 19 in January, 28 in February, and 13 in March.

Therefore the note becomes due Mar. $^{13}/_{16}$. (13 denotes the time it is nominally due, 16 the time it is legally due.)

The time to run is 27 days in February and 16 in March = 43 days.

The discount is the interest on $520.16 for 43 days, at 6%. Therefore,

The discount is .007$\frac{1}{6}$ of $520.16 = $3.73.

The proceeds is $520.16 − $3.73 = $516.43.

(2) $650. CINCINNATI, Nov. 3, 1880.

Six months from date we jointly and severally promise to pay to the order of Charles Fall six hundred and fifty dollars, value received, with interest at six per cent.

Discounted at 7%, Jan. 3, 1881.

JOHN HENDERSON.
JAMES HENDRICKS.

Interest on note for 6 mos. 3 dys., = $19.82.
Amount of note $650 + $19.82 = $669.82.
Day of maturity, May $^{3}/_{6}$, 1881.
Time to run, 4 mos. 3 dys.
Discount on $669.825, at 7%, for 4 mos. 3 dys. = $16.02.
Proceeds is $669.82 − $16.02 = $653.80.

Exercise LXXII.

Find *day of maturity*, the *time to run*, the *discount*, and *proceeds* of the following notes:

1. \$750. New York, Jan. 1, 1881.

Four months from date I promise to pay to the order of James Fay seven hundred and fifty dollars, value received.

Discounted at 7%, Jan. 12. John Pray.

2. \$4325.50. Boston, Jan. 3, 1881.

Sixty days from date I promise to pay to James Finn, or order, four thousand three hundred twenty-five and $\frac{50}{100}$ dollars, value received.

Discounted at $6\frac{1}{2}$%, Jan. 6. George Bellows.

3. \$1340.70. Richmond, Va., Jan. 6, 1881.

Ninety days from date I promise to pay to the order of Peter Bright thirteen hundred forty and $\frac{70}{100}$ dollars, value received.

Discounted at 7%, Jan. 26. George Wright.

4. \$1456.30. Charleston, S.C., Jan. 19, 1881.

Three months after date I promise to pay to the order of John George fourteen hundred fifty-six and $\frac{30}{100}$ dollars, value received.

Discounted at 5%, Feb. 1. John Waldorf.

5. \$4550.36. Detroit, Mich., Feb. 2, 1881.

Four months after date I promise to pay to the order of John Callender four thousand five hundred fifty and $\frac{36}{100}$ dollars, value received.

Discounted at $5\frac{1}{2}$%, Feb. 16. James Barton.

6. \$5000. CHICAGO, Ill., Oct. 4, 1880.

Six months after date I promise to pay to John Adams or order five thousand dollars, value received, with interest at seven per cent.

Discounted at 8%, Dec. 31. WILLIAM DUNN.

7. \$4760. MILWAUKEE, Wis., Jan. 1, 1881.

Ninety days after date I promise to pay to the order of James Pike four thousand seven hundred and sixty dollars, value received.

Discounted at 7½%, Feb. 15. WILLIAM CLEMENT.

8. \$2017.85. KANSAS CITY, Mo., Jan. 14, 1881.

Three months after date I promise to pay to the order of John Brown two thousand seventeen and $\frac{85}{100}$ dollars, value received.

Discounted at 10%, Mar. 1. TIMOTHY BRUCE.

9. \$652.45. CONCORD, N H., Jan. 25, 1881.

Five months after date I promise to pay to the order of Charles Barrett six hundred fifty-two and $\frac{45}{100}$ dollars, value received, with interest at six per cent.

Discounted at 4½%, Mar. 15. WILLIAM KIMBALL.

10. \$9040. BALTIMORE, Md., Jan. 19, 1881.

Sixty days from date I promise to pay to the order of Charles Carroll nine thousand and forty dollars, value received.

Discounted at 5½%, Feb. 16. JAMES MONROE.

11. \$215. AUGUSTA, Me., Jan. 28, 1881.

Thirty days after date, I promise to pay to the order of James Fogg two hundred and fifteen dollars, value received.

Discounted at 6%, Feb. 3. JOHN MOSES.

359. If it be required to determine the face of a note that will yield a given sum when discounted at a bank, the method will be as follows:

Ex. For how much must a 3-months note, without interest, be made, that it may yield $500 when discounted at 6%?

The discount on $1 for 3 mos. 3 dys. is $.0155.
Proceeds on $1 is $1 − $.0155 = $.9845.
Therefore, the face required for $500 is $500 ÷ .9845,
= $507.87. *Ans.*

12. Find the face of a note at 90 dys. that will realize $850 when discounted at 7%.
13. Find the face of a note at 4 mos. that will realize $1600 when discounted at $5\frac{1}{2}$%.
14. Find the face of a note at 30 dys. that will realize $1200 when discounted at $6\frac{1}{2}$%.
15. Find the face of a note at 60 dys. that will realize $4000 when discounted at 8%.
16. Find the face of a note at 2 mos. that will realize $4500 when discounted at $7\frac{3}{10}$%.
17. Find the face of a note at 3 mos. that will realize $1100 when discounted at 7%.

PRESENT WORTH AND DISCOUNT.

360. The *present worth* of a sum of money due at the end of a given time, is the sum that, put at interest for the given time, will amount to the given sum.

Thus, $50 will, in 2 yrs., at 6%, amount to $56. And $56 to be paid at the end of 2 yrs. is equal in value to $50 paid now. Hence, $50 is regarded as the present worth of $56 to be paid in 2 yrs.

361. The difference between the given sum and its present worth is called the true discount.

362. It is evident that this discount of the given sum is the interest of its present worth.

The operation of finding the present worth of a sum of money at a given rate of interest is the same as the operation of finding the principal when the amount, time, and rate per cent are given. § 355.

Ex. Find the present worth and discount of $5000 due at the end of 3 yrs. 6 mos., at 6%.

If 100 be taken to represent the present worth, the discount will be represented by $3\frac{1}{2} \times 6 = 21$.

The given sum will be represented by $21 + 100 = 121$.

Hence, the present worth is $\frac{100}{121}$ of $5000 = $4132.23; and the discount is $5000 − $4132.23 = $867.77.

18. Find the present worth of $500 due in 11 mos., at 5%.

19. Find the present worth and discount of $3334.62 due in 2 yrs., at $4\frac{1}{2}$%.

20. Find the present worth and discount of $4261.33 due at the end of 1 yr. 6 mos. at 6%.

21. Find the present worth and discount of $2416.50 due in 7 mos., at 5%.

22. Find the present worth of $678.40 due in 16 mos., at $4\frac{1}{2}$%.

23. Find the present worth and discount of $574.17 due in 2 yrs. 3 mos., at $5\frac{1}{3}$%.

24. Find the present worth and discount of $625.13 due in 8 mos., at $7\frac{3}{10}$%.

25. Find the present worth and discount of $715.20 due in 1 yr. 4 mos., at $3\frac{1}{2}$%.

NOTE. This discount is called *true* discount to distinguish it from discount used in business transactions. Discount in business always means either so many per cent off without reference to time, as when a merchant is paid before he proposes to demand payment; or so many per cent off reckoned at the current rate of interest for a given time, as when a note is discounted at a bank.

Partial Payments.

363. When settlements of accounts between two merchants are made at the expiration of a *year, or less*, it is customary to reckon interest on the several items in the accounts to the time of settlement. Also, when partial payments are made, and endorsed on a note that contains the words "**with interest,**" and that is paid in full within a year, it is usual to compute the interest on the principal and on each of the payments to the time of settlement.

(1) Andrew Hall buys of John Paul $400 worth of goods, at 30 dys. At the end of 3 mos. he pays $200, and the balance 2 mos. later. Find the balance.

The time between the end of 30 dys. and the time of settlement is 4 mos. Therefore, interest is reckoned on the $400 for 4 mos. and on the $200 for 2 mos.

$400 + 4 mos. interest =	$408
$200 + 2 mos. interest =	202
Therefore, balance due is	$206

(2) A man holds a note for $500, dated Jan. 1, 1880, on which are endorsed partial payments as follows: Mar. 1, 1880, $50; Oct. 1, 1880, $25; Nov. 1, 1880, $400. What is due Jan. 1, 1881, interest at 7%?

Amount of $500 for 1 yr. at 7% is		$535.00
Amount of $50 for 10 mos. at 7% =	$52.92	
Amount of $25 for 3 mos. at 7% =	25.44	
Amount of $400 for 2 mos. at 7% =	404.67	
		483.03
Balance due,		$51.97

364. This method is in accordance with what is called the **Merchants' Rule.**

26. If I buy goods Jan. 10, at 30 dys., for $218; Feb. 5, at 60 dys., for $421; and pay Feb. 10, $200, Mar. 17, $50; what is due Apr. 25, interest at 6%?

27. A note for $618.75, dated Apr. 17, 1880, payable on demand, bears the following endorsements: June 5, $126.50; Aug. 20, $137.25; Nov. 17, $210. What is due Jan. 1, 1881, reckoning interest at 6%?

28. A note for $1000, dated Apr. 1, 1880, payable on demand, with interest at 7%, bears the following endorsements: May 6, $200; July 5, $225.37; Oct. 18, $322. What is due Jan. 1, 1881?

29. A note for $835.25, dated July 1, 1880, payable on demand, with interest at 6½%, bears the following endorsements: Aug. 20, $157.50; Sept. 21, $180.25; Oct. 5, $200; Dec. 1, $80. What is due Jan. 1, 1881?

When a note runs *longer than a year*, and has endorsements, the Merchants' Rule works injustice to the holder of the note, as may be seen by an extreme case: Thus, if the interest due at the end of each year be endorsed on a note of $1000, at 6% interest, the amount of the interest payments in about 24 years will cancel the whole note, without the payment of one cent of the principal. Hence,

365. When a note that contains the words "**with interest**" runs longer than a year, and partial payments have been made, the interest is computed by a rule adopted by the Supreme Court of the United States, and therefore called

THE UNITED STATES RULE.

Find the amount of the principal to the time when the payment, or sum of the payments, equals or exceeds the interest.

From this amount deduct the payment or sum of the payments.

Consider the remainder as a new principal, and proceed as before.

Ex. A note of $1520, dated May 20, 1880, and drawing interest at 6%, had payments endorsed upon it as follows: Oct. 2, 1880, $300; Feb. 26, 1881, $25; Apr. 2, 1881, $570; Aug. 8, 1881, $600. Find the amount due Dec. 6, 1881.

yr.	mos.	dys.				
1880	10	2			$1520	
1880	5	20			.022	
	4	12	.022		$33.44	1st interest.
					1520.00	
$300.					$1553.44	
					300.00	1st payment.
1881	2	26			$1253.44	2d principal.
1880	10	2			.024	
	4	24	.024	$25	$30.08	2d interest.
					$1253.44	2d principal.
$25.					.006	
				$570	$7.52	3d interest.
1881	4	2			30.08	2d interest.
1881	2	26			1253.44	
	1	6	.006		$1291.04	
					595.00	2d & 3d payments.
$570.					$696.04	3d principal.
					.021	
1881	8	8			$14.62	4th interest.
1881	4	2			696.04	
	4	6	.021		$710.66	
					600.00	
$600.					$110.66	4th principal.
					.019⅔	
1881	12	6			$2.18	5th interest.
1881	8	8			110.66	
	3	28	.019⅔		$112.84	*Answer.*

In the first place, find the difference in time between each pair of consecutive dates. At the right of the result in each case put the corresponding decimal multiplier for the interest at 6%, and put the corresponding payment below.

Generally it can be determined *mentally* whether one or more payments must be taken to make a sum equal to or greater than the interest. If two or more payments are required, the corresponding decimal multipliers may be added and the result taken for the multiplier. Thus, it is evident that .024 of $1253.44 is more than $25; therefore, .024 + .006 = .03, may be taken for the multiplier, which will give for the interest $37.60. To this the principal is added, and from the amount the sum of the payments is subtracted.

When the rate is greater or less than 6% the several *interests* must be increased or diminished according to the given rate.

30. A note of $2000, dated Jan. 22, 1880, and drawing interest at 6%, had payments endorsed upon it as follows: May 20, 1880, $100; July 20, 1880, $325; Nov. 2, 1880, $20; Dec. 23, 1880, $125. Find the balance due Mar. 1, 1881.

31. A note of $1662.50, dated Jan. 15, 1880, and drawing interest at 6½%, had payments endorsed upon it as follows: Apr. 30, 1880, $25; June 24, 1880, $25; Sept. 2, 1880, $625; Jan. 31, 1881, $700. Find the balance due May 12, 1881.

32. A note of $4560, dated Jan. 22, 1879, and drawing interest at 7%, had payments endorsed upon it as follows: Jan. 10, 1880, $2000; Aug. 31, 1880, $500; Jan. 15, 1881, $1200; Mar. 4, 1881, $860. Find the balance due June 15, 1881.

33. A note of $785.50, dated Jan. 30, 1879, and drawing interest at 5%, had payments endorsed upon it as follows: July 17, 1879, $100; Jan. 29, 1880, $100; Dec. 31, 1880, $20; Mar. 16, 1881, $300; June 14, 1881, $50. Find the balance due July 23, 1881.

34. A note of $300.25, dated Aug. 4, 1879, and drawing interest at 6½%, had payments endorsed upon it as follows: Oct. 14, 1879, $100; July 21, 1880, $100; Oct. 11, 1880, $50; Jan. 18, 1881, $50. Find the amount due July 22, 1881.

Compound Interest.

366. When a note contains the words "**with interest annually,**" and the interest is not paid at the time it is due, the interest is usually added to the principal; and new principals are thus formed at regular intervals of time.

367. The interest may be compounded with the principal (that is, made a part of the principal), annually, semi-annually, quarterly, etc., according to agreement.

Ex. Find the compound interest of \$800 for 2 yrs. 3 mos. 15 dys., at 7%.

$$
\begin{array}{lrl}
 & \$800 & \\
 & .07 & \\
\hline
 & \$56 & \text{1st interest.} \\
 & 800 & \\
\hline
 & 856 & \text{2d principal.} \\
 & .07 & \\
\hline
 & \$59.92 & \text{2d interest.} \\
 & 856.00 & \\
\hline
 & \$915.92 & \text{3d principal.} \\
 & .0175 & \\
\text{3 mos. 15 dys.} & 6)\overline{16.03} & \\
 & 2.67 & \\
\hline
 & \$18.70 & \text{3d interest.} \\
 & 915.92 & \\
\hline
 & \$934.62 & \text{amount.} \\
 & 800.00 & \\
\hline
 & \$134.62 & \text{interest.}
\end{array}
$$

Hence, the compound interest, at 7%, for the given time is \$134.62.

368. If the given time be not an integral number of years, the amount is found for the number of entire years, and then the amount of this for the fractional part of a year.

35. Find the amount of \$356.25 in 4 yrs., at 5% compound interest.

36. Find the amount of \$637.50 in 2 yrs. 6 mos., at 4% compound interest.

37. Find the compound interest of \$800 in 3 yrs. 9 mos., at 6%.

38. Find the compound interest of \$39.35 in 1 yr. 9 mos., at 5%.

If the interest be payable semi-annually, quarterly, etc., the half, quarter, etc., of the rate per cent must be used, and the amount obtained for each half-year, quarter-year, etc.

39. Find the compound interest of \$300 in 2 yrs., at 4%, interest being payable semi-annually.

40. Find the compound interest of \$525 in 1 yr. 6 mos., at 5%, interest being payable quarterly.

41. Find the compound interest of \$10,000 in 6 mos., at 6%, interest being paid monthly.

Ex. What principal will in 3 yrs. produce \$780.40 compound interest, at 4%?

The amount of \$1 for 1 yr. at 4% is $\$1 \times 1.04$;
for 2 yrs. is $\$1 \times 1.04 \times 1.04 = \$1 \times 1.04^2$;
for 3 yrs. is $\$1 \times 1.04^2 \times 1.04 = \$1 \times 1.04^3$.
Hence, the amount of \$1 for 3 yrs. at 4% is \$1.124864.
The interest is $\$1.124864 - \$1 = \$.124864$.
$\$780.40 \div .124864 = \$6250$.

42. What principal will amount to \$137.81 in 2 yrs., at 5% compound interest?

43. What principal will amount to \$1860.96 in 3 yrs., at 6% compound interest?

44. What principal will amount to \$1500 in 1 yr., at 4% compound interest, payable quarterly?

45. What principal will produce \$100 in 1 yr. 6 mos., at 6% compound interest, payable semi-annually?

Annual Interest.

369. Annual interest is simple interest on the principal *and on each year's* interest from the time each interest is due until settlement.

Ex. Find the interest due July 4, 1881, on a note dated May 4, 1877, for $850, with interest payable annually, at 7%.

yrs.	mos.	dys.			
				$850	
1881	7	4		.25	
1877	5	4		6)212.50	
4	2		.25	35.42	
				$247.92	interest for 4 yrs. 2 mos.

yrs.	mos.		
		$850	
		.07	
3	2	59.50	annual interest.
2	2	.07	
1	2	4.165	
	2	$6\frac{2}{3}$	
6	8 = $6\frac{2}{3}$ yrs.	27.77	interest on annual interest.
		247.92	
		$275.69	total interest due.

The simple interest on the principal is $247.92.

The first year's interest, $59.50, remains overdue 3 yrs. 2 mos.; the second year's, 2 yrs. 2 mos.; the third year's, 1 yr. 2 mos.; the fourth year's, 2 mos. The interest on $59.50 for the sum of these periods, 6 yrs. 8 mos., is $27.77. Hence, the total interest due is $247.92 + $27.77 = $275.69.

46. Find the interest due May 19, 1881, on a note dated Dec. 26, 1877, for $1224.60, with interest payable annually, at 5%, when no interest has been paid.

47. Find the amount due May 27, 1881, on a note dated Jan. 4, 1879, for $215.50, with interest payable annually at $5\frac{1}{2}$%, when no interest has been paid.

48. Find the amount due Jan. 16, 1881, on a note dated Jan. 8, 1879, for $3115.20, with interest payable annually at 5%, when no interest has been paid.

49. Find the amount due Jan. 18, 1881, on a note dated Jan. 8, 1877, for $2875, at 6%: (1) simple interest; (2) annual interest; (3) compound interest.

NOTE. When partial payments have been made on a note, the annual interest, by the law of Vermont, is found to the end of the *first year in which any payments have been made*, and the sum of the payments, with the interest reckoned on each to the end of that year, is applied to cancel any interest that may have accrued on the yearly interest, then to cancel the yearly interest, and then to the payment of the principal.

The law of New Hampshire is the same as that of Vermont; but if at the time of any payment no interest is due, except what is accruing for the year, and the payment is less than the interest due at the end of the year, then no interest is allowed on the payment.

The Connecticut Rule is similar to the United States Rule; but when *less than a year's* interest has accrued at the time of a payment, *except it be the last payment*, the difference between the amount of the principal for *an entire year* and the amount of the payments made that year is taken as the new principal. But if the interest which has accrued at the time of a payment exceeds the payment, no interest is computed on the payment.

These are special rules for the guidance of the State Courts or their agents, whenever it becomes necessary for them to determine the amount due on a note. But it is the general practice among business men to settle notes "*with interest*" that run for a year or less, by the Merchants' Rule; that run for more than a year, by the United States Rule; and to settle notes "*with interest annually*," or "*semi-annually*," if the interest be not paid when due, by compounding the interest with the principal.

The Legislature of each State has fixed the *legal rate of interest* for that State. Practically, however, the rate of interest is determined by the law of supply and demand. When the supply is greater than the demand, the rate of interest is low; when the supply is less than the demand, the rate of interest is high.

CHAPTER XVIII.

STOCKS.

370. The name **stock** is applied to the capital of banks, railroads, and other incorporated companies.

The capital of a company is usually divided into **shares**, of which the *original value* is \$100, or some other fixed sum; but the *market value* at any time is estimated by the current price per share.

371. When the market value of stock is equal to its original value, it is said to be **at par.** In quotations of stocks par is generally represented by 100; and when stock is quoted at above 100, it is said to be at a premium; below 100, at a discount. The premium or discount is the difference between the quotation and 100.

Thus, when the price of a stock on a given day is 91, or, as it is commonly expressed, when the stock is *at* 91, the meaning is, that \$100 stock costs on that day \$91 money. Or that if 100 be the representative of any quantity of stock, 91 will represent the corresponding value in money. In this case the stock is said to be 9% discount.

The buying and selling of stocks is conducted through the agency of stock-brokers, who receive a brokerage on the stock. The brokerage is generally reckoned at $\frac{1}{8}$ of 1% on the *par value* of the stock. Thus, if a broker sells stock for a person at 91, that person receives $90\frac{7}{8}$; and if he buys stock for a person at 91, that person pays $91\frac{1}{8}$.

(1) How much would be received for 52 shares of stock, \$100 each, at $89\frac{1}{2}$?

$\frac{1}{8}$ will represent the brokerage.

$89\frac{1}{2} - \frac{1}{8} = 89\frac{3}{8}$, price to the seller.

Hence, 1 share will bring $\$89\frac{3}{8}$; and 52 shares, $52 \times \$89\frac{3}{8} = \$4647.50$. *Ans*

(2) What amount of stock at $84\frac{1}{2}$ may be bought for $9393.375?

If $\frac{1}{8}$ be the brokerage,
$84\frac{1}{2} + \frac{1}{8} = 84\frac{5}{8}$, price to the buyer.
Hence, $.84\frac{5}{8}$ buys $1 stock, and $9393.375 will buy
$\$9393.375 \div .84\frac{5}{8} = \$11,100$. *Ans.*

(3) What is the quoted price of stock when $42,464.25 is paid for $46,600 stock?

If $46,600 stock cost $42,464.25,
then $1 stock costs $\$42,464.25 \div 46,600 \doteq \$.91\frac{1}{8}$.
Hence, the stock is quoted at $91\frac{1}{8} - \frac{1}{8}$ brokerage $= 91$. *Ans.*

Exercise LXXIII.

In the following examples the prices of stocks include brokerage unless otherwise stated:

1. Find the cost of $4000 stock, at $109\frac{7}{8}$.
2. Find the cost of $2500 stock, at 98.
3. Find the cost of $3900 stock, at $78\frac{1}{8}$.
4. Find the cost of $4700 stock, at $100\frac{1}{2}$.
5. Find the cost of $1250 stock, at $87\frac{5}{8}$, brokerage $\frac{1}{8}$.
6. How much bank stock, at $75\frac{1}{4}$, may be bought for $8729?
7. How much railroad stock, at $91\frac{1}{8}$, may be bought for $\$4237\frac{5}{16}$?
8. How much railroad stock may be bought for $6305, at $121\frac{1}{4}$?
9. How much railroad stock may be bought for $5137.50, at $102\frac{3}{4}$?
10. How many $100 railroad shares, at $68\frac{3}{4}$, may be bought for $1650?
11. What must be the price of stock, in order that $9200 stock may be bought for $8970?

12. If $1500 stock be bought for $1374.375, what is the price of the stock?

(1) What income will be derived from $18,700 6% stock?

.06 of $18,700 = $1122. *Ans.*

(2) How much 5% stock must be bought to give an income of $360?

Since $.05 is derived from $1 stock, $360 will be derived from $360 ÷ .05 = $7200. *Ans.*

13. What income will be derived from $29,700 4% stock?

14. Find the income from $4500 6% stock.

15. How much will a person receive from $9400 railroad stock if a dividend of 3½% be declared?

16. How much 8% stock must be bought to give an income of $2400?

17. A person receives $343 as his semi-annual dividend from a 7% stock. How much stock does he hold?

18. Find the total income of a person whose property consists of $3000 6% stocks and $8200 7% stocks.

19. Find the rate of dividend paid by some stock, when a holder of $24,600 receives $924.50.

20. Find the rate per cent at which $11,100 will yield a semi-annual return of $499.50.

Ex. If $10,250 be invested in 6% stock, at 102½, what income will be obtained?

$102½ is the cost of $100 stock.
Hence, $10,250 = cost of $10,250 ÷ 1.025 = $10,000 stock.
And .06 of $10,000 = $600. *Ans.*

21. If $19,500 be invested in 4% stock, at 91, what income will be received?

22. Find the income on $7000 when invested in 4% stocks, at 103½.

23. What income will be derived from \$6800 if it be invested in 7% stocks, at 130?

24. A person invests \$7650 in railroad stock, at 63¾. What will he receive if a dividend of 3¼% be declared?

Ex. If a person buys 6% stock at 120, what rate of interest does he receive on his money invested?

\$100 stock costs \$120. \$100 stock pays \$6.
Hence, the \$120 invested pays \$6.
Therefore, the rate of interest is $6 \div 120 = .05$ or 5%.

25. If 3% stocks are at 88⅛, what rate per cent interest will a purchaser receive on his money?

26. If an 8% stock is at 150, what rate per cent interest will a purchaser receive on his money?

27. If a 10% stock is at 175, what rate per cent interest will an investor receive on his money?

28. If a 4½% stock is at 85, what rate per cent interest will a purchaser receive on his money?

29. If a 7% stock is at 114, what rate per cent interest will a purchaser receive on his money?

30. If a 6% stock is at 130, what rate per cent interest will a purchaser receive on his money?

31. If an 8% stock is at 140, what rate per cent interest will a purchaser receive on his money?

32. How much money must be invested in 4% stock, at 92, to receive \$245 income?

33. Find the sum required for an investment in a 4% stock, at 98½, to produce an income of \$200 a year.

34. A person bought some bank stock at 107, and received \$384.25 when a dividend of 7¼% was paid. How much had he invested?

35. What must be the price of a 5% stock, in order that a buyer may receive 6% on his money?

36. What must be the price of a 7% stock, in order that a buyer may receive 6% interest on his money?

37. What may be paid for an 8% stock, in order that a buyer may receive 6% interest on his money? for a 9% stock? for a 10% stock?

38. A person invested $2855 in a bank, when the stock was at 142¾. What is the rate per cent of the dividend when he receives $150?

39. How much will be received for some 3% stock, from which an income of $250 has been derived, if sold at 87½?

40. If a 5% stock pays $340 income, and is sold out for $7990, at what price is it sold?

41. On what per cent stock must an investment have been made from which $185.50 was derived yearly, and which, when sold out at 97, brought $5141?

42. A person receives 4½% interest on his money by investing it in some 6% stock. At what price did he buy it?

Ex. What alteration will be made in an income by selling $10,000 4% stocks at 89¼, including brokerage, and buying 5% stock at 105, including brokerage?

It is necessary to determine three things:

(1) The income derived frcm the 4% stock.

(2) The amount received for the 4% stock when sold at 89¼.

(3) The income which this amount will produce when invested in the 5% stock, at 105.

(1) .04 of $10,000 = $400, income from 4% stock.

(2) .89¼ of $10,000 = $8925, amount from the 4% stock.

(3) $1.05 is paid for $1 worth of 5% stock. Hence, $8925 is paid for $8925 ÷ 1.05 = $8500 stock.

.05 of $8500 = $425, income from 5% stock.

$425 − $400 = $25, increase in income.

43. If $4800 3% stocks be sold at 88, and the proceeds be invested in 5% stocks at 105⅗, what additional income will be obtained?

44. If $7800 $3\frac{1}{2}\%$ stocks be sold at 60, and the proceeds be invested in 5% stocks at 90, find the alteration in income.

45. If $10,000 3% stocks be sold at 88, and the proceeds be invested in $3\frac{1}{2}\%$ stocks at par, find the alteration in income.

46. If $10,000 8% stocks be sold at 150, and the proceeds be invested in 6% stocks at par, find the alteration in income.

47. If $8000 10% stocks be sold at 170, and the proceeds be invested in 5% stocks at 68, find the alteration in income.

48. If $7000 8% stocks be sold at 150, and the proceeds be invested in 6% stocks at 105, find the alteration in income.

49. If $1000 8% stocks be sold at 170, and the proceeds be invested in 5% stocks at par, find the alteration in income.

50. If $8000 5% stocks be sold at 90, and the proceeds be invested in $3\frac{1}{2}\%$ stocks at 60, find the alteration in income.

51. If $10,000 $3\frac{1}{2}\%$ stocks be sold at 65, and the proceeds be invested in 8% stocks at 130, find the alteration in income.

52. If $8000 $4\frac{1}{2}\%$ stocks be sold at 70, and the proceeds be invested in 10% stocks at 160, find the alteration in income.

53. If $6000 6% stocks be sold at 90, and the proceeds be invested in 10% stocks at 135, find the alteration in income.

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54. Find the rate of interest obtained by investing in a stock, at 124, paying $6\frac{1}{4}$ per cent per annum.

55. What is the price of stock if $7000 stock can be bought for $5880?

56. Find the price of mining shares issued at $15 a share and sold at 2¼% discount?

57. How much 3½% stock must be sold at 81⅞, in order to buy $5000 4% stock at 94½; brokerage, ⅛ in each transaction.

58. How much stock must be sold at 96⅞ to raise a sufficient sum for discounting a note for $1000, due 49 days hence, and discounted at 5½%?

59. A broker bought $5000 stock at 88½. At what price must he sell it to gain $100?

60. If a broker buy stock at 85, at what price must he sell it to make 12¼% profit; brokerage, ⅛ on each transaction.

61. Which is the more profitable stock for investment, a 4% at 85 or a 3% at 63? a 3½% stock at 67¼ or a 4% stock at 81½?

62. Find the price of a 4½% stock to equal a 8½% stock at 88½?

63. Find the price of a 5% stock to equal a 3% stock at 89½.

64. Find the price of a 3½% stock to equal a 6% stock at par.

65. Find the profit or loss in buying $80,000 stock, at 91⅝, and selling at 90; brokerage, ⅛ on each transaction.

66. Which is the better investment, a 5% stock, at 137¼, or a 3½% stock at 91½? What rate of interest would be received from each investment?

67. A person invests $7370 in the purchase of a stock at 92. What loss will he sustain if he sell at 90, brokerage being ⅛ in each transaction?

68. How much stock must be sold at 90⅝ so that when the proceeds are invested in a mortgage, at 6%, $543.75 a year may be received?

69. A person invests 2/7 of his money at 6%, ⅖ at 4½%, and the rest at 3½%. What per cent will he receive on the whole amount?

CHAPTER XIX.

EXCHANGE.

372. A **draft** or **bill of exchange** is a written order directing one person to pay a specified sum of money to another.

Thus, if B. Smith of Portland owes S. Brown of Boston $500; when the debt becomes due, Smith procures a draft from a bank in Portland on a bank in Boston, and sends it by mail to Brown of Boston. The bank in Portland makes the draft payable to the order of B Smith (unless otherwise directed), and Smith writes on the back of the draft, "Pay to the order of S. Brown," and signs it. On receipt of the draft, Brown takes it to his bank in Boston, writes his name across the back of the draft, and receives his money.

373. A **time draft** is a draft payable at a specified time after sight (or date).

When the person on whom a time draft is drawn accepts a draft, he writes the word "Accepted," with the date, across the *face*, and signs his name. The draft is then called an **acceptance**, and the acceptor is responsible for its payment.

An acceptance is of the nature of a promissory note, and when discounted the discount is calculated for the specified time and *three days of grace.*

374. The system of paying debts due to persons living at a distance, by transmitting drafts instead of money, is called **exchange.**

The rate per cent which the cost of a draft is more or less than its face is called the **rate of exchange.**

The comparative value of the coins of different countries is called the **par of exchange.**

The par of exchange as modified by the rate of exchange is called the **course of exchange.**

(1) Find the cost of the following draft, exchange being ½ of 1% premium, and interest 6%.

$800. CINCINNATI, O., Feb. 8, 1881.

Thirty days after sight pay to the order of S. Clark eight hundred dollars, and place to the account of

To H. WRIGHT, Philadelphia. P. CLEMENT.

	$800.00	
.0055 of $800 =	4.40	discount at 6% for 33 days.
	$795.60	proceeds of draft (cost at par).
.005 of $800 =	4.00	premium at ½ of 1%.
	$799.60	cost of draft.

(2) Find the cost of a draft for $400, payable 60 days after sight, exchange being ¼ of 1% discount, and interest 7%.

	$400.00	
.01225 of $400 =	4.90	discount at 7% for 63 days.
	$395.10	proceeds of draft (cost at par).
.0025 of $400 =	1 00	discount at ¼ of 1%.
	$394.10	cost of draft.

(3) Find the face of a draft, payable 30 days after sight, that can be bought for $1000, exchange being ¼ of 1% premium, and interest 6%.

$.0055 =	Discount on $1 for 33 days, at 6%.	
$1 − $.0055 =	$.9945	proceeds of $1.
	$.0025	premium on $1.
	$.997	cost of $1.

Therefore. $1000 ÷ .997 = $1003.01 face of draft.

EXERCISE LXXIV.

1. Find the cost of a draft on New York for $1100, at ¼ of 1% premium.
2. Find the cost of a draft on New Orleans for $1350, at ¼ of 1% discount.

3. Find the cost of a draft for $1600, payable 30 days after sight, when exchange is $\frac{1}{4}$ of 1% premium, and interest 6%.

4. Find the cost of a draft for $500, payable 60 days after sight, when exchange is $\frac{1}{2}$ of 1% discount, and interest 7%.

5. Find the cost of a draft for $1200, payable in 90 days after sight, when exchange is $\frac{1}{2}$ of 1% premium, and interest 7%.

6. Find the cost of a draft for $950, payable in 30 days, when exchange is at par and interest $4\frac{1}{2}\%$.

7. Find the cost of a draft for $725, payable in 60 days, when exchange is at $\frac{1}{4}$ of 1% discount, and interest 5%.

8. Find the cost of a draft for $810, payable in 90 days, when exchange is at $\frac{1}{4}$ of 1% premium, and interest $5\frac{1}{2}\%$.

9. Find the face of a draft, payable 30 days after sight, that can be bought for $274, when exchange is at par, and interest 6%.

10. Find the face of a draft, payable 60 days after sight, that can be bought for $1250, when exchange is at $\frac{1}{4}$ of 1% premium, and interest 7%.

11. Find the face of a draft, payable 60 days after date, that can be bought for $1125, when exchange is at $\frac{1}{4}$ of 1% discount, and interest $5\frac{1}{2}\%$.

12. Find the face of a draft, payable 30 days after date, that can be bought for $520, when exchange is at $\frac{1}{2}$ of 1% premium, and interest 4%.

13. Find the face of a draft, payable 90 days after date, that can be bought for $10,000, when exchange is at par, and interest $4\frac{1}{2}\%$.

Foreign Exchange.

(1) Find the cost of a draft on London for £502 12*s.*, in New York, when sterling exchange is quoted at 4.85.

£502 12*s.* = £502.6.
$4.85 × 502.6 = $2437.61. *Ans.*

(2) Find the face of a draft on London that can be bought for $2425.77, when sterling exchange is quoted at 4.85.

$2425.77 ÷ $4.85 = 500.2.
£500.2 = £500 4*s.* *Ans.*

Exercise LXXV.

1. Find the cost of a draft on London for £320 10*s.* 6*d.*, when sterling exchange is quoted at 4.83.
2. Find the cost of a thirty-day draft on London for £150, when thirty-day bills are quoted at 4.82, and the broker's commission is $\frac{1}{8}$ of 1% of cost of draft.
3. Find the cost of the following draft, when sixty-day bills are quoted at 4.81, and the broker's commission is $\frac{1}{8}$ of 1% of cost of draft:

£500. New York, Feb. 17, 1881.

Sixty days after sight of this *First of Exchange* (Second and Third of the same tenor and date unpaid), pay to the order of James Wilson five hundred pounds, value received, and charge to account of Simon Morton & Co.

To James Sage & Co.,
London.

Note. Foreign Bills are generally drawn in sets of three, of the same tenor and date, called *First*, *Second*, and *Third* of Exchange. These are sent by different mails to avoid loss or delay. When one is accepted or paid the others are void.

4. Find the face of a draft on Glasgow that can be bought for $2000, when sterling exchange is quoted at 4.84.
5. Find the face of a draft on Dublin that can be bought for $135.24, when sterling exchange is quoted at 4.83.
6. How large a draft on London can be bought for $4000, when exchange is quoted at 4.86$\frac{3}{8}$.

Exchange on Paris is quoted at so many francs for $1.

Ex. What will be the cost of a draft on Paris for 7740 fr., when Paris exchange is quoted at 5.16?

$$7740 \div 5.16 = 1500.$$

Hence, draft of 7740 fr. = $1500.

7. Find the cost of a draft on Paris for 8000 fr., when Paris exchange is quoted at 5.12$\frac{3}{8}$, and brokerage is $\frac{1}{8}$ of 1%.
8. Find the cost of a draft on Paris for 10,000 fr., when Paris exchange is quoted at 5.14.
9. How large a sixty-day draft on Paris can be bought for $1500, when sixty-day bills are quoted at 5.11$\frac{1}{2}$?
10. How large a sight draft on Paris can be bought for $2840, when Paris exchange is quoted at 5.13$\frac{1}{2}$?

Exchange on Germany is quoted at so many cents for four reichmarks.

Ex. What will be the cost of a draft on Berlin for 4460 marks, when German exchange is quoted at .95?

$$\frac{4460}{4} \text{ of } \$.95 = \$1059.25.$$

11. Find the cost of a draft on Hamburg for 2876 marks, when German exchange is quoted at .95$\frac{1}{2}$.
12. Find the cost of a draft on Munich for 12,000 marks, when German exchange is quoted at .94$\frac{5}{8}$.
13. How large a draft on Frankfort can be bought for $1200, when German Exchange is quoted at .95$\frac{1}{2}$?

CHAPTER XX.

AVERAGES.

Ex. If a dozen eggs weigh 666^g, what is their average weight?

Since the 12 eggs weigh 666^g, the average weight of an egg will be $\frac{1}{12}$ of $666^g = 55.5^g$.

The average weight is sometimes called the mean weight.

EXERCISE LXXVI.

1. If a dozen eggs weigh 692^g, what is the mean weight of an egg?
2. Seven boys weigh respectively 119.7 lbs., 105 lbs., 178.3 lbs., 165.3 lbs., 142.8 lbs., 109 lbs., 154.2 lbs. What is their average weight?
3. A merchant mixes 1 lb. of coffee worth 27 cents, 1 lb. worth 35 cents, and 1 lb. worth 40 cents. What are the three pounds together worth? How much a pound is the mixture worth?
4. A merchant mixes 2 lbs. of coffee worth 27 cents a pound, 3 lbs. worth 35 cents a pound, and 1 lb. worth 40 cents. What is a pound of the mixture worth?
5. What is the value per pound of a mixture of coffee containing 7 lbs. worth 26 cents a pound, 4 lbs. worth 31 cents a pound, and 10 lbs. worth 34 cents a pound?
6. What is the cost of a gallon of the mixture in which 7 gals. cost 67 cents a gallon, 5 gals. cost 48 cents a gallon, and water, without cost, was added until there were 15 gals. of the mixture?

7. If 7^l of water be poured into a vessel containing 3^l of sulphuric acid, specific gravity 1.840, and the mixture shrink to 9.972^l, what is the specific gravity of the mixture?

8. If 4^l of water and 1^l of sulphuric acid, specific gravity 1.842, when mixed shrink $\frac{1}{4}$ of 1%, what is the specific gravity of the mixture?

Ex. In what proportion must alcohol of specific gravity .800 be mixed with water to bring the specific gravity to .925, if no allowance be made for condensation?

The specific gravity of the alcohol lacks .125 of the required specific gravity; and specific gravity of the water is 075 above the required specific gravity.

Therefore, the alcohol is to the water in the inverse ratio of 125 to 75. That is,

$$\text{Alcohol} : \text{water} = 75 : 125 = 3 : 5.$$

To test this answer: 3^l of alcohol weigh 2.4^{kg}, 5^l of water weigh 5^{kg}; therefore, 8^l of the mixture weigh 7.4^{kg}, and the specific gravity of the mixture is $\frac{7.4}{8} = .925$.

9. In what proportions must tin of specific gravity 7.29 and lead of specific gravity 11.35 be mixed to make a solder of specific gravity 10.21, if no allowance be made for expansion or condensation? (Give the proportions in bulk.)

10. In what proportions must oils worth \$1.25 and 80 cents a gallon be mixed to make a mixture worth \$1 a gallon? (Test the answer.)

11. In what proportion may oils worth \$1.20, 80 cents, and 60 cents a gallon be mixed so that the mixture shall be worth 70 cents a gallon?

The price of the mixture may be brought to 70 cents by adding to the 60-cent oil either of the better oils, or both mixed in any proportion.

When the 80-cent oil alone is taken, in what ratio to the 60-cent must it be used? When the \$1.20 oil alone is taken, in what ratio to the 60-cent oil must it be used? When the \$1.20 and 80-cent oils are mixed gallon for gallon, how much 60-cent oil must be added? When 1 gal. of the \$1.20 oil and 3 of the 80-cent oil are taken, how much 60-cent oil must be added? If three-fourths of the mixture consist of the 60-cent oil, what per cent of each of the other two must be taken?

12. A solder composed of tin and lead, specific gravities 7.29 and 11.35, has a specific gravity of 10.44. What is the weight of each metal in a kilogram of solder?

Average of Payments.

Ex. John Smith has given to William Jones notes as follows: \$150, due May 15; \$200, due June 30; \$500, due July 21. He wishes to pay them all at one time. When shall they be considered due?

If all the notes were paid May 15, Smith would lose the use of \$200 for 46 days, and of \$500 for 67 days.

The use of \$200 for 46 days is equal to the use of $\$200 \times 46$ for 1 day; and the use of \$500 for 67 days is equal to the use of $\$500 \times 67$ for 1 day. Smith would, therefore, lose what is equivalent to the use of $\$9200 + \$33,500 = \$42,700$ for 1 day, and is entitled to keep the $\$150 + \$200 + \$500 = \$850$ as many days after May 15 as are required for the use of \$850 to equal the use of \$42,700 for 1 day, or $\frac{42700}{850}$ days $= 50.2$ days. Hence, the *equated time* for paying all the notes is 50 days after May 15; that is, July 4.

The work may be arranged as follows:

$$\begin{array}{lr}
\$150 \times 00 = & \\
\$200 \times 46 = & \$9,200 \\
\underline{\$500} \times 67 = & \underline{\$33,500} \\
\$850 &)\underline{\$42,700} \\
 & 50.2
\end{array}$$

13. Find the equated time for the payment of $250 due in 3 mos., $400 due in 6 mos., $700 due in 8 mos.

14. Find the equated time for the payment of $300 due in 30 dys., $500 due in 60 dys., and $200 due in 90 dys.

15. Find the equated time for the payment of $325 due now, $200 due in 30 dys., $460 due in 60 dys., and $150 due in 90 dys.

16. Find the equated time for the payment of $240 due May 10, $420 due July 2, $310 due Sept. 14, and $600 due Oct. 1.

17. Find the equated time for the payment of $275 due June 21, $175 due July 16, $200 due Aug. 6, and $150 due Sept. 3.

18. Find the equated time for the payment of $112.30 due July 6, $115.25 due July 30, $232.15 due Sept. 4, and $102.36 due Oct. 1.

Ex. A owes B $200 due in 10 mos. If he pays $120 in 4 mos. when should he pay the balance?

By paying $120 in 4 months A loses the use of $120 for 6 months, which is equal to the use of $720 for 1 month. Therefore, he is entitled to keep the balance ($80) $\frac{720}{80}$ months = 9 months after its maturity.

19. A owed B $2000 payable in 4 mos., but at the end of 1 mo. he paid him $500, at the end of 2 mos. $500, and at the end of 3 mos. $500. In how many months is the balance due?

20. A merchant bought, Feb. 11, 1881, a bill of goods amounting to $1700, on 4 months' credit; but he paid Mar. 22, $400, Apr. 20, $220, May 10, $300. When is the balance due?

Find the equated time of maturity of each of the following bills, and the amount due at settlement, including interest at 6%:

21. JAMES PRICE, *to* JOHN BATES, *Dr.*

1881.		
Apr. 5.	To mdse. on 4 mos. credit . . .	$120.50
Apr. 15.	" " 3 " " . . .	87.33
May 7.	" " 3 " " . . .	218.17
May 21.	" " 4 " " . . .	317.00
		$743.00

Paid Oct. 18, 1881.

Find the equated time for payment, reckoning from July 15, which is the earliest date that any item becomes due, and find the interest on the whole bill from the equated time to Oct. 18. As this is not banking business, allow no days of grace.

22. HALL & Co. *bought of* BOLES & Co.

1881.		
Feb. 11.	To mdse. on 30 dys.	$250.00
Apr. 20.	" " 2 mos.	500.00
May 31.	" " 3 mos.	150.00
July 6.	" " 60 dys.	1000.00

Paid Nov. 10, 1881.

23. Find the equated time of maturity of each side of the following account:

ADAMS & Co. *in account with* BACON & Co.

DR.					CR.
1881.					
Jan. 3.	To mdse. 90 dys.	$250	Apr. 11.	By cash,	$200
Mar. 7.	" 60 "	150	Apr. 30.	"	100
May 3.	" 60 "	325	May 30.	"	125
June 7.	" 30 "	175	July 2.	"	400

NOTE. On the *Dr.* side is the statement of the goods sold by Bacon & Co. to Adams & Co.; and on the *Cr.* side is the statement of the cash paid by Adams & Co. to Bacon & Co.

Ex. Find the time for payment of the balance of an account, if the debit and credit sides, when equated, stand as follows:

Dr.	Cr.
Due May 29, 1881, \$900	Due May 30, 1881, \$825.

Difference in equated time = 1 day.
Balance of account = \$900 − \$825 = \$75.

If the account were settled at the *later* date, May 30, the \$900 on the *Dr.* side would have been on interest 1 day, and this is equivalent to having the balance, \$75, on interest $\frac{900}{75}$ of 1 day = 12 days. Hence, the balance should begin to draw interest 12 days *before* May 30; that is, May 18.

Ex. Find the time for payment of the balance of an account if the debit and credit sides, when equated, stand as follows:

Dr.	Cr.
Due Feb. 18, 1881, \$950	Due Jan. 20, 1881, \$850

Difference in equated time, 29 days.
Balance of account, \$950 − \$850 = \$100.

If the account were settled at the *later* date, Feb. 18, the \$850 would have been on interest 29 days, which is equivalent to having the balance, \$100, on interest $\frac{850}{100}$ of 29 days = $246\frac{1}{2}$ days. Hence, to increase the *Dr.* side by *an equal amount of interest*, the balance should remain unpaid 247 days; that is, the balance is due Oct. 23, 1881.

375. From the two preceding examples is derived the following

RULE FOR EQUATING ACCOUNTS.

Find the equated time for each side of the account.

Multiply the side of the account that falls due **first** *by the number of days between the dates of the equated times of the two sides, and divide the product by the balance of the account.*

The quotient will give the number of days to the maturity of the balance, to be counted **forward** *from the later date when the* **smaller** *side falls due first, and* **backward** *when the* **larger** *side falls due first.*

Find the time for paying the balance in the following equated bills:

	Average due.	Dr.	*Average due.*	Cr.
24.	May 17, 1881 . .	$950	Apr. 12, 1881	$1000
25.	Apr. 12, 1881 . .	$950	May 17, 1881	$1000
26.	May 30, 1881 . .	$1000	June 23, 1881	$920
27.	July 6, 1881. . .	$500	Apr. 14, 1881	$480

Note. In finding the equated time of accounts it is customary to neglect cents if less than 50, and if 50 or more to consider them as $1. A fraction of a day in the result is rejected if less than $\frac{1}{2}$; if $\frac{1}{2}$ or more it is called a day.

376. When an account is settled by cash at any other date than that at which the balance becomes due, the interest is found on the balance for the interval between the day of settlement and the day the balance is due, and is added to, or deducted from, the balance, according as the settlement is made *after* or *before* the balance is due.

377. Another method is by computing the interest on each item, from its date to the day of settlement. (The time is reckoned in days.)

1881.		Dr.	Int.	1881.		Cr.	Int.
Apr. 8.	To cash,	$250	$7.71	Apr. 4.	By mdse.	$300	$9.45
May 31.	"	380	8.36	May 19.	"	350	8.40
July 20.	"	420	5.74	June 9.	"	610	12.50
Oct. 10.	To bal. acct.	210					
" 10.	" int.		8.54	Settled Oct. 10, 1881.			
		$1260	$30.35			$1260	$30.35

Hence, cash balance = $210 + $8.54 = $218.54. *Ans.*

When the balance of account and the balance of interest fall on *opposite* sides, the cash balance is their *difference.*

Find (by either method) the cash balance in the following bills, reckoning interest at 6%:

28.

1881.		Dr.	1881.		Cr.
Apr. 5.	To mdse.,	$250.00	Apr. 20.	By cash,	$200.00
" 27.	"	610.00	" 30.	"	500.00
June 1.	"	200.00	June 4.	"	400.00

Settled June 20, 1881.

29.

1881.		Dr.	1881.		Cr.
Jan. 15.	To mdse. 3 mos.	$250.00	Apr. 26.	By cash,	$150.00
Feb. 25.	" 3 "	98.50	May 17.	"	150.00
Mar. 7.	" 3 "	300.00	July 7.	"	200.00

Settled Oct. 10, 1881.

NOTE. Since the *Dr.* items have the *same term of credit*, find the equated time of these items, and count forward from that date 3 mos. for the term of credit.

30.

1881.		Dr.	1881.		Cr.
Jan. 3.	To mdse. 30 dys.	$100.00	Feb. 25.	By cash,	$100.00
Mar. 8.	" "	200.00	Mar. 22.	"	150.00
May 10.	" "	150.00	June 20.	"	200.00
June 2.	"	95.00		Settled Aug. 2, 1881.	

31.

1881.		Dr.	1881.		Cr.
Apr. 5.	To mdse.,	$250.00	Apr. 20.	By cash,	$200.00
" 27.	"	670.00	" 30.	"	500.00
June 4.	"	200.00	June 1.	"	400.00

Settled Sept. 1, 1881.

32.

1881.		Dr.	1881.		Cr.
Mar. 10.	To mdse.,	$580.00	Mar. 15.	By cash,	$500.00
Apr. 20.	"	200.00	Apr. 15.	"	300.00
May 5.	"	150.00	" 25.	"	120.00
" 17.	"	325.00	May 20.	"	225.00

Settled June 4, 1881.

CHAPTER XXI.

POWERS AND ROOTS.

378. The *square* of a number is the product of *two* factors, each equal to this number.

Thus, the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

379. The **square root** of a number is one of the *two equal factors* of the number.

Thus, the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

380. The square root of a number is indicated by the *radical sign* $\sqrt{\ }$, or by the fractional exponent $\frac{1}{2}$ written above and to the right of the number.

381. Since $(2^4)^2 = (16)^2 = 256 = 2^8$; and $(2^4)^{\frac{1}{2}} = (16)^{\frac{1}{2}} = 4 = 2^2$, it is evident that,

A power of a power of a number, or a root of a power of a number, is that number with an exponent equal to the product of the given exponents.

382. Since $35 = 30 + 5$, the square of 35 may be obtained as follows:

$$\begin{array}{l} 30 + 5 \\ \underline{30 + 5} \\ 30^2 + (30 \times 5) \\ \underline{\qquad\quad (30 \times 5) + 5^2} \\ 30^2 + 2(30 \times 5) + 5^2 \end{array} \qquad\qquad \begin{array}{r} 30^2 = 900 \\ 2(30 \times 5) = 300 \\ 5^2 = \underline{\ \ 25} \\ = 1225 \end{array}$$

383. Hence, since every number consisting of two or more figures may be regarded as composed of tens and units,

The square of a number will contain the square of the tens + twice the tens × the units + the square of the units.

Square Root.

384. The first step in extracting the square root of a number is to mark off the figures in periods.

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any number expressed by *one* or *two* figures is a number of *one* figure; of any number expressed by *three* or *four* figures is a number of *two* figures; and so on.

If, therefore, an integral number be divided into periods of two figures each, from the right to the left, the number of figures in the root will be equal to the number of periods. The last period at the left may consist of only one figure.

Ex. Find the square root of 1225.

```
   12'25(35
    9
65)3 25
   3 25
```

Since 1225 consists of two periods, the square root will consist of two figures.

The first period, 12, contains the square of the tens' number of the root.

The greatest square in 12 is 9, and the square root of 9 is 3. Hence, 3 is the tens' figure of the root.

The square of the tens is subtracted, and the remainder, 325, is twice the tens × the units + the square of the units.

Twice the 3 tens is 6 tens, and 6 tens is contained in the 32 tens of the remainder 5 times. Hence, 5 is the units' figure of the root.

Since twice the tens × the units + the square of the units is equal to (twice the tens + the units) × the units, the 5 units are annexed to the 6 tens, and the result, 65, is multiplied by 5.

385. The same method will apply to numbers of more than two periods, by considering the part of the root already found as so many tens with respect to the next figure of the root.

Ex. Extract the square root of 7890481.

```
      7'89'04'81(2809
      4
   48)3 89
      3 84
 5609)5 04 81
      5 04 81
```

When the third period, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 56 is not contained in 50. The 0 is then placed both in the root and the divisor, and the next two figures, 81, are brought down.

386. If the square root of a number have decimal places, the number itself will have twice as many.

Thus, if .11 be the square root of some number, the number will be $(.11)^2 = .11 \times .11 = .0121$.

Therefore, the number of *decimals* in every square number will be *even;* and the number of decimal places in the root will be *half* as many as in the given number itself.

Hence, if a given square number contain a decimal, and if it be divided into periods of two figures each, by beginning at the decimal point and marking toward the left for the integral number, and toward the right for the decimal, the number of periods to the *left* of the decimal point will show the number of *integral* places in the root, and the number of periods to the *right* will show the number of *decimal* places in the root.

Ex. Extract the square root of 52.2729.

```
      52.27'29(7.23
      49
  142)3 27
      2 84
 1443)43 29
      43 29
```

It will be seen from the periods that the root will have one integral and two decimal places.

387. If a number contain an *odd* number of decimal places, or if any number give a *remainder*, when as many figures in the root have been obtained as the given number has periods, then its exact square root cannot be found. We may, however, approximate to the exact root as near as

we please, by annexing ciphers and continuing the operation. The result will be a constantly varying succession of figures.

Ex. Extract to six places of decimals the square root of 19.

```
     19.00'00'00(4.358899
     16
    83)3 00
       2 49
   865)  51 00
         43 25
   8708)7 75 00
         6 96 64
      8716)78 360
           69 728
            8 6320
            7 8444
              78760
              78444
```

In this example, after finding four figures of the root, the other three are found by common division. The rule in such cases is, that one less than the number of figures already obtained may be found without error by division, the divisor to be employed being twice the part of the root already found.

388. The square root of a common fraction is found by extracting the square roots of the numerator and denominator. But when the denominator is not a perfect square, it is best to reduce the fraction to a decimal and then extract the root.

Thus, the square root of $\frac{121}{361} = \frac{11}{19}$.

Exercise LXXVII.

Find the square roots of:

1. 2916.	**5.** 3,345,241.	**9.** 53.7289.
2. 7921.	**6.** 125457.64.	**10.** 883.2784.
3. 494,209.	**7.** 47,320,641.	**11.** 1.97262025.
4. 20,164.	**8.** 21,609.	**12.** .0002090916.

13. 2. **14.** 5. **15.** .3. **16.** $3\frac{1}{4}$. **17.** $8\frac{5}{6}$. **18.** .9.

19. Find in yards the side of a square field containing 20 acres.

20. Find the side of a square the area of which is 150 sq. ft. 9 sq. in.

21. Find the side of a square the area of which is 8 sq. yds. 7 sq. ft. 73 sq. in.

Find to six places of decimals the square roots of:

22. $\frac{4}{5}$; $\frac{5}{9}$; $\frac{1}{2}$; $\frac{3}{5}$; $\frac{5}{7}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{5}{6}$.

Sometimes the square root of a number may be most easily found by resolving the number into its prime factors, and taking the roots of the factors separately.

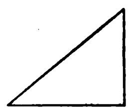
Thus, $1089 = 3^2 \times 11^2$.
Hence, $\sqrt{1089} = 3 \times 11 = 33$.

Find, by factoring, the square roots of:

23. 2025; 17.64; 2.0164; 533.61; 204.49.

The sum of two numbers multiplied by their difference is equal to the difference of their squares.

Thus, the sum of 3 and 5 is 8, their difference is 2, and $2 \times 8 = 16$; and the difference of their squares, $25 - 9$, is 16.



In a right triangle,

The sum of the squares on the two legs is equivalent to the square on the hypotenuse.

Thus, if 3 and 4 represent respectively the legs of a right triangle, the hypotenuse $= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

And if 5 represent the hypotenuse of a right triangle, and 4 one leg, then the other leg is $\sqrt{(5+4)(5-4)} = \sqrt{9} = 3$.

24. A ladder 13 ft. long standing on level ground reaches a window 12 ft. from the ground. How far from the wall is the foot of the ladder?

25. The two legs of a right triangle are 35 in. and 84 in. respectively. Find the hypotenuse.

26. The hypotenuse of a right triangle is 61 in., and one leg 11 in. Find the other leg.
27. Find the longest straight line that can be drawn on the floor of a room 20 ft. by 15 ft.
28. Find the longest line in a box that is 8 ft. long, 4 ft. wide, 1 ft. deep.

Cube Root.

389. The *cube* of a number is the product of *three* factors, each equal to the number.

The cubes of	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
are	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

390. The **cube root** of a number is one of the three equal factors of the number.

Thus, the cube roots of	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000,
are	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.

391. The cube root of a number is indicated by $\sqrt[3]{\ }$, or by the fractional exponent $\frac{1}{3}$ written above and to the right of the number.

Thus, $\sqrt[3]{343}$, or $343^{\frac{1}{3}}$, means the cube root of 343.

392. Since $35 = 30 + 5$, the cube of 35 may be obtained thus:

$$\begin{array}{l}
30 + 5 \\
\underline{30 + 5} \\
30^2 + (30 \times 5) \\
\underline{\quad + (30 \times 5) + 5^2} \\
30^2 + 2(30 \times 5) + 5^2 \\
\underline{30 + 5} \\
30^3 + 2(30^2 \times 5) + (30 \times 5^2) \\
\underline{\quad (30^2 \times 5) + 2(30 \times 5^2) + 5^3} \\
30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3
\end{array}$$

$$\begin{array}{rr}
30^3 = & 27{,}000 \\
3(30^2 \times 5) = & 13{,}500 \\
3(30 \times 5^2) = & 2{,}250 \\
5^3 = & \underline{125} \\
 & 42{,}875
\end{array}$$

Hence, the cube of any number composed of tens and units contains four parts:

I. *The cube of the tens.*

II. *Three times the product of the square of the tens by the units.*

III. *Three times the product of the tens by the square of the units.*

IV. *The cube of the units.*

In extracting the cube root of a number, the first step is to mark it off in periods.

Since $1 = 1^3$, $1000 = 10^3$, $1,000,000 = 100^3$, and so on, it follows that the cube root of any number between 1 and 1000, that is, of any number that has *one*, *two*, or *three* figures, is a number of *one* figure; and that the cube root of any number between 1000 and 1,000,000, that is, of any number that has *four*, *five*, or *six* figures, is a number of *two* figures; and so on.

If, therefore, an integral number be divided into periods of three figures each, from right to left, the number of figures in the root will be equal to the number of periods. The last period may consist of one, two, or three figures.

Ex. Extract the cube root of 42875.

$$
\begin{array}{rr|l}
 & & 42'875\,(35 \\
 & & 27 \\
\hline
3 \times 30^2 = & 2700 & 15\,875 \\
3 \times (30 \times 5) = & 450 & \\
5^2 = & 25 & \\
\hline
 & 3175 & 15\,875
\end{array}
$$

Since 42875 consists of two periods, the cube root will consist of two figures.

The first period, 42, contains the cube of the tens' number of the root.

The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence, 3 is the tens' figure of the root.

The remainder, 15875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Each of these three parts contains the units' number as a factor.

Hence, the 15875 consists of two factors, one of which is the units' number of the root; and the other factor is three times the square of the tens + three times the product of the tens by the units + the square of the units. The larger part of this second factor is three times the square of the tens.

And, if the 158 hundreds of the remainder be divided by the $3 \times 30^2 = 27$ hundreds, the quotient will be the units' number of the root.

The second factor can now be completed by adding to the 2700 $3 \times (30 \times 5) = 450$ and $5^2 = 25$.

393. The same method will apply to numbers of more than two periods, by considering the part of the root already found as so many tens with respect to the next figure of the root.

Ex. Extract the cube root of 57512456.

$$
\begin{array}{rr|l}
 & & 57'512'456(386 \\
 & & 27 \\
\hline
3 \times 30^2 = & 2700 & 30\,512 \\
3 \times (30 \times 8) = & 720 & \\
8^2 = & \underline{64} & \\
 & 3484 & \underline{27\,872} \\
 & & 2\,640\,456 \\
3 \times 380^2 = & 433200 & \\
3 \times (380 \times 6) = & 6840 & \\
6^2 = & \underline{36} & \\
 & 440076 & \underline{2\,640\,456}
\end{array}
$$

394. If the cube root of a number have decimal places, the number itself will have *three times* as many.

Thus, if .11 be the cube root of a number, the number is $.11 \times .11 \times .11 = .001331$. Hence, if a given number contain a decimal, and if it be divided into periods of three figures each, by beginning at the decimal point and marking toward the left for the integral number and toward the right for the decimal, the number of periods toward the *left* from the decimal point will show the number of ***integral*** places in the root, and the number of periods toward the ***right*** will show the number of decimal places in the root.

Ex. Extract the cube root of 187.149248.

$$
\begin{array}{rr|l}
 & & 187.149'248\,(5.72 \\
 & & 125 \\
3 \times 50^2 = & 7500 & 62\,149 \\
3 \times (50 \times 7) = & 1050 & \\
7^2 = & 49 & \\
 & 8599 & 60\,193 \\
 & & 1\,956\,248 \\
3 \times 570^2 = & 974700 & \\
3 \times (570 \times 2) = & 3420 & \\
2^2 = & 4 & \\
 & 978124 & 1\,956\,248
\end{array}
$$

It will be seen from the periods that the root will have one integral and two decimal places, and therefore the decimal point must be placed in the root as soon as one figure of the root is obtained.

395. If the given number be not a perfect cube, ciphers may be annexed, and a value of the root may be found as near to the *true* value as we please.

Ex. Extract the cube root of 1250.6894.

$$
\begin{array}{rr|l}
 & & 1'250.689'400\,(10.77 \\
 & & 1 \\
3 \times 10^2 = & 300 & 250
\end{array}
$$

Since 300 is not contained in 200, the next figure of the root will be 0.

$$
\begin{array}{rr|l}
3 \times 100^2 = & 30000 & 250\,689 \\
3 \times (100 \times 7) = & 2100 & \\
7^2 = & 49 & \\
 & 32149 & 225\,043 \\
 & & 25\,646\,400 \\
3 \times 1070^2 = & 3434700 & \\
3 \times (1070 \times 7) = & 22470 & \\
7^2 = & 49 & \\
 & 3457219 & 24\,200\,533 \\
 & & 1\,445\,867
\end{array}
$$

396. The following method very much shortens the work in long examples.

Ex. Extract the cube root of 5 to five places of decimals.

	5.000(1.70997
	1
$3 \times 10^2 = 300$	4 000
$3(10 \times 7) = 210$	
$7^2 = 49$	
559	3 913
259	87 000 000
$3 \times 1700^2 = 8670000$	
$3(1700 \times 9) = 45900$	
$9^2 = 81$	
8715981	78 443 829
45981	8 556 1710
$3 \times 1709^2 = 8762043$	7 885 8387
	670 33230
	613 34301

After the first two figures of the root are found, the next trial divisor is obtained by bringing down the sum of the 210 and 49 obtained in completing the preceding divisor; then adding the three lines connected by the brace, and annexing two ciphers to the result.

It is seen at a glance that when the trial divisor is increased by 3 times the 17 tens of the root, it will be greater than 87000; so that 0 is placed in the root, and 3×1700^2 is obtained by annexing two ciphers to the 86700. Again the trial divisor is obtained by bringing down the sum of the 45900 and 81 which was obtained in completing the preceding divisor; then adding the three lines connected by the brace, and annexing two ciphers to the result.

The last two figures of the root are found by division. The rule in such cases is, that two less than the number of figures already obtained may be found without error by division, the divisor to be employed being three times the square of the part of the root already found.

397. The cube root of a common fraction is found by taking the cube roots of the numerator and denominator; but if the denominator be not a perfect cube, it is best to reduce the fraction to a decimal and then extract the root.

Exercise LXXVIII.

Find the cube roots of:

1. 1331.	**5.** 148877.	**9.** .007821346625.
2. 1728.	**6.** 2048383.	**10.** 104.600290750613.
3. 12.167.	**7.** 59.776471.	**11.** 17183498535125.
4. 300.763.	**8.** 304957.115891.	**12.** 122615.327232.

13. 10; $3\frac{5}{8}$; $8\frac{1}{3}$ to four places of decimals.

14. 5; $\frac{5}{9}$; $7\frac{3}{5}$; $\frac{3}{4}$ to four places of decimals.

15. Find the entire surface of a cube the volume of which is 14 cu. ft. 705.088 cu. in.

The square of $(30 + 5) = 30^2 + 2(30 \times 5) + 5^2$. § 382.

The 30^2 may be represented by a square (Fig. 1) 30 in. on a side.

The $2(30 \times 5)$ may be represented by two strips 30 in. long and 5 in. wide, of Fig. 2, which are added to two adjacent sides of Fig. 1.

The 5^2 may be represented by the small square of Fig. 3 required to make Fig. 2 a complete square.

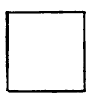


Fig. 1.

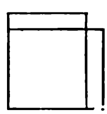


Fig. 2.

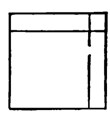


Fig. 3.

In extracting the square root of 1225, the large square, which is 30 in. on a side, is first removed, and a surface of 325 sq. in. remains.

This surface consists of two equal rectangles, each 30 in. long, and a small square whose side is equal to the width of the rectangles.

The width of the rectangles is found by dividing the 325 sq. in. by the sum of their lengths, that is, by 60, which gives 5 in.

Hence, the entire length of the surfaces added is 30 in. + 30 in. + 5 in. = 65 in., and the width is 5 in.

Therefore, the total area is $(65 \times 5) = 325$ sq. in.

The cube of $(30 + 5) = 30^3 + 3\,(30^2 \times 5) + 3\,(30 \times 5^2) + 5^3$. § 392.

The 30^3 may be represented by a cube whose edge is 30 in. (Fig. 1).

The $3\,(30^2 \times 5)$ may be represented by three rectangular solids, each 30 in. long, 30 in. wide, and 5 inches thick, to be added to three adjacent faces of Fig. 1.

The $3\,(30 \times 5^2)$ may be represented by three equal rectangular solids, 30 in. long, 5 in. wide, and 5 in. thick, to be added to Fig. 2.

The 5^3 may be represented by the small cube required to complete the cube of Fig. 3.

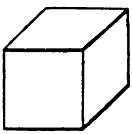


Fig. 1.

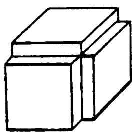


Fig. 2.

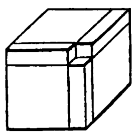


Fig. 3.

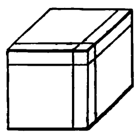


Fig. 4.

In extracting the cube root of 42875, the large cube (Fig. 1), whose edge is 30 in., is first removed.

There remain $(42875 - 27000)$ cu. in. = 15875 cu. in.

The greater part of this is contained in the three rectangular solids which were added to Fig. 1, and which are each 30 in. long and 30 in. wide.

The thickness of these solids is found by dividing the 15875 cu. in. by the sum of the three faces, each of which is 30 in. square; that is, by 2700 sq. in. The result is 5 in.

There are also the three rectangular solids which are added to Fig. 2, and which are 30 in. long and 5 in. wide; and a cube which is added to Fig. 3, and which is 5 in. long and 5 in. wide.

Hence the sum of the products of two dimensions of all these solids is

For the larger rectangular solids,	$3\,(30 \times 30)$ sq. in. =	2700 sq. in.
For the smaller rectangular solids,	$3\,(30 \times 5)$ sq. in. =	450 sq. in.
For the small cube,	(5×5) sq. in. =	25 sq. in.
		3175 sq. in.

This number multiplied by the third dimension gives (5×3175) cu. in. = 15,875 cu. in.

398. In bodies of the same shape,

Two corresponding lines are in the same ratio as any other two.

The ratio of two corresponding surfaces is the square of the ratio of two corresponding lines.

The ratio of two corresponding volumes is the cube of the ratio of two corresponding lines.

Conversely,

The ratio of two corresponding lines is the square root of the ratio of two corresponding surfaces, and the cube root of the ratio of two corresponding volumes.

EXERCISE LXXIX.

1. If the diameter of the moon be reckoned at 2000 mi., and that of the earth at 8000 mi., find the ratio of the surface of the moon to that of the earth. Also, find the ratio of the volume of the moon to that of the earth.
2. If the diameter of the earth be reckoned at 8000 mi., that of Jupiter at 84,000 mi., and that of the sun at 880,000 mi., find the ratios of their volumes.
3. If the diameters of two circles be 20 in. and 40 in. respectively, find the ratio of their circumferences and the ratio of their surfaces.
4. If the areas of two circles be 8000 sq. in. and 36,000 sq. in. respectively, find the ratio of their diameters to the nearest thousandth of an inch.
5. If the volumes of two spheres be 100 cu. in. and 1000 cu. in. respectively, find the ratio of their diameters to the nearest thousandth of an inch.
6. If two stacks of hay of the same shape contain 4 t. 6 cwt. and 1 t. 8 cwt. respectively, find the ratio of their heights.

7. If an ox 7 ft. in girth weigh 1500 lbs., what will be the girth of a similar ox weighing 2500 lbs.?
8. The surface of a pyramid is 560 sq. in. What is the surface of a similar pyramid whose volume is 27 times as great?
9. The volume of a pyramid is 1331 cu. in. What is the volume of a similar pyramid whose surface is 4 times as great?
10. If a well-proportioned man 5 ft. 10 in. high weigh 160 lbs., what should a man 6 ft. high weigh, to the nearest tenth of a pound? What should be the height, to the nearest tenth of an inch, of a man weighing 210 lbs.?
11. A three-gallon jug and a one-gallon jug are of the same shape. What, to the nearest thousandth, is the ratio of their diameters?
12. Two hills have exactly the same shape; one is 900 ft. high, the other 1200 ft. Find the ratio of their surfaces, and also the ratio of their volumes.
13. A ball 3 in. in diameter weighs 4 lbs.; another ball of the same metal weighs 9 lbs. Find the diameter of the second ball to the nearest thousandth of an inch.
14. If Apollo's altar were a perfect cube 10 ft. on a side, what, to the nearest hundredth of an inch, would be the dimensions of a new cubical altar containing twice as much stone?
15. A man standing 40 ft. from a building 24 ft. wide observed that, when he closed one eye, the width of the building hid from view 90 rods of fence which was parallel to the width of the building. Find the distance from the eye of the observer to the fence.
16. A bushel measure and a peck measure are of the same shape. Find the ratio of their heights.

CHAPTER XXII.

LOGARITHMS.

399. In the common system of notation the expression of numbers is founded on their relation to *ten*.

Thus, 3854 indicates that this number contains 10^3 three times, 10^2 eight times, 10 five times, and four units.

400. In this system a number is represented by a series of *different* powers of 10, the exponent of each power being *integral*. But, by employing *fractional* exponents, any number may be represented (approximately) as a *single* power of 10.

401. When numbers are referred in this way to 10, the **exponents of the powers** corresponding to them are called their **logarithms** to the base 10.

For brevity the word "logarithm" is written log.

From § 169 it appears that:

$$10^0 = 1, \qquad 10^{-1} \left(= \tfrac{1}{10}\right) = .1,$$
$$10^1 = 10, \qquad 10^{-2} \left(= \tfrac{1}{100}\right) = .01,$$
$$10^2 = 100, \qquad 10^{-3} \left(= \tfrac{1}{1000}\right) = .001,$$

and so on. Hence,

$$\log 1 = 0, \qquad \log .1 = -1,$$
$$\log 10 = 1, \qquad \log .01 = -2,$$
$$\log 100 = 2, \qquad \log .001 = -3,$$

and so on.

It is evident that the logarithms of all numbers between

1	and	10	will be	$0 +$ a fraction,
10	and	100	will be	$1 +$ a fraction,
100	and	1000	will be	$2 +$ a fraction,
1	and	.1	will be	$-1 +$ a fraction,
.1	and	.01	will be	$-2 +$ a fraction,
.01	and	.001	will be	$-3 +$ a fraction,

402. The fractional part of a logarithm cannot be expressed *exactly* either by common or by decimal fractions; but decimals may be obtained for these fractional parts, true to as many places as may be desired.

If, for instance, the logarithm of 2 be required; log 2 may be supposed to be $\frac{1}{3}$.

Then $10^{\frac{1}{3}} = 2$; or, by raising both sides to the *third* power, $10 = 8$, a result which shows that $\frac{1}{3}$ is too large.

Suppose, then, $\log 2 = \frac{3}{10}$. Then $10^{\frac{3}{10}} = 2$, or by raising both sides to the *tenth* power, $10^3 = 2^{10}$. That is, $1000 = 1024$, a result which shows that $\frac{3}{10}$ is too small.

Since $\frac{1}{3}$ is too large and $\frac{3}{10}$ too small, log 2 lies between $\frac{1}{3}$ and $\frac{3}{10}$; that is, between .33333 and .30000.

In supposing log 2 to be $\frac{1}{3}$, the error of the result is $\frac{10-8}{10} = \frac{2}{10} = .2$. In supposing log 2 to be $\frac{3}{10}$, the error of the result is $\frac{1000-1024}{1000} = \frac{-24}{1000} = -.024$; log 2, therefore, is nearer to $\frac{3}{10}$ than to $\frac{1}{3}$.

The difference between the errors is $.2 - (-.024) = .224$, and the difference between the supposed logarithms is $.33333 - .3 = .03333$.

The last error, therefore, in the supposed logarithm may be considered to be approximately $\frac{24}{224}$ of $.03333 = .0035$ nearly, and this added to .3000 gives .3035, a result a little too large.

By shorter methods of higher mathematics, the logarithm of 2 is known to be 0.3010300, true to the seventh place.

403. The logarithm of a number consists of two parts, an integral part and a fractional part.

Thus, $\log 2 = 0.30103$, in which the integral part is 0, and the fractional part is .30103; $\log 20 = 1.30103$, in which the integral part is 1, and the fractional part is .30103.

404. The integral part of a logarithm is called the **characteristic**; and the fractional part is called the **mantissa.**

405. The mantissa is always made *plus*. Hence, in the case of numbers less than 1 whose logarithms are *minus*, the logarithm is made to consist of a *minus* characteristic and a *plus* mantissa.

406. When a logarithm consists of a *minus* characteristic and a *plus* mantissa, it is usual to write the minus sign *over* the characteristic, or else to add 10 to the characteristic and to indicate the subtraction of 10 from the resulting logarithm.

Thus, $\log .2 = \bar{1}.30103$, and this may be written $9.30103 - 10$.

407. *The characteristic of a logarithm of an integral number, or of a mixed number, is one less than the number of integral digits.*

Thus, from § 401, $\log 1 = 0$, $\log 10 = 1$, $\log 100 = 2$. Hence, the logarithms of all numbers from 1 to 10 (that is, of all numbers consisting of *one* integral digit), will have 0 for characteristic; and the logarithms of all numbers from 10 to 100 (that is, of all numbers consisting of *two* integral digits), will have 1 for characteristic; and so on, the characteristic increasing by 1 for each increase in the number of digits, and therefore always being 1 less than that number.

408. *The characteristic of a logarithm of a decimal fraction is minus, and is equal to the number of the place occupied by the first significant figure of the decimal.*

Thus, from § 401, $\log .1 = -1$, $\log .01 = -2$, $\log .001 = -3$. Hence, the logarithms of all numbers from .1 to 1 will have -1 for a characteristic (the mantissa being *plus*); the logarithms of all numbers from .01 to .1 will have -2 for a characteristic; the logarithms of all numbers from .01 to .001 will have -3 for a characteristic; and so on, the characteristic always being *minus and equal to the number of the place occupied by the first significant figure of the decimal.*

409. *The mantissa of a logarithm of any integral number or decimal fraction depends only upon the digits of the number, and is unchanged so long as the sequence of the digits remains the same.*

For, changing the position of the decimal point in a number is equivalent to multiplying or dividing the number by a power of 10. Its logarithm, therefore, will be increased or diminished by the *exponent* of that power of 10; and, since this exponent is *integral*, the *mantissa* of the logarithm will be unaffected.

Thus, if $27196 = 10^{4.4345}$,

then

$$\begin{aligned}
2719.6 &= 10^{3.4345},\\
27.196 &= 10^{1.4345},\\
2.7196 &= 10^{0.4345},\\
.27196 &= 10^{9.4345-10},\\
.0027196 &= 10^{7.4345-10}.
\end{aligned}$$

410. The advantage of using the number 10 as the base of a system of logarithms consists in the fact that the **mantissa** depends only on the **sequence of digits,** and the **characteristic** on the **position of the decimal point.**

411. As logarithms are simply exponents therefore (§ 148), *The logarithm of a product is the sum of the logarithms of the factors.*

$$\begin{aligned}
\text{Thus, } \log 20 = \log(2 \times 10) &= \log 2 + \log 10\\
&= 0.3010 + 1.0000 = 1.3010;\\
\log 2000 = \log(2 \times 1000) &= \log 2 + \log 1000,\\
&= 0.3010 + 3.0000 = 3.3010;\\
\log .2 = \log(2 \times .1) &= \log 2 + \log .1,\\
&= 0.3010 + 9.0000 - 10 = 9.3010 - 10;\\
\log .02 = \log(2 \times .01) &= \log 2 + \log .01,\\
&= 0.3010 + 8.0000 - 10 = 8.3010 - 10.
\end{aligned}$$

Exercise LXXX.

Given: $\log 2 = 0.3010$; $\log 3 = 0.4771$; $\log 5 = 0.6990$; $\log 7 = 0.8451$.

Find the logarithms of the following numbers by resolv-

ing the numbers into factors, and taking the sum of the logarithms of the factors:

1. log 6.	9. log 25.	17. log .021.	25. log 2.1.
2. log 15.	10. log 30.	18. log .35.	26. log 16.
3. log 21.	11. log 42.	19. log .0035.	27. log .056.
4. log 14.	12. log 420.	20. log .004.	28. log .63.
5. log 35.	13. log 12.	21. log .05.	29. log 1.75.
6. log 9.	14. log 60.	22. log 12.5.	30. log 105.
7. log 8.	15. log 75.	23. log 1.25.	31. log .0105.
8. log 49.	16. log 7.5.	24. log 37.5.	32. log 1.05.

412. As logarithms are simply exponents, therefore (§ 381),

The logarithm of a power of a number is equal to the logarithm of the number multiplied by the exponent of the power.

Thus, $\log 5^7 = 7 \times \log 5 = 7 \times 0.6990 = 4.8930.$
$\log 3^{11} = 11 \times \log 3 = 11 \times 0.4771 = 5.2481.$

413. As logarithms are simply exponents, therefore (§ 381),

The logarithm of a root of a number is equal to the logarithm of the number multiplied by the index of the root.

Thus, $\log 2^{\frac{1}{4}} = \frac{1}{4}$ of $\log 2 = \frac{1}{4} \times 0.3010 = 0.0753.$
$\log .002^{\frac{1}{3}} = \frac{1}{3}$ of $(7.3010 - 10).$

The expression $\frac{1}{3}$ of $(7.3010 - 10)$
may be put in the form of $\frac{1}{3}$ of $(27.3010 - 30)$ which $= 9.1003 - 10$; for, since $20 - 20 = 0$, the addition of 20 to the 7, and of -20 to the -10, produces no change in the *value* of the logarithm

414. *In simplifying the logarithm of a root the equal plus and minus numbers to be added to the logarithm must be such that the resulting minus number, when divided by the denominator of the index of the root, shall give a quotient of* -10.

EXERCISE LXXXI.

Given: $\log 2 = 0.3010$; $\log 3 = 0.4771$; $\log 5 = 0.6990$; $\log 7 = 0.8451$.

Find logarithms of the following:

1. 2^8.	6. 5^5.	11. $5^{\frac{1}{5}}$.	16. $7^{\frac{2}{7}}$.	21. $5^{\frac{7}{2}}$.
2. 5^3.	7. $2^{\frac{1}{3}}$.	12. $7^{\frac{1}{11}}$.	17. $5^{\frac{5}{3}}$.	22. $2^{\frac{11}{7}}$.
3. 7^4.	8. $5^{\frac{1}{2}}$.	13. $2^{\frac{3}{4}}$.	18. $3^{\frac{9}{11}}$.	23. $5^{\frac{3}{4}}$.
4. 3^6.	9. $3^{\frac{1}{6}}$.	14. $5^{\frac{2}{3}}$.	19. $7^{\frac{7}{2}}$.	24. $7^{\frac{11}{7}}$.
5. 7^3.	10. $7^{\frac{1}{5}}$.	15. $3^{\frac{6}{7}}$.	20. $3^{\frac{4}{3}}$.	25. $21^{\frac{7}{8}}$.

415. Since logarithms are simply exponents, therefore (by § 169),

The logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor.

Thus, $\log \frac{3}{2} = \log 3 - \log 2 = 0.4771 - 0.3010 = 0.1761.$
$\log \frac{2}{3} = \log 2 - \log 3 = 0.3010 - 0.4771 = -0.1761.$

To avoid the minus logarithm -0.1761, we subtract the *entire* logarithm 0.1761 from 10, and then indicate the subtraction of 10 from the result.

Thus, $-0.1761 = 9.8239 - 10.$
Hence, $\log \frac{2}{3} = 9.8239 - 10.$

416. The remainder obtained by subtracting the logarithm of a number from 10 is called the **cologarithm** of the number, or **arithmetical complement** of the logarithm of the number.

Cologarithm is usually denoted by colog, and is most easily found by *beginning with the characteristic of the logarithm and subtracting each figure from* 9 *down to the last significant figure, and subtracting that figure from* 10.

Thus, $\log 7 = 0.8451$; and $\text{colog } 7 = 9.1549$. Colog 7 is readily found by subtracting, mentally, 0 from 9, 8 from 9, 4 from 9, 5 from 9, 1 from 10, and writing the resulting figure at each step.

417. Since $\text{colog } 7 = 9.1549,$
and $\log \frac{1}{7} = \log 1 - \log 7 = 0 - 0.8451 = 9.1549 - 10,$
it is evident that,

If 10 *be subtracted from the cologarithm of a number, the result is the logarithm of the reciprocal of that number.*

418. Since
$$\log \tfrac{7}{5} = \log 7 - \log 5,$$
$$= 0.8451 - 0.6990 = 0.1461,$$
and
$$\log 7 + \text{colog } 5 - 10 = 0.8451 + 9.3010 - 10,$$
$$= 0.1461,$$
it is evident that,

The addition of a cologarithm $-$ **10** *is equivalent to the* **subtraction of a logarithm.**

The steps that lead to this result are:

$$\tfrac{7}{5} = 7 \times \tfrac{1}{5},$$

therefore, $\log \frac{7}{5} = \log (7 \times \frac{1}{5}) = \log 7 + \log \frac{1}{5}.$ § 410.

But $\log \frac{1}{5} = \text{colog } 5 - 10.$ § 416.

Hence, $\log \frac{7}{5} = \log 7 + \text{colog } 5 - 10.$

Therefore,

419. *The logarithm of a quotient may be found by adding together the logarithm of the dividend and the cologarithm of the divisor, and subtracting* 10 *from the result.*

In finding a cologarithm when the *characteristic* of the logarithm is a *minus* number, it must be observed that the *subtraction* of a *minus* number is equivalent to the *addition* of an *equal plus* number.

Thus,
$$\log \frac{5}{.002} = \log 5 + \text{colog } .002 - 10,$$
$$= 0.6990 + 12.6990 - 10,$$
$$= 3.3980.$$

Here $\log .002 = \bar{3}.3010$, and in subtracting -3 from 9 the result is the same as adding $+3$ to 9.

Again,
$$\log \frac{2}{.07} = \log 2 + \text{colog } .07 - 10,$$
$$= 0.3010 + 11.1549 - 10,$$
$$= 1.4559.$$

Also, $\log \frac{.07}{2^3} = 8.8451 - 10 + 9.0970 - 10,$

$= 17.9421 - 20,$

$= 7.9421 - 10.$

Here, $\log 2^3 = 3 \log 2 = 3 \times 0.3010 = 0.9030.$

Hence, $\text{colog } 2^3 = 10 - 0.9030 = 9.0970.$

EXERCISE LXXXII.

Given: $\log 2 = 0.3010$; $\log 3 = 0.4771$; $\log 5 = 0.6990$; $\log 7 = 0.8451$.

Find logarithms for the following quotients:

1. $\frac{2}{5}$.	7. $\frac{5}{3}$.	13. $\frac{.05}{3}$.	19. $\frac{.05}{.003}$.	25. $\frac{.02^2}{3^2}$.
2. $\frac{2}{7}$.	8. $\frac{5}{2}$.	14. $\frac{.005}{2}$.	20. $\frac{.007}{.02}$.	26. $\frac{3^3}{.02^2}$.
3. $\frac{3}{5}$.	9. $\frac{7}{3}$.	15. $\frac{.07}{5}$.	21. $\frac{.02}{.007}$.	27. $\frac{7^3}{.02^2}$.
4. $\frac{3}{7}$.	10. $\frac{7}{2}$.	16. $\frac{5}{.07}$.	22. $\frac{.005}{.07}$.	28. $\frac{.07^3}{.003^3}$.
5. $\frac{5}{7}$.	11. $\frac{3}{2}$.	17. $\frac{3}{.007}$.	23. $\frac{.03}{7}$.	29. $\frac{.005^2}{7^3}$.
6. $\frac{7}{5}$.	12. $\frac{7}{.5}$.	18. $\frac{.003}{7}$.	24. $\frac{.0007}{.2}$.	30. $\frac{7^3}{.005^2}$.

420. A table of *four-place* logarithms is here given, which contains logarithms of all numbers under 1000, *the decimal point and characteristic being omitted.* The logarithms of single digits 1, 8, etc., will be found at 10, 80, etc.

Tables containing logarithms of more places can be procured, but this table will serve for many practical uses, and will enable the student to use tables of six-place, seven-place, and ten-place logarithms, in work that requires greater accuracy.

421. In working with a four-place table, the numbers corresponding to the logarithms, that is, the *antilogarithms*, as they are called, may be carried to *four significant digits.*

To Find the Logarithm of a Number in this Table.

422. Suppose it is required to find the logarithm of 65.7. In the column headed "N" look for the first two significant figures, and at the top of the table for the third significant figure. In the line with 65, and in the column headed 7, is seen 8176. To this number prefix the characteristic and insert the decimal point. Thus,

$$\log 65.7 = 1.8176.$$

Suppose it is required to find the logarithm of 20347. In the line with 20, and in the column headed 3, is seen 3075; also in the line with 20, and in the 4 column, is seen 3096, and the difference between these two is 21. The difference between 20300 and 20400 is 100, and the difference between 20300 and 20347 is 47. Hence, $\frac{47}{100}$ of $21 = 10$, nearly, must be added to 3075. That is,

$$\log 20347 = 4.3085.$$

Suppose it is required to find the logarithm of .0005076. In the line with 50, and in the 7 column, is seen 7050; in the 8 column, 7059: the difference is 9. The difference between 5070 and 5080 is 10, and the difference between 5070 and 5076 is 6. Hence, $\frac{6}{10}$ of $9 = 5$ must be added to 7050. That is,

$$\log .0005076 = 6.7055 - 10.$$

To Find a Number when its Logarithm is Given.

423. Suppose it is required to find the number of which the logarithm is 1.9736.

Look for 9736 in the table. In the column headed "N," and in the line with 9736, is seen 94, and at the head of

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

the column in which 9736 stands is seen 1. Therefore, write 941, and insert the decimal point as the characteristic directs. That is, the number required is 94.1.

Suppose it is required to find the number of which the logarithm is 3.7936.

Look for 7936 in the table. It cannot be found, but the two adjacent mantissas between which it lies are seen to be 7931 and 7938; their difference is 7, and the difference between 7931 and 7936 is 5. Therefore, $\frac{5}{7}$ of the difference between the numbers corresponding to the mantissas, 7931 and 7938, must be added to the number corresponding to the mantissa 7931.

The number corresponding to the mantissa 7938 is 6220.
The number corresponding to the mantissa 7931 is 6210.
The difference between these numbers is 10,

and $$6210 + \tfrac{5}{7} \text{ of } 10 = 6217.$$

Therefore, the number required is 6217.

Suppose it is required to find the number of which the logarithm is $7.3882 - 10$.

Look for 3882 in the table. It cannot be found, but the two adjacent mantissas between which it lies are seen to be 3874 and 3892; their difference is 18, and the difference between 3874 and 3882 is 8. Therefore, $\frac{8}{18}$ of the difference between the numbers corresponding to the mantissas, 3874 and 3892, must be added to the number corresponding to the mantissa 3874.

The number corresponding to the mantissa 3892 is 2450.
The number corresponding to the mantissa 3874 is 2440.
The difference between these numbers is 10,

and $$2440 + \tfrac{4}{9} \text{ of } 10 = 2444.$$

Therefore, the number required is .002444.

EXERCISE LXXXIII.

Find logarithms of the following numbers:

1. 70.	6. 6897.	11. 77860.	16. 5.0009.
2. 101.	7. 9901.	12. 30127.	17. 0.3769.
3. 333.	8. 4389.	13. 730.84.	18. 0.070707.
4. 3491.	9. 1111.	14. 0.008765.	19. 0.03723.
5. 1866.	10. 58343.	15. 8.0808.	20. 98.871.

Find antilogarithms to the following logarithms:

21. 3.9017.	25. 2.9850.	29. 8.7324 − 10.
22. 1.2076.	26. 4.5388.	30. 9.5555 − 10.
23. 0.4442.	27. 0.8550.	31. 6.0216 − 10.
24. 1.0090.	28. 9.9992 − 10.	32. 7.0080 − 10

Ex. Find the product of $908.4 \times .05392 \times 2.117$.

$$\begin{aligned} \log 908.4 &= 2.9583 \\ \log .05392 &= 8.7318 - 10 \\ \log 2.117 &= \underline{0.3257} \\ & 2.0158 = \log 103.7. \quad \textit{Ans.} \end{aligned}$$

Find by logarithms the following products:

33. 948.22×0.4387.	40. 270.05×0.0087.
34. 1.9704×0.0786.	41. 11.163×0.3333.
35. 380.25×0.00673.	42. 777.78×0.0787.
36. 216.21×0.76312.	43. 2.6537×0.2313.
37. $.56127 \times 1.2312$.	44. 37.587×12.371.
38. $.86311 \times 56.371$.	45. 89.313×2.3781.
39. 59.795×0.7955.	46. 9.1765×0.089.

Ex. Find the quotient of $\dfrac{8.371 \times 834.64}{7309}$.

$$\begin{aligned} \log\ 8.371 &= 0.9227 \\ \log\ 834.64 &= 2.9215 \\ \text{colog}\ 7309 &= \underline{6.1362-10} \\ & \ 9.9804-10 = \log .9558. \ \textit{Ans.} \end{aligned}$$

Find the quotients of:

47. $\dfrac{56.407}{13.045}$.

48. $\dfrac{857.06}{3079.8}$.

49. $\dfrac{.9387}{598.6}$.

50. $\dfrac{3069}{.7891}$.

51. $\dfrac{75.46 \times 0.0765}{93.08 \times 98.071}$.

52. $\dfrac{98 \times 537 \times 0.0079}{67309 \times 0.0947}$.

53. $\dfrac{314 \times 7.18 \times 8132}{519 \times 827 \times 3.215}$.

54. $\dfrac{212 \times 2.16 \times 8002}{536 \times 351 \times 7.256}$.

Ex. Find the cube of 0.0497.

$$\begin{aligned} \log .0497 &= 8.6964-10 \\ & \ \underline{3} \\ & \ 6.0892-10 = \log\ .0001228. \ \textit{Ans.} \end{aligned}$$

Find by logarithms:

55. 5.06^3.
56. 2.501^5.
57. 1.716^7.
58. 1.178^{10}.
59. 0.7685^6.
60. 0.9611^3.
61. 0.0231^2.
62. 0.8567^3.
63. $(\frac{61}{73})^4$.
64. $(\frac{13}{71})^3$.
65. $(\frac{16}{9})^5$.
66. $(\frac{35}{4})^3$.
67. $(5\frac{5}{11})^2$.
68. $(4\frac{4}{31})^3$.
69. $(2\frac{29}{37})^5$.
70. $(\frac{371}{711})^3$.

Ex. Find the fourth root of 0.00862.

$$\begin{aligned} \log 0.00862 &= 7.9355-10 \\ & \ \underline{30 \qquad -30} \\ & 4\overline{)37.9355-40} \\ & \ 9.4839-10 = \log .3047. \ \textit{Ans.} \end{aligned}$$

Find by logarithms:

71. $13^{\frac{1}{3}}$.
72. $29^{\frac{1}{5}}$.
73. $879^{\frac{1}{10}}$.
74. $.609^{\frac{1}{4}}$.
75. $93.73^{\frac{1}{2}}$.
76. $21.97^{\frac{5}{6}}$.
77. $7.935^{\frac{5}{7}}$.
78. $0.815^{\frac{2}{3}}$.

(1) Find the weight of a leaden brick 113^{mm} long, 76^{mm} wide, and 38^{mm} thick, of specific gravity 11.36.

$$\begin{aligned}
\log\ 113 &= 2.0531\\
\log\ 76 &= 1.8808\\
\log\ 38 &= 1.5798\\
\log\ 11.36 &= \underline{1.0554}\\
&\ 6.5691 = \log 3{,}707{,}500.
\end{aligned}$$

That is, 3.7075^{kg}. *Ans.*

(2) What is the capacity of a bin 315^{cm} long, 93.5^{cm} wide, and 47.8^{cm} deep?

$$\begin{aligned}
\log\ 315 &= 2.4983\\
\log\ 93.5 &= 1.9708\\
\log\ 47.8 &= \underline{1.6794}\\
&\ 6.1485 = \log 1{,}408{,}000.
\end{aligned}$$

That is, 1408^{l}. *Ans.*

(3) How many bushels will it take to fill a globe 43.8 in. in diameter?

$$\begin{aligned}
\log\ 43.8^3 &= 4.9245\\
\log\ 0.5236 &= 9.7190 - 10\\
\operatorname{colog}\ 2150.42 &= \underline{6.6675 - 10}\\
&\ 1.3110 \qquad = \log 20.47.
\end{aligned}$$

That is, 20.47 bu. *Ans.*

(4) Find the weight of a sphere of solid tin 6.35^{cm} in diameter, of specific gravity 7.29.

$$\begin{aligned}
\log\ 6.35^3 &= 2.4084\\
\log\ 0.5236 &= 9.7190 - 10\\
\log\ 7.29 &= \underline{0.8627}\\
&\ 2.9901 \qquad = \log 977.5.
\end{aligned}$$

That is, 977.5^{g}. *Ans.*

79. What weight of sulphuric acid, specific gravity 1.841, will fill a silver sphere 138^{mm} in diameter?

80. What is the area of a circle 13.75 in. in diameter?

81. Find the depth of a cubical bin that holds 75 bu.

82. Find the diameter of a 24-lb. shot, specific gravity 7.6.

CHAPTER XXIII.

APPROXIMATIONS.

424. What number exceeds its square root by 3?

Make two suppositions, 5 and 6.

By logarithms, $5 - \sqrt{5} = 2.764$, an error of $-.236$; and $6 - \sqrt{6} = 3.551$, an error of $+.551$.

The difference of the assumed numbers is 1, and the difference of the resulting errors is .787.

The errors of results are approximately in proportion to the errors of the assumed numbers.

Therefore, in assuming 5, the error is to 1 as .236 is to .787; and in assuming 6 the error is to 1 as .551 is to .787.

Hence, 5 is nearly .3 too small, and 6 is nearly .7 too large.

Therefore, assume 5.3 and 5.4.

By logarithms, $5.3 - \sqrt{5.3} = 2.998$, an error of $-.002$; and $5.4 - \sqrt{5.4} = 3.077$, an error of $+.077$.

The difference of the assumed numbers is .1, and the difference of the errors is .079.

Therefore, the error of 5.3 is to .1 as .002 is to .079; or, error of $5.3 : .1 = 2 : 79$.

From this proportion the error is found to be .0025.

$5.3 + .0025 = 5.3025$, and this result is the nearest approximation attainable by four-place logarithms.

425. Hence, in solving questions of this kind,

Assume two answers, test each, and note the errors of the results.

Calculate the error of either supposition by assuming that it is in the same ratio to the difference of the two suppositions as the error of its result is to the difference of the two results.

EXERCISE LXXXIV.

1. What number is 3 less than its square?

   Assume 2.3 and 2.4.

2. A flag-staff 50 ft. high broke, and the top falling over rested one end on the stump and the other 17 ft. from its base. How high was the stump?

   Assume 22 ft. and 23 ft.

3. What number added to eight times its reciprocal is equal to 8?

   Two answers are required: one between 1 and 2, the other between 6 and 7.

4. Find a number whose reciprocal is equal to 4 minus the number.

   Two answers are required: one between 0 and 1, the other between 3 and 4.

5. What number is ten times its own logarithm?
6. What number is double its own cube-root?
7. What number exceeds its cube root by $6\frac{1}{4}$?
8. What is the number which added to its own square makes 11?
9. What is the number which multiplied by 10 makes 8 more than the square of the number?
10. A certain number is equal to the sum of $\frac{1}{8}$ its own cube plus $\frac{1}{4}$ its own square. What is the number?
11. What number is equal to its square minus three times its logarithm?

    Assume 1.1 and 1.2.

12. The sum of the square, and the square root of a number, being divided by 1 plus the number gives a quotient of $2\frac{1}{2}$. What is the number?

CONTINUED FRACTIONS.

426. As in decimal fractions a result accurate to a given number of places is often required, so in common fractions it is often required to find the most accurate value of a ratio that can be given with denominators limited to a certain size.

Ex. Find the most accurate ratios of a circumference to a diameter expressed by a fraction with a denominator under 10; with a denominator under 100; with a denominator under 1000.

The ratio 3.1416 is true to the nearest ten-thousandth; and, therefore, it is required to find values of $\frac{1416}{10000}$ within the prescribed limits.

The first step is to reduce the fraction $\frac{1416}{10000}$ to its lowest terms.

$$\tfrac{1416}{10000} = \tfrac{177}{1250}.$$

Then, divide the denominator by the numerator; the last divisor by the last remainder; and so on, as in finding the greatest common measure.

```
177)1250(7
    1239
      11)177(16
         176
           1)11(11
             11
```

If, therefore, both terms of the fraction $\frac{177}{1250}$ be divided by the numerator, the result is

$$\frac{1}{7\frac{11}{177}};$$

and if the fraction in the denominator be omitted, the required ratio, with a denominator less than 10, is $3\frac{1}{7} = \frac{22}{7}$.

But, if the fraction $\frac{11}{177}$ be put in the form of

$$\frac{1}{16\frac{1}{11}},$$

and the fraction in the denominator be omitted, the ratio becomes

$$3\frac{1}{7\frac{1}{16}} = 3\tfrac{16}{113} = \tfrac{355}{113};$$

a result which shows that $\frac{22}{7}$ is the nearest ratio expressed by a fraction with a denominator under 100, and that $\frac{355}{113}$ is the nearest ratio expressed by a fraction with a denominator under 1000.

427. After the quotients have been found, the results may be written as follows:

$$3+\cfrac{1}{7+\cfrac{1}{16+\cfrac{1}{11}}}$$

428. The successive approximate values of a continued fraction are found by beginning at the top and taking first one, then two, then three, and so on, of its parts. Thus:

The first approximate value is 3.

The second is $3 + \frac{1}{7} = \frac{22}{7}$.

The third is $3 + \cfrac{1}{7+\cfrac{1}{16}} = 3 + \frac{16}{113} = \frac{355}{113}$.

The fourth is $3 + \cfrac{1}{7+\cfrac{1}{16+\cfrac{1}{11}}}$

and this $= 3 + \dfrac{1}{7\frac{11}{177}} = 3 + \frac{177}{1250} = \frac{3927}{1250}$, or 3.1416.

429. In reducing the part of a continued fraction selected for an approximate value, begin with the last fraction.

Ex. Find the value of the continued fraction

$$\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5}}}}$$

$$\frac{1}{4\frac{1}{5}} = \frac{5}{21};\quad \frac{1}{3\frac{5}{21}} = \frac{21}{68};\quad \frac{1}{2\frac{21}{68}} = \frac{68}{157}.$$

EXERCISE LXXXV.

1. Convert $\frac{8}{11}$, $\frac{18}{75}$, $\frac{29}{127}$, $\frac{135}{64}$ into continued fractions.
2. Find the approximate values of $\frac{20}{27}$; $\frac{58}{47}$; $\frac{734}{851}$.
3. Find common fractions approximating to .236; .2361; 1.609.
4. Find common fractions approximating to .382; 1.732; .6253.
5. Find approximate values of $\frac{171}{457}$; $\frac{613}{757}$; $\frac{711}{813}$; $\frac{937}{113}$.
6. Find the proper fraction that, when reduced to a continued fraction, will have 2, 3, 5, 6, 7 as quotients.
7. Find a series of fractions approximating to the ratio of the pound troy (5760 grs.) to the pound avoirdupois (7000 grs.).
8. Find a series of fractions approximating to the ratio of the side of a square to its diagonal; that ratio being $1:1.414214$ nearly.
9. Find a series of fractions approximating to the ratio of the ar to the square chain, from the equality

   $$1 \text{ ar} = .2471 \text{ of a square chain.}$$

10. Find a series of fractions approximating to the ratio of the 48-pound shot to the weight of the French shot of 24^{kg}.
11. If the mean diameter of the Earth is reckoned at 7912 mi., and that of Mars 4189 mi., find a series of fractions approximating to the ratio of the mean diameters of these two planets.
12. Find a series of fractions approximating to the ratio of a cubic yard to a cubic meter from the equality

    $$1 \text{ cu. yd.} = .76453 \text{ of a cubic meter.}$$

13. Find a series of fractions approximating to the ratio of the kilometer to the mile, from the equality

    $$1^{\text{m}} = 1.09362 \text{ yds.}$$

CHAPTER XXIV.

PROGRESSIONS.

ARITHMETICAL PROGRESSION.

430. A series of numbers that increase or decrease by a common difference is called an **Arithmetical Progression.**

Thus, the numbers 5, 8, 11, 14 are an arithmetical progression, since they increase by a common difference, 3; and the numbers 12, 10, 8, 6 are an arithmetical progression, since they decrease by a common difference, 2.

The several numbers of a series are called its **terms.**

431. In the series 2, 5, 8, 11, 14, 17, 20 it is obvious that the second term, 5, is $2+(3\times 1)$; the third term, 8, is $2+(3\times 2)$; the fourth term, 11, is $2+(3\times 3)$; the fifth term, 14, is $2+(3\times 4)$; and so on. Hence,

Any term may be found by multiplying the common difference by 1 *less than the number of the term, and adding the product to the first term.*

Thus, the twelfth term of the series 2, 5, 8..... will be $2+(3\times 11) = 35$.

In like manner, any term of a decreasing series will be found by *subtracting* this multiple of the difference from the first term.

Thus, the eleventh term of the series 50, 46, 42..... will be $50-(10\times 4)=10$.

Exercise LXXXVI.

Find:

1. The seventh term of the series 3, 5, 7.....
2. The fifteenth term of the series 2, 7, 12.....
3. The sixth term of the series 2, $2\frac{5}{7}$, $3\frac{3}{7}$.....
4. The twentieth term of the series 2, $3\frac{1}{4}$, $4\frac{1}{2}$.....
5. The seventh term of the series 21, 19, 17.....
6. The twelfth term of the series 18, $17\frac{1}{3}$, $16\frac{2}{3}$.....
7. When the first term of a series is 5, and the common difference $2\frac{1}{4}$, find the thirteenth and eighteenth terms.

Ex. When the fourth term of a series is 14, and the twelfth term 38, find the common difference.

The difference between the fourth and twelfth terms will evidently be eight times the common difference. Hence, the common difference will be $\frac{38-14}{8}=3$.

Find the common difference in a series:

8. Whose fourth term is 12 and seventh term 27.
9. Whose first term is 20 and fourth term 40.
10. Whose first term is 2 and eleventh term 20.
11. Whose third term is 7 and eighth term $12\frac{1}{2}$.
12. Whose first term is 1 and fourth term 19.

432. The sum of seven terms of the series 3, 5, 7..... will be $3+5+7+9+11+13+15$, or written in reverse order, $15+13+11+9+7+5+3$ Therefore,

twice the sum $=18+18+18+18+18+18+18=18\times 7$.

Hence, the sum $=\dfrac{18\times 7}{2}$.

433. It will be seen that 18 is the sum of 3 and 15; that is, the sum of the first and last terms; and that 7 is the number of terms. Hence,

The sum of an arithmetical series may be found by multiplying one-half the sum of the first and last terms by the number of terms.

Thus, the sum of eight terms of the series whose first term is 3, and last term 38, is

$$8 \times \left(\frac{3+38}{2}\right) = 164.$$

EXERCISE LXXXVII.

Find the sum of:

1. $1 + 5 + 9 + \cdots\cdots$ to twenty terms.
2. $4 + 5\frac{1}{2} + 7 + \cdots\cdots$ to eight terms.
3. $8 + 7\frac{2}{3} + 7\frac{1}{3} + \cdots\cdots$ to sixteen terms.
4. $20 + 18\frac{1}{4} + 16\frac{1}{2} + \cdots\cdots$ to seven terms.
5. The first twenty natural numbers.
6. The natural numbers from 37 to 53 inclusive.
7. A series of thirty terms, of which the first is 21 and the last 59.
8. The series whose first two terms are 3 and 9 and last 75.
9. A series of twenty terms whose third and fifth terms are 10 and 15.
10. A stone, when dropped from a height, falls through 16.1 ft. in the first second, 48.3 in the next, 80.5 in the third, and so on, in arithmetical progression. How far will it fall in the seventh second? and how far in 7 sec.?
11. A, who travels 8 mi. the first day, 11 the second, 14 the third, and so on, overtakes in 17 dys. B, who started at the same time, and travelled uniformly. What is B's rate per day?
12. One hundred stones lie in a straight line, 1 yd. apart. A boy starts at the first stone, brings each of the others in separately, and piles them with the first stone. How far does he travel?

434. If a represent the first term, d the common difference, l the last term, n the number of terms, and s the sum of the terms, when any three of the five quantities are given, the other two may be found by means of the following expressions, called **formulas**:

$$\text{I.} \quad l = a + (n-1) \times d.$$

$$\text{II.} \quad s = \frac{n}{2} \times (a + l).$$

Ex. If the common difference is $\frac{1}{2}$, the first term 13, and the last term 41, how many terms are there?

Here d, a, and l are given, and n is required.

If for d, l, and a in the first formula, $\frac{1}{2}$, 41, and 13, respectively, be substituted, the result is $41 = 13 + (n-1) \times \frac{1}{2}$.

By subtracting 13 from each side, $28 = (n-1) \times \frac{1}{2}$.

By multiplying by 2, $56 = n - 1$.

Therefore, $n = 57$.

Geometrical Progression.

435. A series in which each term is obtained from the preceding term by multiplying it by a constant multiplier is called a **Geometrical Progression.**

Thus, the numbers 3, 6, 12, 24..... are a geometrical progression, since each term is twice the preceding term; and the numbers 9, 3, 1, $\frac{1}{3}$..... are a geometrical progression, since each term is one-third of the preceding term.

The constant multiplier is called the **ratio** of the progression.

436. In the series 3, 6, 12, 24, 48, 96, ... it is obvious that the second term, 6, is 3×2; the third term, 12, is 3×2^2; the fourth term, 24, is 3×2^3; the fifth term, 48, is 3×2^4; and so on. Hence,

[illegible]

[illegible]7. [illegible]

[illegible]

Find : [illegible]

1. The [illegible] term of the [illegible]
2. The [illegible] term of the [illegible]
3. The seventh term of the [illegible] 11
4. The sixth term of the [illegible]
5. The eighth term of the [illegible]
6. Write the first three [illegible] fifth and sixth terms are [illegible]
7. The seventh and ninth [illegible] of a series are 100 and 144, respectively. Find the twelfth term.
8. A capital of $1000 is increased by $\frac{1}{10}$ of itself each year. What will it be at the beginning of the fifth year?
9. A capital of $1000 is increased by $\frac{6}{100}$ of itself each year. What will it be at the beginning of the sixth year?

438. If the sum of the series 3, 3×2, 3×2^2, 3×2^3, 3×2^4 be represented by s, then

$$s = 3 + 3\times 2 + 3\times 2^2 + 3\times 2^3 + 3\times 2^4.$$

And, if both sides be multiplied by the ratio 2, the result is

$$2s = 3\times 2 + 3\times 2^2 + 3\times 2^3 + 3\times 2^4 + 3\times 2^5.$$

If the subtraction of the first equation from the second be indicated, then

$$2s - s = 3\times 2^5 - 3,$$

and this may be put in the form of

$$s\,(2-1) = 3\,(2^5 - 1),$$

or

$$s = \frac{3\,(2^5-1)}{(2-1)}.$$

By noticing that 2 is the ratio and 3 the first term, it will be seen that,

The sum of a geometrical series may be found as follows:

Raise the ratio to a power equal to the number of terms.

Divide the difference between that power and 1 *by the difference between the ratio and* 1.

Multiply the quotient by the first term.

439. When the ratio is less than 1, as in the ratio

$$s = 32 + 32\times \tfrac{1}{2} + 32\times (\tfrac{1}{2})^2 + 32\times (\tfrac{1}{2})^3 + 32\times (\tfrac{1}{2})^4,$$

if both sides be multiplied by the ratio $\frac{1}{2}$, the result is

$$\tfrac{1}{2}s = 32\times \tfrac{1}{2} + 32\times (\tfrac{1}{2})^2 + 32\times (\tfrac{1}{2})^3 + 32\times (\tfrac{1}{2})^4 + 32\times (\tfrac{1}{2})^5.$$

Subtract the second equation from the first, then

$$s - \tfrac{1}{2}s = 32 - 32\times (\tfrac{1}{2})^5;$$

and this may be put in the form of

$$s\,(1-\tfrac{1}{2}) = 32\times \{1 - (\tfrac{1}{2})^5\}.$$

By noticing that $\frac{1}{2}$ is the ratio and 32 the first term, it will be seen that,

When the ratio is less than 1, the sum of the series may be found as follows:

Raise the ratio to a power equal to the number of terms.

Divide the difference between 1 *and that power of the ratio by the difference between* 1 *and the ratio.*

Multiply the quotient by the first term.

(1) Find the sum of five terms of the series 2, 6, 18.....

Here the ratio is 3 and the first term 2. Hence,

$$s = 2 \times \frac{3^5 - 1}{3 - 1} = 2 \times \frac{242}{2} = 242.$$

(2) Find the sum of seven terms of the series 3, 1, $\frac{1}{3}$.....

Here the ratio is $\frac{1}{3}$ and the first term 3. Hence,

$$s = 3 \times \frac{1 - (\frac{1}{3})^7}{1 - \frac{1}{3}} = 3 \times \frac{1 - \frac{1}{2187}}{1 - \frac{1}{3}} = 3 \times \frac{2186}{2187} \times \frac{3}{2} = 4\tfrac{121}{243}.$$

Exercise LXXXIX.

Find the sum of:

1. $2 + 6 + 18 + \cdots$ to six terms.
2. $1 + 2 + 4 + \cdots$ to nine terms.
3. $3 + 9 + 27 + \cdots$ to five terms.
4. $2 + 3 + 4\frac{1}{2} + \cdots$ to eight terms.
5. $1 + \frac{1}{3} + \frac{1}{9} + \cdots$ to eight terms.
6. $1 + \frac{1}{2} + \frac{1}{4} + \cdots$ to ten terms.
7. $\frac{1}{2} + \frac{1}{3} + \frac{2}{9} + \cdots$ to eight terms.
8. Find the sum of the first six terms of the series whose first term is 3 and ratio 5.
9. Find the sum of the first eight terms of the series whose first term is 3 and ratio $\frac{1}{3}$.
10. A person saved in one year \$64, and in each succeeding year, for 9 years more, $1\frac{1}{2}$ times as much as in the preceding year. Find the whole amount saved.

440. If a represent the first term, r the ratio, l the last term, n the number of terms, and s the sum of the terms, when any three of these five quantities are given, the other two may be found by means of the following formulas:

I. $l = a \times r^{n-1}$.

II. $s = \dfrac{r^n - 1}{r - 1} \times a$, when the ratio is greater than 1.

III. $s = \dfrac{1 - r^n}{1 - r} \times a$, when the ratio is less than 1.

441. In formula III., r^n becomes smaller as n becomes larger; and when n is too large to be counted, r^n is too small to be considered. The formula, therefore, becomes:

$$s = \frac{a}{1 - r}.$$ Hence,

The sum of a decreasing series, with an unlimited number of terms, is the quotient obtained by dividing the first term by the difference between 1 *and the ratio.*

(1) Find the sum of the infinite series $1 + \frac{1}{3} + \frac{1}{9} + \cdots$

$$s = \frac{1}{1 - \frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}.$$

(2) Find the sum of the infinite series $.1 + .01 + .001 + \cdots$ $= .111 \ldots$

$$s = \frac{.1}{1 - .1} = \frac{.1}{.9} = \frac{1}{9}.$$

Exercise XC.

Find the sum of the infinite series:

1. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$
2. $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots$
3. $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$
4. $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \cdots$
5. .171717.
6. .212121.....
7. .9999.....
8. .232323.....
9. .36848484.....
10. .15272727.

COMPOUND INTEREST

442. If P represent the principal ... the number of years ... the amount ...

$1 + r$ will represent the amount ...
$(1 + r)^2$ will represent the amount ...
$(1 + r)^3$ will represent the amount ...
$(1 + r)^n$ will represent the amount ...
$P \times (1 + r)^n$ will represent the amount ...

Therefore, $A = P \ldots$

$\log A = \log P \ldots$

By means of this formula ... est can be solved.

A deposits $15 in a sav... the end of 10 year... compounded annua...

In the formula ...

That is, the amount ...

In how many years ... at 4% ...

Here ...

Hence ...

Therefore ...

Exercise XCI.

1. A deposits $60 in a savings bank, and draws it out at the end of 8 yrs., with 4% compound interest. What does he receive?
2. What will $100 amount to in 7 yrs., interest at 8% per annum, compounded semi-annually?

Note. Interest for 7 yrs. at 8% per annum, compounded semi-annually, is the same as interest for 14 yrs. at 4%, compounded annually.

3. In how many years will a sum of money double itself at 6%, compounded annually?
4. In how many years will a sum of money treble itself at 6%, compounded annually?
5. In how many years will $87 amount to $99 at 3%, compounded annually?
6. In how many years will $100 amount to $175 at 4%, compounded annually?
7. At what rate per cent will a sum of money double itself in 12 yrs., compound interest?

Note. Consider $1 + r$ the quantity sought, and its logarithm will be $\frac{1}{12}$ of log 2.

8. At what rate will a sum of money treble itself in 15 yrs. at compound interest?
9. At what rate will $80 at compound interest amount to $110 in 8 yrs.?
10. What sum must be invested at 5%, compound interest to amount to $1200 in 7 yrs.?
11. What sum must be invested at 4%, compound interest, to amount to $2000 in 10 yrs.? To amount to $5000 in 8 yrs.?
12. If A puts $100 a year into a savings bank that pays 4% per annum, compound interest, what will he have in the bank at the end of 10 years?

13. What will be the amount in the last problem if the bank pays $4\frac{1}{2}\%$ per annum?

14. What should be paid to-day for an annuity of $500 a year, for 12 years, if money is worth $3\frac{1}{2}\%$, compound interest?

NOTE. First find the sum of the payments and interest at the end of the 12 yrs., and then the present worth of that sum.

15. What should be paid to-day for an annuity of $300 a year, for 10 years, if money is worth 4%, compound interest?

16. What should be paid to-day for the assurance that 5 yrs. hence I shall begin to receive $500 a year, for 8 yrs., if money is worth $4\frac{1}{2}\%$, compound interest?

NOTE. Find what should be paid, if paid all at once, 5 yrs. hence; then find the present worth of that sum.

17. If interest is reckoned at 6%, what sum of money must be paid annually, beginning a year hence, to clear off a debt of $10,000 in 5 equal payments?

18. If interest is reckoned at 6%, what is the amount of each of 12 equal semi-annual payments, the first to be paid 6 mos. hence, required to clear off a debt of $24,000?

CHAPTER XXV.

MISCELLANEOUS PROBLEMS.

1. Make six different numbers with the digits 1, 2, 3, and find their sum.

2. Make six different numbers with the digits 2, 3, 5, and find, by logarithms, their continued product.

3. Make six different numbers with the digits 8, 7, 3, and find, by logarithms, their continued product.

4. Find, by logarithms, the missing term in each of the following proportions:

(i.) 7.13 : 3.57 : : 4.18 : ? (iii.) 7.37 : ? : : 86.1 : 43.7.

(ii.) 5.89 : 76.3 : : ? : 38.7. (iv.) ? : 69.7 : : 3.79 : 29.4.

5. Find, by logarithms, the values of $.08^{1/\text{[illegible]}}$; $2734^{1/\text{[illegible]}}$; $21.97^{1/\text{[illegible]}}$. $7^{3.6}$.

6. Find, by logarithms, the values of $9.71^{7/\text{[illegible]}}$; $7.935^{\text{[illegible]}}$.

NOTE. In solving the following problems use logarithms whenever they can be used with advantage.

7. What is the horizontal distance between two points, when the air-line distance is 1534 ft., and the difference of level 34 ft.?

8. Find the horizontal distance when the road distance is 1 mile, and the rise 347 ft.

9. If the road distance is half a mile, and the horizontal distance 2513 ft., find the difference of level.

10. The diagonal of a rectangular floor is 34.6 ft., and the width is 17.8 ft. Find the length of the floor.

11. The height of a tower on a river's bank is 55 ft., the length of a line from the top to the opposite bank is 78 ft. Find the breadth of the river.

12. The number of seamen at Portsmouth is 800, at Charlestown 404, and at Brooklyn 756. A ship is commissioned whose complement is 490 seamen. Determine the number to be drafted from each place in order to obtain a proportionate number from each.

13. Show, without division, that 36,432 contains 8, 9, 11 as factors.

14. Find the smallest multiplier that will make 47,250 a perfect cube.

15. Find the proper fraction which, when reduced to a continued fraction, has for quotients 1, 3, 5, 7, 2, 4.

16. If the meter is equal to 1.09362 yds. find a series of four fractions that will express more and more nearly the true ratio of the meter to the yard.

17. Find the square factors contained in 33,075.

18. The top of St. Peter's, Rome, is $\frac{9}{110}$ of a mile above the ground, and that of St. Paul's, London, is $\frac{17}{264}$ of a mile. By how many feet does the height of St. Peter's exceed that of St. Paul's?

19. How many days elapsed between the annular eclipse of May 15, 1836, and that of March 15, 1858?

20. In a gale, a flag-staff 60 ft. high snaps 28.8 ft. from the bottom; and, not being wholly broken off, the top touches the ground. If the ground is level, how far is the top from the bottom?

21. Seventeen trees are standing in a line, 20 yds. apart from each other; a person walks from the first to the second and back, then to the third and back, and so on to the end. How far does he walk?

22. A level reach in a canal is $14\frac{3}{4}$ mi. long and 48 ft. broad. At one end is a lock 80 ft. long, 12 ft. broad, and with a fall of 8 ft. 6 in. How many barges can pass through the lock before the water in the canal is lowered 1 in.?

23. Find the capacity, in liters and in bushels, of a box 1.7^{m} long, 87^{cm} wide, and 31^{cm} deep.

24. Find the number of kilograms of olive oil, specific gravity .915, to fill a vessel 2.3^{m} long, 1.8^{m} wide, and 74^{cm} deep.

25. How many tons in a block of marble 4 ft. long, 34 in. wide, 17.3 in. thick, if its specific gravity is 2.73?

26. Find the surface of a sphere 18.3 in. in diameter.

NOTE. The area of a circle is 3.1416 times the square of the radius; and the surface of a sphere is 4 times the area of a circle of the same radius as the sphere.

27. Find the number of acres in a circular field 213 yds. 2 ft. across.

28. How many cubic inches in a 10-inch globe? in a 20-inch globe? What is the ratio of their volumes?

29. How many balls 3 in. in diameter can be cast from a pig of iron 7 ft. long, 6.7 in. wide, 3.8 in. thick, if the waste in melting and casting is reckoned at $3\frac{1}{4}\%$?

30. Find the difference in length, at 80° F., of a glass and a steel rod, each 3 ft. long at freezing point, if the expansion at 100° C. is .00085 for glass and .0012 for steel.

31. A grain of gold is beaten out in leaf to cover 56 sq. in. What weight will be required for gilding the faces of a cube whose edge is $3\frac{1}{2}$ ft.?

32. What premium must be paid, at the rate of $1\frac{1}{4}\%$, for insuring a vessel worth $117,750, in order that in the event of loss the owner may receive both the value of the ship and the premium?

33. By selling goods at 60 cts. a pound, 8% on the cost is lost; what advance must be made in the price in order to gain 15% on the cost?

34. Divide $$27.12\frac{1}{2}$ among three persons, giving the second $5 less than the first, and twice as much as the third.

35. The population of a city in 1880 was 12,298, showing a decrease of $8\frac{1}{3}\%$ on its population in 1870; in 1870 there was an increase of $7\frac{1}{2}\%$ on the census of 1860. What was its population in 1860?

36. Find the increase of income obtained by transferring $2500 from 3% stocks at $94\frac{1}{2}$ to 4% stocks at 105.

37. Each person breathing in a closed room spoils the air at the rate of about 8 cu. ft. a minute. A congregation of 400 persons enter a closed room 70 ft. by 40 ft. and 20 ft. high. How long will it take them to spoil the air?

38. How long can the windows and doors of a school-room be safely kept closed when occupied by 50 children, if the room is 25 ft. by 20 ft. and 10 ft. high?

39. Find the square root, to four decimal places, of the reciprocal of .0043.

40. A pays B $230 as the present value of $300 due in 5 yrs. Which gains by the payment, and how much, if interest is reckoned at 5%?

41. Find the quantity of coal required by a steamer for a voyage of 4043 mi., if her rate per hour is 14.04 knots, and her consumption of coal 87 t. per day.

Reckon 2240 lbs. to the ton, and a knot 6086 ft.

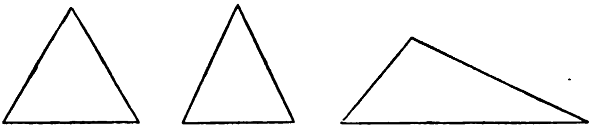
42. Find the area of a circular ring of which the inner and outer diameters are 7.36 and 10.64 in.

43. A and B can do a piece of work in $13\frac{1}{3}$ dys., A and C in $10\frac{2}{7}$ dys., A, B, and C in $7\frac{1}{2}$ dys. In how many days can A do it alone?

44. If 3 men working 11 hrs. a day can reap 20 A. in 11 dys., how many men working 12 hrs. a day can reap a field 360 yds. long and 320 yds. broad in 4 dys.?

45. Find the area of a triangle whose sides are 12, 5, and 13 in.

443. When the three sides of a triangle are known, the area is found as follows:



Subtract each side separately from half the sum of the three sides.

Find the continued product of the half-sum and the three remainders, and extract the square root of that product.

Ex. Find the area of a triangle whose sides are 3, 4, and 5 ft. respectively.

The half-sum of the sides is $\frac{3+4+5}{2} = 6$ ft., and the three remainders are 3 ft., 2 ft., 1 ft.

The area is $\sqrt{6 \times 3 \times 2 \times 1}$ sq. ft. $= 6$ sq. ft.

46. Find the area of a triangle whose sides are 73, 57, and 48 ft.

47. Find the number of hektars in a triangular field whose sides are 37.5^m, 91.7^m, and 78.9^m.

48. Find the number of hektars in a triangular field whose sides are 67.5^m, 81.2^m, and 102.7^m.

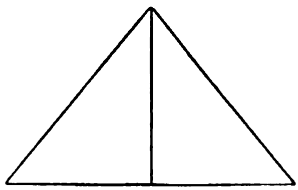
49. Find the number of acres in a triangular field whose sides are 227, 342, and 416 ft.

50. Find the number of acres in a triangular field whose sides are 79 chains 8 links, 57 chains 3 links, and 102 chains 19 links.

51. Find the number of square rods in a triangle whose sides are 7 rds. 2 yds., 6 rds. 5 yds., and 9 rds. $4\frac{1}{2}$ ft.

52. Find the number of acres in a four-sided field, the sides of which are in order 361, 561, 443, and 357 ft.; and the distance from the beginning of the first side to the end of the second side is 682 ft.

444. When one side of a triangle is regarded as a base, the triangle may be imagined as resting with that base on a horizontal line. The distance of the highest point of the triangle above that line is called the *altitude* of the triangle.



When the altitude and base of a triangle are known, the area is found as follows:

Multiply the altitude by the base, and take one-half the product.

Ex. Find the area of a triangle of which the base is 3 ft. and altitude 4 ft.

$$\left(\frac{3 \times 4}{2}\right) \text{sq. ft.} = 6 \text{ sq. ft.}$$

53. Find the number of hektars in a field of three sides, one of which is 82.1^{m}, and the distance from this side to the opposite corner 47.3^{m}.

54. Find the number of acres in a triangular lot, one side of which is 343.6 ft., and the distance from this side to the opposite corner is 163.2 ft.

When the three sides of a triangle are known, and the altitude is required:

Find the area of the triangle, and divide the result by half the side that is taken as the base.

55. Find the altitude of a triangle, if each side is 1000 ft.

56. Find the distances of the vertices from the opposite sides of a triangle, when these sides are 17.8^{mm}, 23.6^{mm}, and 31.5^{mm}.

57. If the four sides of a field measured in succession are 237, 253, 244, and 261 ft., and the diagonal measured from the end of the first side to the end of the third side is 351 ft.; find its area.

58. If the four sides of a field are 237, 253, 244, and 261 ft., taken in order, and if the corner formed by the second and third sides is a square corner; find the diagonal from the beginning of the second side to the end of the third side, and also find the area of the field.

59. Find the area of a circle that has a radius of 10 in.; of a circle that has a diameter of 10 ft.; of a circle that has a circumference of 30 in.

60. A horse is tied by a rope 27.8^{m} long; what part of a hektar can he graze?

61. How many square feet in a circle that has a diameter of $17\frac{2}{3}$ yds.?

62. How many square feet in a circle that has a circumference of 117 yds.?

63. How many square inches in the surface of a globe that has a radius of 12.37 in.?

64. Find the area of the surface of the largest globe that can be turned out from a joist 4 in. by 6 in.

65. How many cubic inches in a globe that has a diameter of 10 in.?

66. If a tree be round, and the girt is 17 ft. 6 in., find its diameter. Find the area of a cross-section, and find the number of cubic feet in the largest sphere that can be cut from it.

67. Find the weight in kilograms and in pounds of an iron ball 21.5^{cm} in diameter, specific gravity 7.47; of a tin ball 13^{cm} in diameter, specific gravity 7.29; of a lead ball 17.3^{cm} in diameter, specific gravity 11.35; of a silver ball 1.31^{cm} in diameter, specific gravity 10.47.

68. A slab of cast-iron 4 ft. $2\frac{1}{2}$ in. long, 17 in. wide, and $8\frac{1}{3}$ in. thick, specific gravity 7.31, is cast into 2-lb. balls. If there is a loss of 5% in melting, how many balls are obtained, and what is the diameter of each?

69. How many pounds avoirdupois would a ball of such iron 30 in. in diameter weigh?

70. If the specific gravity of ice is .921, find the weight and the surface of each of three spheres of ice whose diameters are 1^{cm}, 10^{cm}, and 1^{m}. Which of these spheres would roll first on a plain, in a gradually-increasing wind?

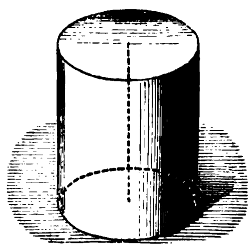
445. A straight, round stick, cut off square at each end, is called a **cylinder.**

The area of the convex surface of a cylinder is obtained as follows:

Multiply the circumference of one end by the length of the cylinder.

The *volume* of a cylinder is obtained as follows:

Multiply the area of one end by the length of the cylinder.



Cylinder.

71. Given a cylinder 10 in. in diameter and 12 in. long; required the area of each end, the convex area, the total area, and the contents in gallons.

72. Find the capacity in gallons of a round cistern 13 ft. in diameter and 9 ft. deep.

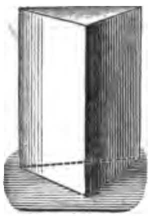
73. What must be the diameter of a cylinder 10 in. deep, in order that it may hold 1 gallon?

74. Find the volume of a cylinder 8 in. in diameter and 11 in. high.

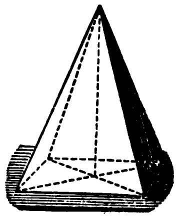
75. Find the dimensions of three cylinders that have the diameters equal to the heights, and hold 1 gal., 1 qt., and 1^{l} respectively.

446. A solid with two equal polygonal ends, connected by plane faces at right angles to the ends, is called a **prism.** The volume of a prism is found as follows:

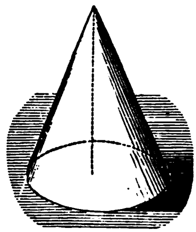
Multiply the area of one end by the length of the prism.



Prism.



Pyramid.



Cone.

76. Find the volume of a triangular prism 11 in. long, the sides of the ends being 2, 3, and 4 in. long.
77. Find the capacity in bushels of a bin 6 ft. long, and the end of which is a square measuring 3 ft. 3 in. on a side.
78. Find the number of cubic yards in a square prism 200 ft. on a side, and 40 ft. long.

447. A solid with a polygonal base, and plane faces meeting in a point, is a **pyramid.** The volume of a pyramid is one-third of that of a prism of the same base and height.

79. How many cubic yards in a square pyramid 210 ft. on a side, and 123 ft. high?
80. Find the capacity of a cup, the mouth of which is a square 4 in. on a side, and the sides of which are four equilateral triangles.
81. The largest of the Egyptian pyramids is 147^{m} high, with a base 231^{m} square. Find its volume in cubic meters.

448. A body whose base is a circle, and whose convex surface tapers uniformly to a point, is called a **cone.**

The volume of a cone is one-third the volume of a cylinder of the same base and height.

82. The slant depth of a conically-shaped drinking-cup is 93^{mm}, and the diameter at the top 8^{cm}. What is its capacity?

83. The volume of a cone is 1^{cbm}; its height is equal to the radius of its base. Find the dimensions of the cone.

449. The capacity of a round vessel, that is not hemispherical, cylindrical, or conical, may be estimated as follows:

Add one-fourth of the square of the diameter to one-third of the square of the depth, and multiply the result by eleven-sevenths of the depth.

84. Find the capacity of a wash-bowl 30^{cm} in diameter and 5^{cm} deep.

85. Find the capacity in liters of a boiler 89^{cm} in diameter and 31^{cm} deep.

86. Find the capacity in quarts of a bowl 10 in. in diameter and 4 in. deep.

87. Find the capacity in pints of a saucer 6 in. across and $1\frac{1}{2}$ in. deep; of a bowl 7 in. across and 3 in. deep; of a bowl 8 in. across and $3\frac{1}{2}$ in. deep.

88. How many gallons will a boiler 5 ft. in diameter and 2 ft. deep hold?

89. How many gallons will a boiler 30 in. in diameter and 1 ft. deep hold?

90. Find the capacity in pints of a cylinder 1.9375 in. in diameter, 2.4375 in. high; of a cylinder $3\frac{1}{8}$ in. in diameter, $3\frac{5}{8}$ in. high; of a cylinder $3\frac{13}{16}$ in. in diameter, $5\frac{1}{16}$ in. high.

91. Find the capacity in pecks of a cylinder 15.865 in. in diameter, 12.5 in. high; of a cylinder 9.25 in. in diameter, 4.25 in. deep; of a cylinder 18.5 in. in diameter, 8 in. deep.

92. What must be the diameter of a circle, in order that it may contain 78.54 sq. ft.? to contain 314.16 sq. ft.?

93. What must be the diameter of a circle to contain 1 A.? to contain 9 A.?

94. What must be the diameter of a circle to contain 1^{ha}? to contain 25^{ha}?

95. Find the number that exceeds its square root by 20.

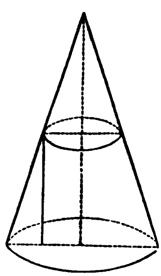
96. How much water will a hemispherical bowl hold that is 10 in. in diameter?

97. What will it cost to gild a hemispherical dome 10 ft. in diameter, at 50 cents a square foot?

98. If the moon is a sphere 2170 miles in diameter, about how many million bushels would she hold if hollow? and how many yards of cloth a yard wide would it take to cover her?

99. If the earth is 7920 miles in diameter, and the air is 40 miles deep, how many cubic miles of air are there about the planet?

100. What is the difference between 2 feet square and 2 square feet? between a foot square and a square foot? between half a foot square and 6 in. square?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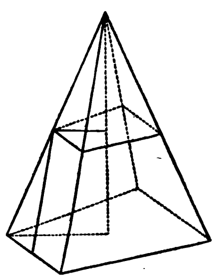

Pyramid

450. When the top of a cone or pyramid is cut off parallel to the base, the volume of the remaining frustum may be found as follows:

Find the volume of the whole, and also of the part cut off. The difference between the two volumes is the volume of the frustum; or,

Multiply the area of the base of the frustum by that of the top; extract the square root of the product; to the result add the areas of the base and top, and multiply one-third this sum by the height of the frustum.

101. Find the volume of a square frustum of which the base is 3 ft. square, top 2 ft. square, and height 4 ft.

102. Find the capacity in liquid quarts of a tin pan 10 in. in diameter at top, 8 in. in diameter at bottom, and 4 in. deep.

103. How many hektoliters will a circular vat hold 5^{m} in diameter at the top, 4.57^{m} at the bottom, and 1.17^{m} deep?

451. The oval made by the shadow of a circular plate is called an **ellipse.**

The area of an ellipse is .7854 *of the product of its longest and shortest diameters.*

104. Find the area of an ellipse 8 in. by 11 in.; of an ellipse 15 in. by 21 in.

105. The ends of a cord 100 ft. long are fastened to stakes placed 80 ft. apart on level ground. A ring, to which a kid is tied, plays freely on the cord. How far from the straight line joining the stakes can the ring be pulled? What are the diameters of the ellipse which the kid can graze? How many square feet in the ellipse?

106. Using the same rope as in the last problem, but putting the stakes 25 ft. apart, how many per cent is the kid's pasturage increased?

107. A cylindrical log, 11 in. in diameter, is sawed off on such a slant that the pieces are 8 in. longer on the longest than on the shortest side. Find the dimensions of the ellipse thus made, and its area.

452. *The number of vibrations that pendulums make in a given time is inversely as the square root of their lengths.*

A pendulum passing its central point of rest once every mean solar second is 39.138 in. long.

108. Find the length of a pendulum beating half-seconds; of a pendulum beating quarter-seconds.

109. How many centimeters long is a pendulum swinging 80 times a minute? a pendulum swinging 30 times a minute?

110. If a cannon-ball be suspended by a fine wire 176 ft. long in the central well of the Bunker Hill Monument, how many times a minute will it swing?

453. *If a plunger fits tightly in a small cylinder, and by it water is forced into a large cylinder, the plunger in the large cylinder is lifted with a force nearly equal to the product of the force with which the little plunger is driven in multiplied by the square of the ratio of the diameters of the two cylinders.*

111. Find the lifting-power of a hydraulic press, the plunger being 1^{cm} in diameter and driven with a force of 100^{kg}, if the lifting-piston is 1^{m} in diameter.

112. If the plunger is ½ in. in diameter, and is driven with a force of 1000 lbs., how much can it lift with a lifting-piston 4 ft. in diameter?

113. If the plunger is 2 in. in diameter, and is driven with a force of 1000 lbs., how much can it lift with a lifting-piston 2 ft. in diameter?

114. The water stands in a fissure in a rock 10^m high and 12^m long. What pressure is exerted to split the rock on the lowest meter's width? on the highest meter's width? in the whole fissure?

NOTE. This pressure is found by multiplying the surface upon one side by the height of water above the centre-line and counting the product as volume of water, and then finding the weight of this volume of water. The principle is precisely the same as in the hydraulic press.

115. A dam is 100 ft. long and 10 ft. deep, and the water is just flowing over it. What pressure is exerted over the lowest two feet of the dam?

454. A body falling in a vacuum falls 4.903^m in the first second; it then has acquired a velocity of 9.806^m.

A falling body increases its velocity in proportion to the time it is falling; and the distance fallen is in proportion to the square of the number of seconds of time it is falling.

Thus, a body falling from a state of rest in a vacuum will in half a second have fallen 1.2257^m, and have acquired a velocity of 4.903^m; in 3 sec. it will have fallen 44.127^m, and have acquired a velocity of 29.418^m per second.

The velocity of heavy bodies falling short distances in air will not be much less.

116. What velocity in meters a second will a cannon-ball acquire in falling three-quarters of a second? in falling three and a quarter seconds?

117. How long will it take a leaden ball, rolling off a table 29 in. high, to reach the floor?

118. What velocity will a crowbar attain in falling endwise from a balloon 2000^m high? How long will it be in coming down?

119. What velocity will a crowbar attain in falling endwise from a balloon one mile and a quarter high? How long will it be coming down?

120. If Carisbrook Well is 210 ft. deep, how long after a pebble is dropped will it be before it is heard to strike the bottom, if its velocity is reckoned at 32 ft. at the end of a second, and the velocity of sound is 1120 ft. a second?

121. On the same suppositions as in Ex. 120, how long after a pebble is dropped will it be heard to strike the bottom of a ventilating shaft 1600 ft. deep?

122. If a pebble is dropped over a precipice, and is heard to strike the bottom in $7\frac{1}{2}$ sec., how far has it fallen, on the same suppositions as in Ex. 120?

123. A pebble dropped down a shaft is heard to strike the bottom in 3 sec. after it begins to fall. Find the depth of the shaft.

124. How long will it take a ball, rolling off a table, to drop 1^{cm}? 1 in.? 10^{cm}? 6 in.?

455. *The velocity with which water will flow out of a hole in the side of a reservoir is nearly proportional to the square root of the depth of the hole below the surface of the water; and is about* 32 *ft. a second at the depth of* 16 *ft.*

125. With what velocity will water flow through a hole 9 ft. below the surface?

126. With what velocity will water leave a fountain having free play, and a head of 25 ft.? a head of 100 ft.?

127. If a hole in the side of a cistern 4 ft. below the surface of the water is delivering 10 gals. an hour, how many gallons would it deliver with 5 ft. more head?

128. If a pipe 2 in. in diameter, and 1 ft. long, inserted in a dam, the head of water being kept constant, delivers 4 gals. a minute, how many gallons a minute may be expected when another pipe of the same length, but $2\frac{1}{2}$ in. in diameter, is substituted for the two-inch pipe?

129. If a one-inch pipe, 20 in. long, is substituted for the two-inch pipe, 1 ft. long, in Ex. 128, and the flow is found to be 5 pts. a minute, what part of the diminution is due to the smaller area of the orifice, and what part to the increased friction on the sides of the longer pipe?

456. *The quantity of water issuing from a hole is in proportion to the square root of the head; and the velocity is in proportion to the square root of the head.*

The work which the water can do is in proportion to the quantity multiplied by the square of the velocity; that is,

The work is in proportion to the square root of the cube of the head.

130. A miller is using water flowing through the gate-way under 4 ft. head. How much more work could he do if the head was raised to 9 ft.? how much more if the head was raised to 25 ft.?

457. *When a body is moving in a circle, the centrifugal force is about* 1.227 *of the continued product of the weight of the body, the number of feet in the radius of the circle, and the square of the number of revolutions in a second.*

Thus, a body going round a circle of 5 ft. radius once a minute, presses away from the centre with a force equal to $1.227 \times 5 \times \frac{1}{60^2}$ of the weight of the body.

NOTE. When the radius is measured in meters, the multiplier 4.025 must be used in place of 1.227.

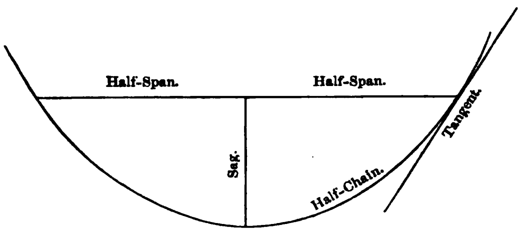
131. If a top 3 in. in diameter is making 200 revolutions a second, with what force does the outer layer pull away from the centre?

132. If a sling 30 in. long contains a stone, and is whirled round 80 times a minute, what is the force pulling on the string?

133. With what force does a locomotive running at 30 mi. an hour, on a curve of 800 ft. radius, bear against the outer rail?

134. If washed wool is put wet into a wire basket 1.2^{m} in diameter, and the basket is set to spinning at the rate of 180 revolutions a minute, with what force is water wrung out of the wool?

135. If steel pens are revolved in a basket 32^{cm} in diameter, 17 revolutions a second, with what force is the oil drained from them?



When a chain of uniform thickness hangs from two points not in the same vertical line, it hangs in a curve called a **common catenary.**

The length of the chain from the lowest point to any point selected may be called **half-chain.** The height of the point selected above the lowest point is called **sag.** The horizontal distance of the point selected from the lowest point is called **half-span.** The horizontal force with which the point selected is drawn inward is called **tension.** The radius of the circle which will fit the curve at its lowest point is called **radius.** The straight line which touches the curve at the point selected is called **tangent.**

The following propositions have been proved by higher mathematics:

I. Tension = weight of a piece of the same chain as long as the radius.

II. Radius = sum of half-chain and sag multiplied by difference of half-chain and sag, and divided by twice sag.

III. The log of half-span = log of sum of half-chain and sag plus log of their difference, plus log of the difference of these two logarithms plus colog of sag plus 0.0612.

IV. Radius divided by half-chain measures the "batter" at the point selected; that is, measures the horizontal falling back for every unit of vertical ascent in a straight line tangent at that point.

136. How strong a horizontal pull on a chain, weighing half a pound to the yard, is required to make the lowest part curve with an 18-in. radius? with a 6-ft. radius?

137. A $\frac{3}{4}$-in. rope, weighing $\frac{1}{4}$ of a pound to the yard, is fastened at one end to a staple, and near the other end, on the same level, runs over a pulley, and has a 25-lb. weight hung to it. What is the radius of its curvature at the middle?

138. A shower wets the rope of Ex. 137, and increases its weight 40%; what does its radius now become?

139. A steam-tug, in attempting to move a ship, straightened her hawser until the radius of the lowest point was 1980 ft. The rope was wet, and weighed $3\frac{1}{4}$ lbs. to the yard. With what force was it stretched?

140. A chain 31 ft. long hangs between points on a level, and sags 4 ft.? What is the radius at the lowest point?

141. The whole chain, in Ex. 140, weighs 18 lbs. What is the horizontal tension? What is the distance of the points apart? What is the slant, or batter, of the end of the chain?

142. A chain weighing 1^{kg} to the meter is suspended from points on a level; the length of chain is 31^{m}, and it sags 1.3^{m}. Find all the conditions, and find how much it falls below a level at 10^{cm} from each end.

143. A chain 100^{m} long, weighing 14 oz. to the foot, is suspended from points on a level 80^{m} apart. What is the sag, the batter at the ends, and the horizontal tension?

163. The diameters of a lamp-shade are 25^{cm} and 7^{cm}; its slant height is 134^{mm}. Give its curved surface in square centimeters.

164. A niche is formed like a half-cylinder surmounted by a quarter of a sphere. The height of the cylinder is 1.2^{m}, the diameter $.8^{m}$. Find the volume of the niche, and the area of its interior surface.

165. A steam-boiler is formed of a cylinder terminated at each end by a hemispherical cap of the same diameter. The length of the cylinder is 3.4^{m}, interior diameter $.8^{m}$. Find the number of hektoliters of water required to fill the boiler half full.

458. In the average state of the atmosphere, the distance at which an object is visible at sea is found by the following formula:

The square of the distance in English miles is seven-fourths the height of the object in English feet; or,

$$\log \text{miles} = 0.1215 + \tfrac{1}{2} \log \text{feet},$$
$$\log \text{feet} = 2 \log \text{miles} - 0.2431.$$

166. A hill 482 ft. high is 8 mi. from the shore. How many miles out at sea is it visible?

167. A sailor at the topmast 80 ft. above the sea can just see a sailor at the topmast of a similar ship. How many miles apart are the vessels?

168. A vessel approaching Valparaiso at day-break just makes out the peak of Aconcagua, 23,000 ft. high and 140 mi. back from the coast. How far is the vessel from land if the eye of the observer is 30 ft. above the water?

169. If Mount Washington is 6240 ft. hi[illegible] 76 mi. in an air-line from Cape Elizabeth, [illegible] from the Cape will its peak be visible [illegible] state of the atmosphere?

156. The top of a wheel is at each instant moving with twice the velocity of the carriage, and is moving in a curve whose centre, at the instant, is as far below ground as the point is above ground. What, then, is the force exerted to separate the mud from the top of a wheel 3 ft. 2 in. in diameter, when the carriage is moving at the rate of 10 mi. an hour?

157. A point in the tire, as a spike-head, moves, while the wheel rolls over once, just four times the diameter of the wheel. How far does the point travel while the wheel, 3 ft. 2 in. in diameter, travels 1 mi.?

158. An oil-can is formed of two cylinders connected by a frustum of a cone. The upper cylinder, or neck, is 6^{cm} in diameter, and 75^{mm} high; the lower cylinder is 13^{cm} in diameter, and 153^{mm} high; the total length of the can is 30^{cm}. Find its capacity in liters.

159. A common tunnel is formed of a frustum of a cone terminated with a cylinder. The height of the frustum is 14^{cm}, and the diameters of the two bases are 175^{mm} and 16^{mm} respectively. The cylinder is 8^{cm} long. Find the capacity of the tunnel in liters.

160. A pan is in the form of a frustum of a cone. The interior measurement is 10^{cm} deep, 12^{cm} across the bottom, and 23^{cm} across the top. Find the capacity of the pan in liters.

161. A stove-pipe is 4^{m} long, 26^{cm} in diameter, and 1^{mm} thick. Find how many square centimeters of sheet-iron it has taken to make it, if the edges lap one centimeter; and give the weight of the pipe, if the specific gravity of the sheet-iron is 7.8.

162. A spherical bomb is 32^{cm} in diameter, and the sides 38^{mm} thick. The specific gravity of the metal of which it is made is 7.2. Find its weight and interior capacity.

163. The diameters of a lamp-shade are 25^{cm} and 7^{cm}; its slant height is 134^{mm}. Give its curved surface in square centimeters.

164. A niche is formed like a half-cylinder surmounted by a quarter of a sphere. The height of the cylinder is 1.2^{m}, the diameter $.8^{m}$. Find the volume of the niche, and the area of its interior surface.

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168. A vessel approaching Valparaiso at day-break just makes out the peak of Aconcagua, 23,000 ft. high and 140 mi. back from the coast. How far is the vessel from land if the eye of the observer is 30 ft. above the water?

169. If Mount Washington is 6240 ft. high and 76 mi. in an air-line from Cape Elizabeth, how far out from the Cape will its peak be visible in the ordinary state of the atmosphere?

170. How many acres of water can a man see, standing on a raft with his eyes just 6 ft. above water, and no land in sight?

171. How far would a mountain 30,000 ft. high be visible? one of 5000 ft. high? one of 1000 ft. high?

172. How high must a mountain be in order to be visible at sea-level 50 mi.? 100 mi.? 150 mi.?

459. When the distance is given in kilometers, and the height in meters,

The square of the distance in kilometers is 15 *times the height in meters;* or,

$$\log \text{kilometers} = 0.5880 + \tfrac{1}{2} \log \text{meters},$$
$$\log \text{meters} \quad = 2 \log \text{kilometers} - 1.1761.$$

173. How far is a mountain 1000^{m} high visible? 2000^{m} high?

174. How far can a man see from the shore, if he stands on a bluff that raises his eye 11^{m} above the sea?

175. If in steaming away from a mountainous island a sailor estimates his distance at 171^{km}, when the island disappears beneath the wave, how high shall he estimate the mountains?

460. Sound travels in still air at 57° F. 1114 ft. a second, and 1 ft. a second faster for every degree above 57°; so that at 63° it goes 1120 ft. a second; at 53°, 1110 ft. a second.

176. The flash of a gun is seen $7\frac{1}{2}$ sec. before the report of the gun is heard; there is no wind, and the thermometer is 73° F. How far off was the gun?

177. A meteor was seen to burst; the report followed in 4 min. 17 sec. What was its distance if the average temperature of the intervening air was 50° F.?

461. Sound travels in air at 21° C. 345^{m} a second, and 53^{cm} a second faster for 1° C. increase in temperature.

178. Find the distances of the last two examples by this rule.

179. How long will it take for an explosion at the equator to be heard at the antipodes of the place, if the circumference of the earth at the equator be reckoned at 40,000^{km}, and the average temperature at the equator at 23° C.?

180. If an explosion at the equator occurs at sunset and the average temperature east of the spot is 22° C., and that to the west 24° C., how far from the antipodes would the sounds meet?

181. How far off is the lightning when the thunder follows in 13 sec., the temperature being 76° F.?

182. How long would it take sound to go through a whispering-tube 3 mi. long, temperature 61° F.?

183. Sound travels in iron about 10½ times as fast as in air. How long, then, after seeing the blow of a sledge-hammer given on the other end of an iron pipe 1½ mi. long, may I expect to hear the sound by the iron; and how long after, to hear the sound through the air in the pipe; thermometer 63° F.?

184. Two gunners fire at each other simultaneously from forts 1½ mi. apart; the wind, at 70° F., blows steadily from one fort to the other, at 11 mi. an hour. How soon will each hear the report of the other's gun? Suppose one ball flies on the average 987 ft. a second, the other 818 ft. a second; when will each receive the other's shot?

185. Sound travels in water about 4.26 times as fast as in air. How many seconds sooner would the sound of a torpedo exploded under water 2 mi. off reach you by water than by air, at 68° F.?

186. Required the expense of painting the walls and ceiling of a room 22 ft. 6 in. long, 13 ft. 6 in. wide, and 10 ft. high, at 30 cents a square yard.

187. In what time would a cistern be filled by three pipes whose diameters are $\frac{1}{2}$ in., $\frac{3}{4}$ in., and 1 in., when the largest alone would fill it in 40 min.; the rates of flow being proportional to the squares of the diameters?

188. How many gallons of water are contained in a length of 50 yds. of a canal, if its width at the top is 8 yds. and at the bottom 7 yds., and its depth 5 ft.?

189. Find the weight supported by each of two men, A and B, who carry a hundred-weight suspended on a pole 6 ft. long, at a point 2 ft. 3 in. from A's end, if the pole weighs 16 lbs.

NOTE. One-half the pole is supported by each man, and the part of the weight supported by each is inversely proportional to its distance from him.

190. A man who rows 4 mi. an hour in still water takes 1 hr. 12 min. to row the same distance up a river. How long will it take him to row down again?

191. How long a ladder will be required to reach a window 40 ft. from the ground, if the distance of the foot of the ladder from the wall is one-fifth of the length of the ladder?

192. If 3 oz. of gold 15 carats fine are mixed with 7 oz. 12 carats fine, what will be the fineness of the compound? What must be the fineness of 11 oz. so that, when added to this compound, the whole may be 14 carats fine?

193. Find the surface of each face of a cube whose volume is 14 cu. ft. 705.088 cu. in.

194. What must be the length of spermaceti candles $\frac{7}{8}$ of an inch in diameter, that 6 of them may weigh a pound, if the specific gravity of spermaceti is .943?

195. A cylinder 10 in. across and 10 in. high contains .3927 of a cubic foot of water. How many shot .1 in. in diameter must be poured in to raise the water to the top?

196. Determine the depth of conical-shaped wine-glasses $2\frac{1}{2}$ in. across the top, that 60 of them may hold a gallon.

197. A train approaching a station at the rate of 25 mi. an hour passes over two signals placed half a mile apart. Find the interval between the times at which their reports are heard at the station, sound travelling at the rate of 1090.2 ft. per second.

198. The apparent intensity of light varies inversely as the square of its distance. Find the point between two lamps 50 yds. apart at which one appears twice as bright as the other.

199. How deep may a round cistern 4 ft. across be made so as to be lined with the same lead as a cubical cistern 4 ft. each way? Compare their capacities.

200. The material for lining a cubical cistern cost \$10; find the cost of material for lining two similar cisterns which shall each hold one-half as much.

201. What distance can be seen from the top of a mountain 4 mi. high.

202. If 5 excavators sink a circular shaft 8 ft. in diameter and 125 fathoms deep in 100 dys. of 10 hrs. each, how many nights of 7 hrs. each will 4 excavators be in sinking a shaft 6 ft. in diameter and 75 fathoms deep; if the difficulty of working by night is one-seventh greater than by day, and the hardness of the ground in the smaller shaft is to that in the larger shaft as 7 is to 5?

203. Find the length of a pendulum that oscillates 80 times a minute?

204. A man whose weight is 160 lbs., wishing to raise a rock, leans with his whole weight on a horizontal crowbar 5 ft. long, which is propped at the distance of 4 in. from the end in contact with the rock. Find what force he exerts on the rock, and what pressure the prop has to sustain, if the weight of the crowbar is not reckoned.

$$\frac{\text{Power}}{\text{Weight}} = \frac{\text{Distance of weight from fulcrum}}{\text{Distance of power from fulcrum}}.$$

205. A child weighing 56 lbs. is at one end of a plank 16 ft. long, and a child weighing 72 lbs. is at the other end. Find the distance of each child from the fulcrum when the plank is used for a see-saw.

206. In a pair of nut-crackers if the nut be placed at a distance of 1 in. from the hinge, and the hand presses at a distance of 8 in. from the hinge, find the pressure upon the nut for every ounce of pressure exerted by the hand.

207. A substance is weighed from both arms of a false balance, and its apparent weights are 9 lbs. and 4 lbs. Find the true weight.

208. A rope passes over a single pulley. How much force is required to raise 180 lbs. attached to one end of a rope if 1% of the force applied is required to overcome friction?

209. A cask of lamp-black weighing 160^{kg} is attached to a rope wound on an axle 19^{cm} in diameter; at one end of the axle is a wheel 175^{cm} in diameter. With what force must a man pull down on a rope passing over the wheel to raise the cask if $1\frac{3}{4}$% of the pull is required to overcome friction?

$$\frac{\text{Force}}{\text{Weight}} = \frac{\text{Diameter of axle}}{\text{Diameter of wheel}}.$$

133. With what force does a locomotive running at 30 mi. an hour, on a curve of 800 ft. radius, bear against the outer rail?

134. If washed wool is put wet into a wire basket 1.2^m in diameter, and the basket is set to spinning at the rate of 180 revolutions a minute, with what force is water wrung out of the wool?

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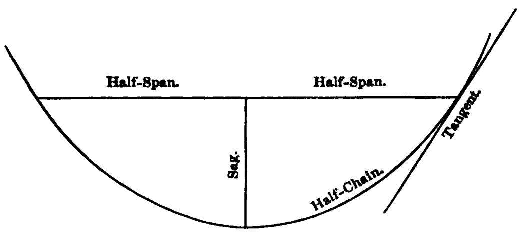

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The length of the chain from the lowest point to any point selected may be called **half-chain.** The height of the point selected above the lowest point is called **sag.** The horizontal distance of the point selected from the lowest point is called **half-span.** The horizontal force with which the point selected is drawn inward is called **tension.** The radius of the circle which will fit the curve at its lowest point is called **radius.** The straight line which touches the curve at the point selected is called **tangent.**

The following propositions have been proved by higher mathematics:

I. Tension = weight of a piece of the same chain as long as the radius.

II. Radius = sum of half-chain and sag multiplied by difference of half-chain and sag, and divided by twice sag.

215. What per cent of pure marble, $CaCO_3$, is oxygen?

216. What per cent of gypsum (plaster of Paris), $CaSO_4 + 2H_2O$, is sulphur?

217. What per cent of washing-soda, $Na_2CO_3 + 10H_2O$, is carbon?

218. In 118 lbs. of Glauber salts, $Na_2SO_4 + 10H_2O$, how many ounces of sulphur?

219. How many ounces of soda, $Na_2O + H_2O$, in 7 lbs. of borax, $Na_2B_4O_7 + 10H_2O$?

220. What per cent of pure alcohol, C_2H_6O, is carbon? What per cent of pure white marble, $CaCO_3$, is carbon?

221. What per cent of pure acetic acid (the acid of vinegar), is carbon, the formula being $C_2H_4O_2$?

222. How much acetic acid can be obtained from 12 lbs. of alcohol, C_2H_6O, if there is no waste?

223. How many grains of carbon in 1 oz. avoirdupois of oxalic acid, $C_2H_2O_4 + 2H_2O$?

224. How many milligrams of carbon in 3^g of tartaric acid, $C_4H_6O_6$?

225. How many kilograms of carbon in 95^{kg} of white sugar, $C_{12}H_{22}O_{11}$?

226. The formula of camphor is $C_{10}H_{16}O$. How many grams of carbon in 14^{kg} of camphor?

227. In 20^{kg} of oil of vitriol, H_2SO_4, how many grams of sulphur?

228. What per cent of oil of vitriol is water? what per cent sulphuric acid, SO_3?

229. In 3.5^g of black oxide of iron, FeO, how many milligrams of iron?

230. Red iron-rust consists of 70% iron and 30% oxygen. Find its formula.

231. The choking vapor of burning sulphur is equal parts sulphur and oxygen. What is its chemical formula?

144. Suppose the points of suspension in Ex. 143 to remain unchanged, and the chain to be shortened 5^m. What does the tension become?

145. How long a rope is required between points 100 ft. apart to sag 30 ft.? 20 ft.? 10 ft.?

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146. If 4 cu. in. of iron weigh 1 lb. avoirdupois, what is the weight of 1 cu. in. in grains? What is the specific gravity of the iron?

147. If 4 cu. in. of iron weigh 1 lb., what is the diameter of a 6-lb. ball? of a 32-lb. ball?

148. If a block of iron (with all its corners square) measures 17.36 in. by 8.7 in. by 1.76 in., what does it weigh at ¼ lb. to the cubic inch? and what would be the diameter if cast into a ball, if 11% is allowed for waste?

149. Answer the same questions as in the last example, for a block of which the dimensions are 71.4 in. by 8¾ in. by 3⅓ in.

150. What is the diameter of a cylinder 11 in. long that holds 2 gals.

151. What is the diameter of a cylinder 9 in. long that holds 2 gals.?

152. What is the diameter of a cylinder 30^{cm} long that holds 10^l?

153. What is the circumference of a globe if the square centimeters of its surface are three times the cubic centimeters of its volume?

154. Find the diameter of a circle of which the number of inches in its circumference is equal to the square feet of its area?

155. How many times does a carriage-wheel 3 ft. 2 in. in diameter turn in going a mile on a smooth road?

APPENDIX:

CONTAINING TABLES USEFUL FOR REFERENCE.

163. The diameters of a lamp-shade are 25^{cm} and 7^{cm}; its slant height is 134^{mm}. Give its curved surface in square centimeters.

164. A niche is formed like a half-cylinder surmounted by a quarter of a sphere. The height of the cylinder is 1.2^{m}, the diameter $.8^{m}$. Find the volume of the niche, and the area of its interior surface.

165. A steam-boiler is formed of a cylinder terminated at each end by a hemispherical cap of the same diameter. The length of the cylinder is 3.4^{m}, interior diameter $.8^{m}$. Find the number of hektoliters of water required to fill the boiler half full.

458. In the average state of the atmosphere, the distance at which an object is visible at sea is found by the following formula:

The square of the distance in English miles is seven-fourths the height of the object in English feet; or,

$$\begin{aligned} \log \text{miles} &= 0.1215 + \tfrac{1}{2} \log \text{feet}, \\ \log \text{feet} &= 2 \log \text{miles} - 0.2431. \end{aligned}$$

166. A hill 482 ft. high is 8 mi. from the shore. How many miles out at sea is it visible?

167. A sailor at the topmast 80 ft. above the sea can just see a sailor at the topmast of a similar ship. How many miles apart are the vessels?

168. A vessel approaching Valparaiso at day-break just makes out the peak of Aconcagua, 23,000 ft. high and 140 mi. back from the coast. How far is the vessel from land if the eye of the observer is 30 ft. above the water?

169. If Mount Washington is 6240 ft. high and 76 mi. in an air-line from Cape Elizabeth, how far out from the Cape will its peak be visible in the ordinary state of the atmosphere?

170. How many acres of water can a man see, standing on a raft with his eyes just 6 ft. above water, and no land in sight?

171. How far would a mountain 30,000 ft. high be visible? one of 5000 ft. high? one of 1000 ft. high?

172. How high must a mountain be in order to be visible at sea-level 50 mi.? 100 mi.? 150 mi.?

459. When the distance is given in kilometers, and the height in meters,

The square of the distance in kilometers is 15 *times the height in meters;* or,

$$\log \text{kilometers} = 0.5880 + \tfrac{1}{2} \log \text{meters},$$
$$\log \text{meters} = 2 \log \text{kilometers} - 1.1761.$$

173. How far is a mountain 1000^{m} high visible? 2000^{m} high?

174. How far can a man see from the shore, if he stands on a bluff that raises his eye 11^{m} above the sea?

175. If in steaming away from a mountainous island a sailor estimates his distance at 171^{km}, when the island disappears beneath the wave, how high shall he estimate the mountains?

460. Sound travels in still air at 57° F. 1114 ft. a second, and 1 ft. a second faster for every degree above 57°; so that at 63° it goes 1120 ft. a second; at 53°, 1110 ft. a second.

176. The flash of a gun is seen $7\frac{1}{2}$ sec. before the report of the gun is heard; there is no wind, and the thermometer is 73° F. How far off was the gun?

177. A meteor was seen to burst; the report followed in 4 min. 17 sec. What was its distance if the average temperature of the intervening air was 50° F.?

461. Sound travels in air at 21° C. 345^{m} a second, and 53^{cm} a second faster for 1° C. increase in temperature.

178. Find the distances of the last two examples by this rule.

179. How long will it take for an explosion at the equator to be heard at the antipodes of the place, if the circumference of the earth at the equator be reckoned at $40{,}000^{km}$, and the average temperature at the equator at 23° C.?

180. If an explosion at the equator occurs at sunset and the average temperature east of the spot is 22° C., and that to the west 24° C., how far from the antipodes would the sounds meet?

181. How far off is the lightning when the thunder follows in 13 sec., the temperature being 76° F.?

182. How long would it take sound to go through a whispering-tube 3 mi. long, temperature 61° F.?

183. Sound travels in iron about 10½ times as fast as in air. How long, then, after seeing the blow of a sledge-hammer given on the other end of an iron pipe 1½ mi. long, may I expect to hear the sound by the iron; and how long after, to hear the sound through the air in the pipe; thermometer 63° F.?

184. Two gunners fire at each other simultaneously from forts 1½ mi. apart; the wind, at 70° F., blows steadily from one fort to the other, at 11 mi. an hour. How soon will each hear the report of the other's gun? Suppose one ball flies on the average 987 ft. a second, the other 818 ft. a second; when will each receive the other's shot?

185. Sound travels in water about 4.26 times as fast as in air. How many seconds sooner would the sound of a torpedo exploded under water 2 mi. off reach you by water than by air, at 68° F.?

186. Required the expense of painting the walls and ceiling of a room 22 ft. 6 in. long, 13 ft. 6 in. wide, and 10 ft. high, at 30 cents a square yard.

187. In what time would a cistern be filled by three pipes whose diameters are $\frac{1}{2}$ in., $\frac{3}{4}$ in., and 1 in., when the largest alone would fill it in 40 min.; the rates of flow being proportional to the squares of the diameters?

188. How many gallons of water are contained in a length of 50 yds. of a canal, if its width at the top is 8 yds. and at the bottom 7 yds., and its depth 5 ft.?

189. Find the weight supported by each of two men, A and B, who carry a hundred-weight suspended on a pole 6 ft. long, at a point 2 ft. 3 in. from A's end, if the pole weighs 16 lbs.

NOTE. One-half the pole is supported by each man, and the part of the weight supported by each is inversely proportional to its distance from him.

190. A man who rows 4 mi. an hour in still water takes 1 hr. 12 min. to row the same distance up a river. How long will it take him to row down again?

191. How long a ladder will be required to reach a window 40 ft. from the ground, if the distance of the foot of the ladder from the wall is one-fifth of the length of the ladder?

192. If 3 oz. of gold 15 carats fine are mixed with 7 oz. 12 carats fine, what will be the fineness of the compound? What must be the fineness of 11 oz. so that, when added to this compound, the whole may be 14 carats fine?

193. Find the surface of each face of a cube whose volume is 14 cu. ft. 705.088 cu. in.

194. What must be the length of spermaceti candles $\frac{7}{8}$ of an inch in diameter, that 6 of them may weigh a pound, if the specific gravity of spermaceti is .943?